Performance Evaluation of an OBS Network as a IPP/M/W/W Network

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Abstract
We develop an analytical method for calculating the burst loss probabilities in a tandem Optical Burst Switched (OBS) network with a bursty arrival process, depicted by an Interrupted Poison Process (IPP). The OBS network is modelled as a tandem network of loss nodes which is analyzed using single-node decomposition, whereby each node is studied in isolation as an IPP/M/W/W. For this, we need the departure process from an IPP/M/W/W which we obtain using binomial moment techniques. Performance evaluation of an OBS network shows that our method gives results which are closer to simulation results than a Poisson arrival process.

Keywords. Optical Burst Switched network, performance evaluation, Interrupted Poison Process, loss nodes.

1 Introduction
Most of the wavelength division multiplexed (WDM) optical networks currently operate over point-to-point links, where optical-to-electrical-to-optical conversion is required at each network node. Future WDM designs envision a hybrid optical network where the data plane remains end-to-end in the optical domain while the control and management planes are in the optical domain during transmission and in the electrical domain for processing. The switching architectures of these future optical networks can be classified into circuit-switched (also known as wavelength-routed), packet switched and burst switched. In a circuit switched network, long-term circuit connections (lightpaths), are setup between the source and destination
nodes. These networks underutilize the available resources because of their quasi-static nature. In contrast, an optical packet switched (OPS) network has high throughput and can easily adapt to traffic congestion or failure. As the name suggests, in an OPS network the user traffic is carried in optical packets along with the in-band control information. The control info is extracted and electronically processed at each node. The successful operation of an OPS network requires the availability of practical optical buffers and practical optical header processing. Unfortunately, both of these technologies are years away.

Given the current state of the optical technology, an Optical Burst Switched (OBS) network is a feasible solution, see Battestilli and Perros (2003). The name optical burst switching comes from the fact that the data is transported in variable size units, called bursts. Due to the great variability of the duration of a burst, they can be viewed as lying between OPS and circuit-switched networks. That is, when all bursts durations are very short, equal to the duration of an optical packet, then the OBS network can be seen as resembling an OPS network. On the other hand, when all burst durations are extremely long, then the OBS network can be seen as resembling a circuit switched optical network. The OBS idea combines the best features of circuit switching and packet switching, see Wei and McFarland (2000). Its dynamic nature allows for network adaptability and scalability, which makes it very suitable for the transmission of Internet traffic.

In order to decrease the end-to-end transmission delay, most OBS architectures use a one-way signaling scheme. The transmitting end-devices do not wait for a positive acknowledgment from the destination end-devices that the control packet has been successful at reserving resources at each hop along the route. Instead, they wait an offset time and then transmit the burst. Therefore a burst may be dropped if it arrives at an OBS node, where the control packet was unsuccessful at reserving a wavelength on its desired output port. In view of this, the calculation of the burst loss probability is an important measure of the performance of an OBS network.

The popular OBS protocols such as Just-Enough-Time (JET), see Qiao and Yoo (1999), Garg and Kaler (2010a) and Lee et al. (2010); Just-In-Time (JIT), see Wei and McFarland (2000), and Xu and Perros (2004); and Horizon, see Xiong et al. (2000), and Gurel and Karasan (2009), all use this one-way signaling scheme but employ different resource scheduling algorithms. The performance of these OBS protocols has been studied both through simulation and analytical models. The analytical studies mostly focus on a single output port of an OBS node and assume Poisson arrivals and full wavelength conversion, see Dolzer et al. (2001a), Yoo et al. (2000), and Vu and Zukerman (2002). Since these studies assume no buffers, the OBS output port is modeled as an M/G/W loss system, where W is the number of wavelengths, and the Erlang-B formula is used to calculate the burst loss probability. There are also studies of OBS with fiber delay lines, see Lu and Mark (2003), Yoo et al. (2000), and Lee et al. (2008), or deflection routing, see Hsu et al. (2002), Chen et al. (2003), and Pedrola et al. (2011). OBS with QoS
classes has been investigated in Dolzer et al. (2001b), Yoo et al. (2000), and Barakat and Sargent (2005), and an OBS architecture with centralized-signaling and guaranteed QoS classes is proposed in Duser and Bayvel (2002).

All of the previously cited analytical models focus on a single OBS node. These models provide a limited insight into the overall performance of an OBS network. An analytical model of an OBS network is proposed in Rosberg et al. (2003), where the OBS network is modeled by a network of loss nodes, each representing a link of W wavelengths. In Garg and Kaler (2010b) a deflection routing scheme is proposed in order to reduce the blocking probability in an OBS network. In Charbonneau and Vokkarane (2011), the performance of Reordering-Robust TCP over OBS networks is evaluated. In Choudhury et al. (2010) an analytical approach to study the impact of burst assembly algorithms on the byte loss rate of an OBS network under bursty traffic input is suggested. In Sahasrabudhe and Manjunath (2006) a two-moment approximation to calculate the burst dropping probabilities of optical burst switched (OBS) networks is proposed. In Abd El-Rahman et al. (2012) a new contention resolution technique based on control packet buffering is proposed. The authors develop a mathematical model to analyze the performance of an OBS network core node employing this technique along with the just-in-time one-way reservation protocol. In McArdle, Tafani and Barry (2011) the authors propose an approximate model for Gamma-distributed interarrival times and show that the traffic variance affects significantly the performance of related OBS network. In McArdle, Tafani, Curran, Holohan and Barry (2011) an asynchronous optical burst switch with wavelength conversion and a multi-channel feed-back fibre delay line is considered. Under renewal type burst arrivals and exponential burst lengths an analytic framework for estimating blocking probability is proposed. Oberoi et al. (2012) focuses on the two components of an OBS network: data network and a control plane network. They provide detailed analysis of the control plane and its impact on OBS performance with a view to selecting of the optimal value of parameters: such as, the average burst length and offset-time duration.

