

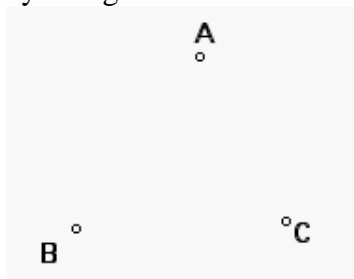



Exploring Triangle Centers

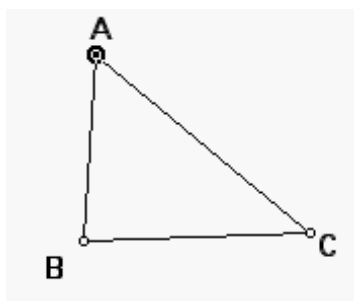
EXPLORATION I: Angle Bisectors

1. Open a new sketch and draw an arbitrary triangle ABC.

Use the Point tool  to create three points. Use the Text tool  to label the points A, B, and C. [Select the Text Tool then click on each point.]



Use the Selection Arrow  to select the three points. Choose **Segment** from the **Construct** menu. This will create triangle ABC.



2. We will be constructing the bisectors of angles A, B, and C. Obviously any two of the angle bisectors will intersect. Hypothesize where you think the third bisector will lie.
3. Construct the bisectors of angles A, B, and C to test your prediction. How does this construction compare with your previous prediction?

To construct an angle bisector :
Select the three points (point, vertex, point – in that order) that define the angle.
Choose the **Angle Bisector** command in the **Construct** menu.

To find the point of intersection of two rays:
Select both rays.
Choose **Point At Intersection** from the **Construct** menu.

4. What do you notice about the three angle bisectors? Do you think this observation will always be true?

Drag any vertex or side of your triangle. Observe what happens to the intersection of the angle bisectors. Discuss and create a conjecture that describes your findings.

5. Focus your attention on the distances from the point where the angle bisectors intersect to each of the three sides of the triangle. Make a general statement about their relationship.

6. Measure the **Distance** from the point where the angle bisectors intersect to each of the three sides along the angle bisector. To do this you will need to first find the **Point of Intersection** of each angle bisector and the corresponding opposite side.

To Measure a Distance:

Select the point at the bisector intersection and point of intersection of a bisector ray with its opposite side.

Press Measure in the main menu.

Choose Distance from the menu.

Repeat for all three sides

Manipulate your triangle by dragging any vertex or side. How are these distances related?

7. Explain why, based on the mathematical relationships you have observed, the point at which the angle bisectors intersect is called the *incenter*.

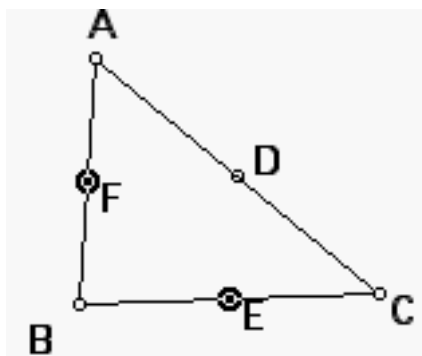
EXPLORATION II: Medians

Construction Instructions:

Step 1: In a new Sketch, Construct a new triangle. Label it ABC.


Step 2: Find the midpoints of sides AB, BC, and AC. Label them D, E, and F, respectively.

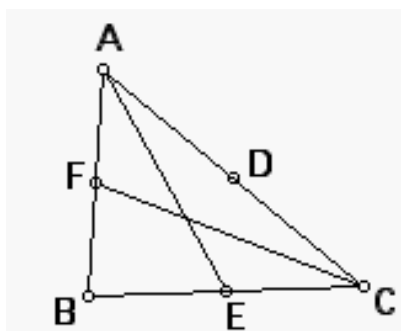
To create midpoints on the sides, click one segment with the Selection Arrow, then click on the other two segments. Choose **Point At Midpoint** from the **Construct** menu.



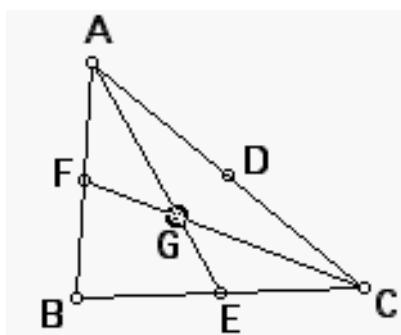
Step 3: Construct the medians of the triangle by connecting each of the triangle's vertex to the midpoint of the side opposite the vertex.

Use the Segment tool to construct just two of the triangle's three medians.

Use the Selection Arrow  to select vertex A and the midpoint of the opposite side (E). Choose **Segment** from the **Construct** menu. Repeat for vertex C and midpoint F.



Select the two medians and choose Point at Intersection from the Construct menu. Then Construct the third median with points B and D.



Investigate:

1. Drag the vertices or sides of triangle ABC. Make sure you drag it into a variety of sizes and shapes. What do you notice about the medians and the point of intersection?
NOTE: The point of intersection of the three medians is called the *centroid*.

2. Investigate the relationship between the areas of the interior triangles formed by the three medians of the triangle. State a conjecture about this relationship. **(You will need to construct the polygon interior of the six smaller triangles and measure each area)**

To construct the polygon interior of a closed figure:

- Select the vertices of the closed figure, in order.
- Choose from the Construct menu, the Polygon Interior command.
- Click anywhere inside the sketch window to de-select the polygon interior.

To measure the area of a polygon:

- Select the polygon interior.
- Choose from the Measure menu, the Area command.

3. Measure the distances from the vertex to the centroid and from the centroid to the side of the triangle. Manipulate your triangle, and make a conjecture about this relationship.

4. What is the relationship between the distance from the vertex to the centroid and the distance from the centroid to the side of the triangle? Use the Calculator tool to compare measurements to help establish this relationship.