Typically the arrival traffic to a network is modeled by the Poisson process due to its mathematical tractability. However, the Poisson process is smooth and as a result it under-estimates the burst loss probability. Smooth traffic is one where the variance of the arrivals is close to the mean of the arrivals. In this paper we assume that the traffic is bursty. The term bursty traffic is not OBS specific and it does not refer to the fact that the arrival process is made of data bursts. In any network, the traffic is considered bursty if large number of arrivals are followed by long idle periods.

The focus of this paper is the end-to-end performance evaluation of an OBS network with bursty traffic, where the OBS bursts dynamically acquire and release wavelengths from link to link as they travel from their source to their destination. Our analytical method transforms a loss node with bursty arrivals into an equivalent node with Poisson arrivals. For this we model the source-destination transmission path of an OBS network by a tandem queueing
network of loss nodes. The queueing network is analysed by studying each node in isolation as an IPP/M/W/W loss node. For this, we characterised the departure process from an IPP/M/W/W by an IPP, which is offered to the next node.

This paper is organized as follows. In Section 2 we describe the queueing network under study. In Section 3 we describe two methods that we use to characterise an Interrupted Poisson Process (IPP) used to model a bursty arrival process. The first method is based on using statistics of the interarrival times and the second method is based on the infinite server description used in teletraffic theory. In Section 4, we characterize the departure process from a loss node with IPP arrivals and exponential holding times as an IPP. In Section 5 we propose an algorithm for obtaining analytically the blocking probabilities in an OBS transmission path with IPP arrivals and we present an example. In Section 6, we validate the accuracy of our analytical method. We conclude in Section 7.

2 The Queueing Network Under Study

We study an OBS network, where the nodes are built from an Optical Cross Connect (OXC) and an electronic control unit. Two adjacent network nodes are linked by a single WDM link (fiber), which has $W + 1$ transmission wavelengths. The first $W$ wavelengths are used for burst transmission while the $(W + 1)^{st}$ wavelength is used to transmit control information. Each OBS node has a full wavelength conversion capability, i.e., in the case of contention at an output port it can optically convert an optical signal from one wavelength to another. There are no fiber delay lines available at the network nodes and thus a burst is lost if it arrives at an output port where all the wavelengths are busy.

In this OBS network we analyze the performance of a specific source-destination transmission path, made of $K$ links. Therefore, we consider $(K + 1)$ network nodes connected in
tandem, as shown in Figure 1. We consider the traffic flow from left to right. The end-devices, linked to OXC 1, transmit bursts to a number of end-devices, linked to OXC \((K + 1)\). We refer to the traffic generated from the transmitting end-devices as the cross traffic. In addition, to the cross traffic we consider traffic generated by other sources in the OBS network. This traffic arrives at the intermediate links of the considered path. We refer to this traffic as the local burst traffic. The local traffic is routed to the same destination end-devices as the cross traffic.

This OBS network can be modeled as a tandem queueing network of IPP/M/W/W loss nodes, where each loss node represent one of the WDM links. We assume a bursty arrival process, which we describe in Section 3.

### 3 Characterising an Interrupted Poisson Process

For the bursty arrival process we choose a 2-state Markov Modulated Poisson Process (MMPP), which modulates between two exponentially distributed states, an ON and an OFF state. The transition rates for the ON and OFF states are respectively \(\phi\) and \(\psi\), see Figure 2. While in the ON state, the process generates Poisson arrivals with rate \(\lambda_{on}\) whereas in the OFF state there are no arrivals. This process is known as the Interrupted Poisson Process (IPP). Intuitively, an IPP is bursty because the arrivals are batched together during the ON period and no arrivals occur during the OFF period. The IPP becomes burstier when long OFF periods are followed by short ON periods with a large arrival rate \(\lambda_{on}\). An IPP is uniquely characterized through the parameters \(\phi\), \(\psi\) and \(\lambda_{on}\).

In this paper we make use of two methods of characterising an IPP. The first method is based on using the statistics of the interarrival times of a process. The second method is based on using the Infinite Server Effect (ISE) principal from teletraffic theory. Below we review these two methods.
3.1 Interarrival Time Description of an IPP

In this section we describe how an IPP can be characterised using the statistics of the interarrival time. For this, we use the squared coefficient of variation $c^2$, which is defined as the ratio of the variance to the squared mean of the interarrival time. The $c^2$ is a dimensionless number that represents the relative variation about the average. A small $c^2$ represents interarrival times that concentrate mostly around the mean. For a Poisson process the $c^2$ is equal to 1. For an IPP the $c^2$ is always greater than 1 if $\phi > 0$, see Girard (1990).

The mean, variance and $c^2$ of an IPP are (see Fischer and Meier-Hellstern (1993)):

$$\text{mean} = \frac{\phi + \psi}{\lambda_{on}\psi}, \quad (1)$$

$$\text{var} = \frac{2\lambda_{on}\phi + (\phi + \psi)^2}{(\lambda_{on}\psi)^2}, \quad (2)$$

$$c^2 = \frac{\text{var}}{\text{mean}^2} = 1 + \frac{2\lambda_{on}\phi}{(\phi + \psi)^2}. \quad (3)$$

In addition, we define the coefficient of burstiness $r$ of an IPP to be the ratio of the average ON time to the sum of the average ON and OFF time:

$$r = \frac{1/\phi}{1/\phi + 1/\psi} = \frac{\psi}{\psi + \phi}, \quad 0 < r < 1. \quad (4)$$

An IPP is more bursty if $r$ is close to 0 and it gets less bursty as $r$ approaches 1. In other words, if short, active, transmission periods are followed by long idle periods then the arrival process is considered more bursty.

Let us denote the IPP average arrival rate with $\lambda_{avg} = 1/\text{mean}$. Now given $\lambda_{avg}$, $c^2$ and $r$ we have enough information to define an unique IPP and determine its three parameters:

$$\lambda_{on} = \frac{\lambda_{avg}}{r}, \quad (5)$$

$$\phi = \frac{2\lambda_{avg}(1-r)^2}{r(c^2-1)}, \quad c^2 > 1, \quad (6)$$

$$\psi = \frac{2\lambda_{avg}(1-r)}{c^2-1}, \quad c^2 > 1. \quad (7)$$

Next, we discuss the effects of the squared coefficient of variation $c^2$ and the coefficient of burstiness $r$ on the blocking probability. We consider 16 different IPPs for a given average arrival rate $\lambda_{avg} = 180$ while varying the $c^2$ and $r$. The most bursty IPP has $r = 0.2$, $c^2 = 20$ and the smoothest one has $r = 0.8$, $c^2 = 5$. Using simulation we apply these IPPs to a single link with $W = 200$ wavelengths. The resulting blocking probabilities are shown in Figure 3. As expected, the IPPs with the highest $c^2$ have the highest blocking probability. For comparison we have also plotted the blocking probability assuming a Poisson arrival process with the same average arrival rate of 180. Under the Poisson process, the blocking probability is much lower.
than in the IPP cases. We also find that $c^2$ has greater impact on the blocking probability than $r$. The coefficient of burstiness affects the blocking probability for $c^2 > 10$. As $r$ approaches 1 and the process becomes less bursty, the blocking probability decreases. For a low value of the $c^2$ the blocking is almost constant with respect to $r$.

3.2 Infinite Server Description of an IPP

We now describe an IPP process using the Infinite Server Effect (ISE) principle from teletraffic theory. Using the ISE principle, a traffic stream is offered to an infinite-server system in order to calculate the mean number $m$ and the variance $v$ of the number of busy servers.

In our case, the traffic stream is an IPP with given parameters. It is offered to an infinite-server system with service rate $\mu$ and the corresponding $m$ and $v$ are calculated. Using the $m$ and $v$, we can estimate the burstiness of an IPP using the teletraffic notion of peakedness, see Jagerman et al. (1996), defined as:

$$Z = \frac{v}{m}. \quad (8)$$

Recall that the peakedness $Z$ depends on the service rate $\mu$ of the infinite-server system. Burstier traffic has a higher peakedness $Z$ and it experiences higher blocking probability when offered to a loss system. For a Poisson process $Z = 1$. For a smoother process, the peakedness is less than 1. In this paper, we use a burstier IPP and thus $Z > 1$. 

Figure 3: Blocking probability as a function of $r$ and $c^2$
In order to calculate the $m$ and $v$ for an IPP with given parameters $\psi$, $\phi$ and $\lambda_{on}$ we use, see Kuczura (1973) and Milne (1982), the factorial moments of the number of busy servers for an IPP/M/$\infty$ system. The $k^{th}$ factorial moment of a random variable with a probability density function $f(x)$ is defined as:

$$M_k = \sum_{x} x^{(k)} f(x),$$

where $x^{(k)} \equiv x(x-1)\ldots(x-k+1)$, $k = 1, 2, \ldots$

We are not interested in the factorial moments but the moments about the mean so that we can calculate the $m$ and $v$. In the case of the mean $m$, we have that $m = M_1$. The variance $v$ can be written as $v = \sum_x x^2 f(x) - m^2$ and thus $v = M_2 - m^2 + m$.

The first factorial moment, and thus $m$, see Girard (1990), p.90, is:

$$m = M_1 = \left( \frac{\lambda_{on}}{\mu} \right) \left( \frac{\psi}{\phi + \psi} \right).$$

(10)

The second factorial moment is:

$$M_2 = \left( \frac{\lambda_{on}}{\mu} \right)^2 \left( \frac{\psi + \mu}{\phi + \psi + \mu} \right) \left( \frac{\psi}{\phi + \psi} \right)$$

and thus

$$v = \left( \frac{\lambda_{on}}{\mu} \right)^2 \left( \frac{\psi + \mu}{\phi + \psi + \mu} \right) \left( \frac{\psi}{\phi + \psi} \right) - m^2 + m.$$  

(12)

Any traffic stream with a given $m$ and $v$ can be modeled by an IPP. In Kuczura (1973), a three-moment and a two-moment match for modeling any traffic stream, as an IPP, are presented. In this paper we use the two-moment match. An IPP has three parameters but we only set the first two moments and thus it is necessary to fix one of the parameters $\phi$, $\psi$ or $\lambda_{on}$ of the process. For more details see Girard (1990), p. 118. We set $\lambda_{on}$ to be:

$$\lambda_{on} = v + 3 \frac{v}{m} \left( \frac{v}{m} - 1 \right).$$

(13)

Then the other two IPP parameters can be obtained by:

$$\phi = \left( \frac{\lambda_{on}}{m} - 1 \right) \psi,$$

(14)

$$\psi = \frac{m}{\lambda_{on}} \left( \frac{\lambda_{on} - m}{Z - 1} - 1 \right).$$

(15)
4 The Departure Process from an IPP/M/W/W Loss Node

In this section, we characterize the departure process from an IPP/M/W/W loss node, shown in Figure 4. An IPP process, characterized by its \( m_{in} \) and \( v_{in} \), is offered to loss node 1 which has \( W \) servers with service rate \( \mu \). Using the ISE principle, the departure process from loss node 1 is then offered to the infinite server node 2, with the same service rate, in order to estimate its \( m_{out} \) and \( v_{out} \).

The problem of characterizing the departure process from an M/M/W/W loss node, i.e, Poisson arrivals and exponential service times, was studied by Rajaratnam and Takawira, see Rajaratnam and Takawira (2000, 2001). They developed a method of calculating the \( m \) and \( v \) of the departure process by using binomial moments and solving a system of equations. Later, van Doorn and Kieu, see van Doorn and Kieu (2003), were able to derive explicit formulas for \( m \) and \( v \). Our method of characterizing the departure process from an IPP/M/W/W loss node follows the Rajaratnam and Takawira.

Let us begin by defining the joint probability distribution \( p_{a,w,c} \), where:

- \( a \) is the state of the IPP, \( a = 1 \) is for the ON state and \( a = 0 \) is for the OFF state
- \( w, 0 \leq w \leq W \), is the number of busy servers in node 1
- \( c, 0 \leq c < \infty \), is the number of busy servers in node 2

The rate transitions for any state of this distribution are shown in Figure 5 and the local balance equations are:
Figure 5: Transition Rate Diagram for \( p_{a,w,c} \) for \( w = 0 \)

\[
\begin{align*}
(\lambda_{on} + c\mu + \phi)p_{10c} &= \psi p_{00c} + \mu p_{11,c-1} + (c + 1)\mu p_{1,0,c+1} \quad (16) \\
(c\mu + \psi)p_{00c} &= \phi p_{10c} + (c + 1)\mu p_{0,0,c+1} + \mu p_{0,1,c-1} \quad (17)
\end{align*}
\]

for \( 1 \leq w \leq W \)

\[
\begin{align*}
(w\mu + \lambda_{on} + c\mu + \phi)p_{1wc} &= \psi p_{0wc} + (w + 1)\mu p_{1,w+1,c-1} + (c + 1)\mu p_{1,w,c+1} + \\
&\quad + \lambda_{on} p_{1,w-1,c} \quad (18) \\
(w\mu + c\mu + \psi)p_{0wc} &= \phi p_{1wc} + (w + 1)\mu p_{0,w+1,c-1} + (c + 1)\mu p_{0,w,c+1} \quad (19)
\end{align*}
\]

for \( w = W \)

\[
\begin{align*}
(W\mu + c\mu + \phi)p_{1Wc} &= \psi p_{0Wc} + (c + 1)\mu p_{1,W,c+1} + \lambda_{on} p_{1,W-1,c} \quad (20) \\
(W\mu + c\mu + \psi)p_{0Wc} &= \phi p_{1Wc} + (c + 1)\mu p_{0,W,c+1} \quad (21)
\end{align*}
\]

where \( 1/\mu \) is the mean of the exponential service time in both node 1 and node 2.

The partial binomial moments of this probability distribution are given by the expression (see Girard (1990) ):

\[
\beta_{awj} = \sum_{c=j}^{\infty} \binom{c}{j} p_{a,w,c}, \quad a = 0, 1; \quad 0 \leq w \leq W \quad (22)
\]

and the \( j^{th} \) binomial moment is

\[
\beta_j = \sum_{w=0}^{W} [\beta_{1wj} + \beta_{0wj}] \quad (23)
\]
Therefore, the first binomial moment is:

\[
\beta_1 = \sum_{w=0}^{W} \left[ \beta_{1w1} + \beta_{0w1} \right]
\]

\[
= \sum_{w=0}^{W} \left[ \sum_{c=1}^{\infty} c (p_{1wc} + p_{0wc}) \right].
\]

which is simply the mean number of busy wavelengths in the infinite node 2 and thus

\[
m_{out} = \beta_1.
\]  

(25)

The second binomial moment is:

\[
\beta_2 = \sum_{w=0}^{W} \left[ \beta_{1w2} + \beta_{0w2} \right]
\]

\[
= \sum_{w=0}^{W} \left[ \sum_{c=1}^{\infty} \frac{c}{2} (p_{1wc} + p_{0wc}) \right]
\]

\[
= \sum_{w=0}^{W} \left[ \sum_{c=1}^{\infty} \frac{c(c-1)}{2} (p_{1wc} + p_{0wc}) \right]
\]

\[
= \frac{1}{2} \sum_{w=0}^{W} \left[ \sum_{c=1}^{\infty} c^2 (p_{1wc} + p_{0wc}) - \sum_{c=1}^{\infty} c (p_{1wc} + p_{0wc}) \right]
\]

and since \( v_{out} = E[c^2] - E^2[c] \), where \( E[\cdot] \) denotes the expected value, we have:

\[
v_{out} = 2\beta_2 - \beta_1^2 + \beta_1.
\]  

(27)

Next, let the probability generating functions for the binomial moments and for the probability distribution \( p_{a,w,c} \) respectively be, see Girard (1990):

\[
F_{aw}(x) = \sum_{j=0}^{\infty} \beta_{awj} x^j, \quad a = 1, 0; \quad 0 \leq w \leq W
\]

(28)

and

\[
G_{aw}(z) = \sum_{c=0}^{\infty} p_{a,w,c} z^c, \quad a = 1, 0; \quad 0 \leq w \leq W
\]

(29)

Note that there is the following relationship between these two probability generating functions, see Girard (1990):

\[
F_{aw}(x) = G_{aw}(x + 1).
\]

(30)

Now let us return to our local balance equations (16) - (21). Using the transformations described Appendix A, these equations become:
for $w = 0$

\[
(\mu + \lambda + \phi)\beta_{101} = \mu\beta_{111} + \mu\beta_{110} + \psi\beta_{001} \\
(\mu + \psi)\beta_{001} = \mu\beta_{010} + \mu\beta_{011} + \phi\beta_{101}
\]  

(31) (32)

for $1 \leq w \leq W$

\[
(\mu + w\mu + \lambda + \phi)\beta_{1w1} = (w + 1)\mu\beta_{1,w+1,1} + (w + 1)\mu\beta_{1,w+1,0} + \\
+ \lambda\beta_{1,w-1,1} + \psi\beta_{0,w,1}
\]

(33)

\[
(\mu + w\mu + \psi)\beta_{0w1} = (w + 1)\mu\beta_{0,w+1,0} + (w + 1)\mu\beta_{0,w+1,1} + \phi\beta_{1,w,1}
\]

(34)

for $w = W$

\[
(\mu + W\mu + \phi)\beta_{1W1} = \lambda\beta_{1,W-1,1} + \psi\beta_{0W1} \\
(\mu + W\mu + \psi)\beta_{0W1} = \phi\beta_{1W1}
\]

(35) (36)

As seen from equations (25) and (27), we only need to determine $\beta_1$ and $\beta_2$ in order to compute $m_{out}$ and $v_{out}$. In Appendix A we find that these two parameters are given by:

\[
\beta_1 = \sum_{w=0}^{W} w(\beta_{1w0} + \beta_{0w0})
\]

(37)

and

\[
\beta_2 = \frac{1}{2} \sum_{w=0}^{W} w(\beta_{1w1} + \beta_{0w1})
\]

(38)

For $\beta_1$ we need $\beta_{1w0}$ and $\beta_{0w0}$. We have that

\[
\beta_{1w0} = \sum_{c=0}^{\infty} \binom{c}{0} p_{1wc},
\]

(39)

which is simply the steady-state probability that there are $w$ busy wavelengths in node 1 and the IPP is in the ON state. Similarly, $\beta_{0w0}$ is the steady state probability that there are $w$ busy wavelengths in node 1 and the IPP is in the OFF state. So, in order to find $\beta_1$ we need the steady-state probabilities of node 1. If we consider only node 1, the corresponding probability distribution is $p_{a,w}$, $a = 0, 1$ and $0 \leq w \leq W$. The transition rate diagram for node 1 is shown in Figure 6. We solve for the steady-state probabilities $\pi_1$ at node 1 numerically. There is a total of $2(W + 1)$ states and thus the numerical solution requires multiplication of the $2(W + 1) \times 2(W + 1)$ rate matrix, i.e. $O(W^2)$ complexity. However this is not a limitation because the maximum number of wavelengths that vendors have been able to transmit over a single fiber is around 128. This number will not grow significantly as the high-precision lasers and filters are expensive, see Ciena: the network specialist (accessed Oct 14, 2013). In addition, we did not encounter any computational complexity issues when experimented with values of $W$ as high as 500.
Next, we find \( \beta_2 \) numerically by solving for \( \beta_{aww}, a = 0, 1; 0 \leq w \leq W \), using the system of \( 2(W + 1) \) equations (31) to (36). Now, that we have \( \beta_1 \) and \( \beta_2 \) we can calculate \( m_{out} \) and \( v_{out} \) of the departure process using (25) and (27).

5 The Algorithm

Now, let us recall the problem at hand, i.e., IPP bursty arrivals offered to an OBS path with large number of wavelengths per link. We begin with node 1, assuming unit service rate, depicted in Figure 4. The arrival traffic to node 1 is an IPP specified by the parameters \( c_1, r_1 \) and \( \lambda_{avg1} \) from which we obtain \( \lambda_{on}, \phi \) and \( \psi \), using (5) - (7) and subsequently \( m_1 \) and \( v_1 \) using (10) and (12). Next, we analyse node 1 numerically and calculate the steady state probability \( \pi_1 \).

In order to estimate the arrival process to node 2, we need to characterize the departure process from node 1, i.e., calculate \( m_{out_1} \) and \( v_{out_1} \). This is done using the technique from Section 4.

We assume that the local arrivals at each intermediate links are also bursty, modeled by an IPP, which is characterized by \( c_{loc}, r_{loc} \) and \( \lambda_{avgloc} \). Again, \( m_{loc} \) and \( v_{loc} \) are obtained using (5) - (7) and (10) and (12). Therefore, the arrival rate to node 2 is

\[
\begin{align*}
    m_2 &= m_{out_1} + m_{loc} \\
    v_2 &= v_{out_1} + v_{loc},
\end{align*}
\]

where the variances are added since the cross traffic and the local traffic are independent.

Next, we construct the IPP arrival process to node 2 that corresponds to a traffic stream characterized by the ISE parameters \( m_2 \) and \( v_2 \). This is done using (13) - (15). Now, we can solve numerically for the steady state probabilities at node 2, i.e, \( \pi_2 \).
The same steps are repeated for the rest of the nodes in the queueing network.

We illustrate our algorithm through an example. We consider an OBS path of $K = 2$ links and $W = 3$ wavelengths per link. Each burst occupies a wavelength on only one link at a time for an exponential amount of time with mean $1/\mu = 1$. Both the cross and the local traffic are modeled as IPPs. For the cross traffic is described by $\lambda_{avg}^{cross}$, $r$ and $c^2$ and the local traffic by $\lambda_{avg}^{loc}$, $r_{loc}$ and $c_{loc}^2$. The three parameters of these IPPs are obtained with (5)-(7).

Analysis of link 1

For the cross traffic IPP at link 1, the steady-state probabilities $\pi_1(w)$, $0 \leq w \leq W$ are found numerically. The state of node 1 is described by $(a, w)$, where $a = 0, 1$ and $0 \leq w \leq W$. If $a = 1$ the IPP is in the ON state and when $a = 0$ the IPP is in the OFF state. Therefore, the state space is

$$(10), (11), (12), (13), (00), (01), (02), (03).$$

We write the transition rate matrix $Q$:

$$Q = \begin{bmatrix}
* & \lambda_{on} & 0 & 0 & \phi & 0 & 0 & 0 \\
\mu & * & \lambda_{on} & 0 & 0 & \phi & 0 & 0 \\
0 & 2\mu & * & \lambda_{on} & 0 & 0 & \phi & 0 \\
0 & 0 & 3\mu & * & 0 & 0 & 0 & \phi \\
\beta & 0 & 0 & 0 & * & 0 & 0 & 0 \\
0 & \beta & 0 & 0 & \mu & * & 0 & 0 \\
0 & 0 & \beta & 0 & 0 & 2\mu & * & 0 \\
0 & 0 & 0 & \beta & 0 & 0 & 3\mu & * 
\end{bmatrix}$$

and solve for the steady-state probabilities $\pi_1$ using

$$\pi_1 Q = 0.$$

The blocking probability at node 1 is obtained from $\pi_1$.

Analysis of link 2

The arrival process to link 2 is the departure process from link 1 in addition to the local arrivals. The departure process from link 1 is characterized using the techniques from Section 4. In this example, the equations (31)-(36) become
\[(\mu + \lambda_{on} + \phi)\beta_{101} = \mu\beta_{111} + \mu\beta_{110} + \psi\beta_{001}\]
\[(\mu + \psi)\beta_{001} = \mu\beta_{010} + \mu\beta_{011} + \phi\beta_{101}\]
\[(2\mu + \lambda_{on} + \phi)\beta_{111} = 2\mu\beta_{121} + 2\mu\beta_{120} + \lambda_{on}\beta_{101} + \psi\beta_{011}\]
\[(2\mu + \psi)\beta_{011} = 2\mu\beta_{020} + 2\mu\beta_{021} + \phi\beta_{111}\]
\[(3\mu + \lambda_{on} + \phi)\beta_{121} = 3\mu\beta_{131} + 3\mu\beta_{130} + \lambda_{on}\beta_{111} + \psi\beta_{021}\]
\[(3\mu + \psi)\beta_{021} = 3\mu\beta_{030} + 3\mu\beta_{031} + \phi\beta_{121}\]
\[(4\mu + \phi)\beta_{131} = \lambda_{on}\beta_{121} + \psi\beta_{031}\]
\[(4\mu + \psi)\beta_{031} = \phi\beta_{131}\]

where the terms \(\beta_{aw0}, a = 0, 1\) and \(0 \leq w \leq 3\) are the steady-state probabilities \(\pi_1\) at loss node 1 when it is offered an IPP, characterized by \(m_1\) and \(v_1\).

The system of equations (40) is solved for \(\beta_{aw1}, a = 0, 1\) and \(0 \leq w \leq 3\). The first and the second binomial moments \(\beta_1\) and \(\beta_2\) are found using (24) and (26) respectively. The \(m_{1out}\) and \(v_{1out}\) are found with (25) and (27).

For the local arrivals at node 2, the \(m_{loc}\) and \(v_{loc}\) are obtained with (10) and (12). Therefore, the arrival process to node 2 is characterized by

\[m_2 = m_{1out} + m_{loc}\]
\[v_2 = v_{1out} + v_{loc}\]

Now, construct an IPP arrival stream to link 2 using \(m_2, v_2\) and equations (13) and (15). Again, we solve numerically for \(\pi_2\) in order to get the blocking at node 2.

### 6 Numerical Results

In this section we examine the accuracy of our algorithm by comparing it to simulation. The analytical results are obtained using Matlab. The simulation results are obtained using a custom event-driven C++ simulator, where the burst arrival process is set to be an IPP. For the simulation results, 95% confidence intervals are very small and consequently they have not been plotted.

Next, we verify the accuracy of the algorithm in an OBS path of \(K=5\) links. We present the results for six experiments. For all experiments, the average arrival rate of the IPP cross arrivals is \(\lambda_{avg}^{cross} = 0.9W\) and the average arrival rate of the IPP local arrivals is \(\lambda_{avg}^{local} = 0.2W\).

In the first three experiments we fix the value of the coefficient of burstiness \(r = 0.6\) and vary the number of wavelengths per link: \(W = 50\) in Figure 7, \(W = 200\) in Figure 8 and \(W = 15\) in Figure 9. The results are compared with simulation and show that our analytical results are accurate.
In Figure 9. In each figure we present three plots, each for a different squared coefficient of variation \(c^2\). The first plot in each figure is for \(c^2 = 5\), the second plot is for \(c^2 = 15\) and the third plot is for \(c^2 = 25\). In each experiment, we observe that the analytical results are a close upper bound for the simulation results. As expected, \(c^2\) increases so does the blocking probability in the OBS path. For comparison, in each plot we have also plotted the simulation results for a Poisson-distributed traffic with the same average rate as the IPP, both for the cross and local traffic. For most links, the results found with our analytical solution with IPP arrivals are better than the Poisson results. Moreover, the Poisson results are not consistent. At some links they are higher than the simulation results and at other links they are lower.

In the next three experiments we fix the value of the squared coefficient of variation \(c^2\) and we vary the coefficient of burstiness \(r\). As shown in Figure 3, the burst coefficient \(r\) affects the burst loss probability when the squared coefficient of variation is greater than 10. Therefore, we set \(c^2 = 20\). We vary the number of wavelengths per link: \(W = 50\) in Figure 10, \(W = 200\) in Figure 11 and \(W = 500\) in Figure 12. Again, in each figure we present three plots, for different values of the coefficient of burstiness \(r\). The first plot is for \(r = 0.1\), the second plot is for \(r = 0.5\) and the third plot is for \(r = 0.8\). Recall that the process is considered bursty as \(r\) approaches 0 and less bursty as \(r\) approaches 1. For comparison, in each plot we have plotted again the simulation results for a Poisson-distributed traffic with the same average rate as the IPP, both for the cross and local traffic.

Based on the above results, we observe that the numerical results are a close upper bound to the simulation results. When \(c^2\) is varied, the loss probability increases as we move away from the first node, which is due to the fact that the local traffic accumulates towards the end node. For high \(c^2\) and low number of wavelengths \(W\), there is a high loss probability of the cross traffic at the first node which has a dampening effect on subsequent nodes. Similar conclusions hold when \(c^2\) is constant and \(r\) is varied. The loss probability increases as we go towards the end node, except when the number of wavelengths \(W\) is small where we observe the same dampening effect of the cross traffic.

Through various numerical experiments, regardless of the value of \(c^2\), we find that the analytical solution deviates from the simulation results when \(r \geq 0.8\), i.e., the IPP has a long ON period and a very small OFF period. For example, we observe this phenomenon in Figure 10 when \(W = 50\), \(c^2 = 20\) and \(r = 0.8\). This is because in these cases, the IPP is not a good model for the arrival process and a more sophisticated stochastic process should be used to describe the arrival stream.

We also find that if the arrival process is characterized as bursty by both the squared coefficient of variation and the coefficient of burstiness, our analytical algorithm produces results much better than a simple Poisson approximation. In Table 5.2, we show that the relative error between the IPP analytical approximation and the simulation results is much better than the relative error for a Poisson approximation. For example, when \(c^2 = 5\) and \(W = 50\) the IPP
Figure 7: Blocking Probability for $W=50$, $\lambda_{cross}^{avg} = 45$, $\lambda_{local}^{avg} = 10$, $r = 0.6$
Figure 8: Blocking Probability for $W=200$, $\lambda_{cross}^\text{avg} = 180$, $\lambda_{local}^\text{avg} = 40$, $r = 0.6$
Figure 9: Blocking Probability for $W=500$, $\lambda_{\text{cross}}^{\text{avg}} = 450$, $\lambda_{\text{local}}^{\text{avg}} = 100$, $r = 0.6$
Figure 10: Blocking Probability for \( W=50, \lambda_{\text{cross}}^{\text{avg}} = 45, \lambda_{\text{local}}^{\text{avg}} = 10, \, \epsilon^2 = 20 \)
Figure 11: Blocking Probability for $W=200$, $\lambda_{avg}^{cross} = 180$, $\lambda_{avg}^{local} = 40$, $c^2 = 20$
Figure 12: Blocking Probability for W=500, $\lambda_{avg}^{cross} = 450$, $\lambda_{avg}^{local} = 100$, $e^2 = 20$
<table>
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<th>$c^2$</th>
<th>$W$</th>
<th>Error</th>
<th>Poisson Error</th>
</tr>
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<td>1.7</td>
<td>13.6</td>
</tr>
<tr>
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<td>200</td>
<td>3</td>
<td>15.9</td>
</tr>
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<td></td>
<td>500</td>
<td>5.7</td>
<td>20.8</td>
</tr>
<tr>
<td>20</td>
<td>50</td>
<td>13.7</td>
<td>31.96</td>
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<tr>
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<td>200</td>
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<td>36.02</td>
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<td>200</td>
<td>11.26</td>
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<tr>
<td></td>
<td>500</td>
<td>12.3</td>
<td>35.7</td>
</tr>
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</table>

Table 1: Relative Error as a function of $c^2, W$ for a fixed $r = 0.1$

relative error is almost six times better than the Poisson error.

7 Conclusions

In this paper, we presented a novel approach for the analysis of a tandem OBS network, assuming that bursts arrive according to an Interrupted Poisson Process (IPP). The OBS network was modelled by a tandem queueing network of loss nodes, where each loss node represented a link between two adjacent OBS nodes.

The queueing network was analyzed by a single-node decomposition, whereby each node was studied in isolation as a loss node with an IPP arrival, i.e., as an IPP/M/W/W node. In order to do this, we developed a method to characterize the mean and the variance of the departure process from an IPP/M/W/W node using binomial moment techniques. We found that the blocking probabilities for an IPP/M/W/W node using the Equivalent Random Method (ERM) gives upper bound values. Also, if the arrival process is characterized as bursty, our analytical algorithm produces results better than a simple Poisson approximation.

A Appendix: Derivation of the Departure Process from a Loss Node with IPP arrivals

Let us re-write equations (16)-(21) using the $z$-transform (29). We begin with equation (16), which becomes:

$$
\mu(z-1)G'_{10}(z) + (\lambda_{on} + \phi)G_{10}(z) = \mu zG_{11}(z) + \psi G_{00}(z). \tag{41}
$$

With a change of variable $z = x + 1$ we obtain:
\[ \mu x G'_{10}(x + 1) + (\lambda_{on} + \phi) G_{10}(x + 1) = \mu (x + 1) G_{11}(x + 1) + \psi G_{00}(x + 1). \tag{42} \]

and using (30) we get

\[ \mu x F'_{10}(x) + (\lambda_{on} + \phi) F_{10}(x) = \mu (x + 1) F_{11}(x) + \psi F_{00}(x) \tag{43} \]

Similarly we convert (17) into (44), (18) into (45), (19) into (46), (20) into (47) and (21) into (48).

for \( w = 0 \)

\[ \mu x F'_{10}(x) + \psi F_{00}(x) = \mu (x + 1) F_{01}(x) + \phi F_{10}(x) \tag{44} \]

for \( 1 \leq w \leq W \)

\[ \mu x F'_{1w}(x) + (w \mu + \lambda_{on} + \phi) F_{1w}(x) = (w + 1) \mu (x + 1) F_{1,w+1}(x) + \lambda_{on} F_{1,w-1}(x) + \psi F_{0w}(x) \tag{45} \]

\[ \mu x F'_{0w}(x) + (w \mu + \psi) F_{0w}(x) = (w + 1) \mu (x + 1) F_{0,w+1}(x) + \phi F_{1w}(x) \tag{46} \]

for \( w = W \)

\[ \mu x F'_{1W}(x) + (W \mu + \phi) F_{1W}(x) = \lambda_{on} F_{1,W-1}(x) + \psi F_{0W}(x) \tag{47} \]

\[ \mu x F'_{0W}(x) + (W \mu + \psi) F_{0W}(x) = \phi F_{1W}(x) \tag{48} \]

Now, we can write equations (43)-(48) in terms of the binomial moments (22) and equate the coefficients of equal power of \( x \). By equating the coefficients of \( x \) to the first power we get

for \( w = 0 \)

\[ (\mu + \lambda_{on} + \phi) \beta_{101} = \mu \beta_{111} + \mu \beta_{110} + \psi \beta_{001} \tag{49} \]

\[ (\mu + \psi) \beta_{001} = \mu \beta_{010} + \mu \beta_{011} + \phi \beta_{101} \tag{50} \]

for \( 1 \leq w \leq W \)

\[ (\mu + w \mu + \lambda_{on} + \phi) \beta_{1w1} = (w + 1) \mu \beta_{1,w+1,1} + (w + 1) \mu \beta_{1,w+1,0} + \lambda_{on} \beta_{1,w-1,1} + \psi \beta_{0,w,1} \tag{51} \]

\[ (\mu + w \mu + \psi) \beta_{0w1} = (w + 1) \mu \beta_{0,w+1,0} + (w + 1) \mu \beta_{0,w+1,1} + \phi \beta_{1,w,1} \tag{52} \]

for \( w = W \)

\[ (\mu + W \mu + \phi) \beta_{1W1} = \lambda_{on} \beta_{1,W-1,1} + \psi \beta_{0W1} \tag{53} \]

\[ (\mu + W \mu + \psi) \beta_{0W1} = \phi \beta_{1W1}. \tag{54} \]

By equating the coefficients of \( x^2 \) we get
for \( w = 0 \)

\[
(2\mu + \lambda_{on} + \phi)\beta_{10} = \mu \beta_{112} + \mu \beta_{111} + \psi \beta_{002}
\]

(55)

\[
(2\mu + \psi)\beta_{002} = \mu \beta_{011} + \mu \beta_{012} + \phi \beta_{102}
\]

(56)

for \( 1 \leq w \leq W \)

\[
(2\mu + w\mu + \lambda_{on} + \phi)\beta_{1w} = (w + 1)\mu \beta_{1,w+1,1} + (w + 1)\mu \beta_{1,w+1,2} + \lambda_{on} \beta_{1,w-1,2} + \psi \beta_{0,w,2}
\]

(57)

\[
(2\mu + w\mu + \psi)\beta_{0w} = (w + 1)\mu \beta_{0,w+1,1} + (w + 1)\mu \beta_{0,w+1,2} + \phi \beta_{1,w,2}
\]

(58)

for \( w = W \)

\[
(2\mu + W\mu + \phi)\beta_{1W} = \lambda_{on} \beta_{1,W-1,2} + \psi \beta_{0W2}
\]

(59)

\[
(2\mu + W\mu + \psi)\beta_{0W2} = \phi \beta_{1W2}.
\]

(60)

Now adding the 2\((W+1)\) equations (49) to (54) results in

\[
\beta_1 = \sum_{w=0}^{W} w(\beta_{1w0} + \beta_{0w0}).
\]

(61)

and adding the 2\((W+1)\) equations (55) to (60) gives

\[
2\beta_2 = \sum_{w=0}^{W} w(\beta_{1w1} + \beta_{0w1}).
\]

(62)

References


URL: [www.csc.ncsu.edu/faculty/perros/recentpapers.html](http://www.csc.ncsu.edu/faculty/perros/recentpapers.html)