Compress/Decompress the image A using 2 singular values

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 4 & 3 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

1. SVD of A

$$U = \begin{bmatrix} 0.653 & -0.707 & 0.272 \\ 0.653 & 0.707 & 0.272 \\ 0.385 & 0.000 & -0.923 \end{bmatrix}, \Sigma = \begin{bmatrix} 7.736 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 0.388 \end{bmatrix}, V$$

$$= \begin{bmatrix} 0.690 & 0.707 & 0.154 \\ 0.690 & -0.707 & 0.154 \\ 0.218 & 0.000 & -0.976 \end{bmatrix}$$

2. Compress

$$U_c = \begin{bmatrix} 0.653 & -0.707 & X \\ 0.653 & X & X \\ X & X & X \end{bmatrix}, \Sigma_c = \begin{bmatrix} 7.736 & X & X \\ X & 1.000 & X \\ X & X & X \end{bmatrix}, V_c = \begin{bmatrix} 0.690 & 0.707 & X \\ 0.690 & X & X \\ X & X & X \end{bmatrix}$$

3. Decompressing

$$U_c = \begin{bmatrix} 0.653 & -0.707 \\ 0.653 & 0.707 \\ 0.385 & 0.000 \end{bmatrix}, \Sigma_c = \begin{bmatrix} 7.736 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}, V_c = \begin{bmatrix} 0.690 & 0.707 \\ 0.690 & -0.707 \\ 0.218 & 0.000 \end{bmatrix}$$

$$A_c = \begin{bmatrix} 2.984 & 3.984 & 1.103 \\ 3.984 & 2.984 & 1.103 \\ 2.055 & 2.055 & 0.651 \end{bmatrix}$$

Compress/Decompress the image A using 2 singular values

$$A=\begin{bmatrix}4&3&2&3\\4&3&3&2\\4&4&2&1\\3&1&3&2\end{bmatrix}$$

1. SVD of A

$$U=\begin{bmatrix}0.534&-0.058&0.840&-0.081\\0.541&-0.068&-0.272&0.793\\0.516&0.685&-0.320&-0.403\\0.395&-0.723&-0.345&-0.450\end{bmatrix},\ \Sigma=\begin{bmatrix}11.371&0.000&0.000&0.000\\0.000&2.256&0.000&0.000\\0.000&0.000&1.264&0.000\\0.000&0.000&0.000&0.154\end{bmatrix},\ V$$

$$=\begin{bmatrix}0.664&0.028&-0.034&-0.747\\0.500&0.725&0.063&0.469\\0.432&-0.497&-0.641&0.394\\0.351&-0.476&0.764&0.259\end{bmatrix}$$

2. Compress

$$U_c=\begin{bmatrix}0.534&-0.058&X&X\\0.541&-0.068&X&X\\0.516&X&X&X\\X&X&X&X\end{bmatrix},\ \Sigma_c=\begin{bmatrix}11.371&X&X&X\\X&2.256&X&X\\X&X&X&X\\X&X&X&X\end{bmatrix},\ V_c=\begin{bmatrix}0.664&0.028&X&X\\0.500&0.725&X&X\\0.432&X&X&X\\X&X&X&X\end{bmatrix}$$

3. Decompressing

$$U_c=\begin{bmatrix}0.534&-0.058\\0.541&-0.068\\0.516&0.685\\0.395&-0.723\end{bmatrix},\ \Sigma_c=\begin{bmatrix}11.371&0.000\\0.000&2.256\end{bmatrix},\ V_c=\begin{bmatrix}0.664&0.028\\0.500&0.725\\0.432&-0.497\\0.351&-0.476\end{bmatrix}$$

$$A_c=\begin{bmatrix}4.027&2.939&2.686&2.193\\4.080&2.964&2.731&2.231\\3.940&4.054&1.765&1.325\\2.933&1.060&2.748&2.351\end{bmatrix}$$

Watermarking via SVD

Images: original A and watermark W

$$A = \begin{bmatrix} 0.000 & 3.000 & 7.000 & 4.000 \\ 1.000 & 1.000 & 3.000 & 8.000 \\ 0.000 & 8.000 & 6.000 & 3.000 \\ 3.000 & 9.000 & 8.000 & 1.000 \end{bmatrix}$$

$$W = \begin{bmatrix} 0.000 & 2.000 & 4.000 & 6.000 \\ 2.000 & 4.000 & 6.000 & 4.000 \\ 4.000 & 6.000 & 4.000 & 2.000 \\ 6.000 & 4.000 & 2.000 & 0.000 \end{bmatrix}$$

1. Embed the watermark

   a. SVD of A

$$U = \begin{bmatrix} -0.435 & -0.267 & 0.841 & -0.179 \\ -0.302 & -0.838 & -0.365 & 0.271 \\ -0.553 & 0.140 & -0.395 & -0.720 \\ -0.643 & 0.455 & -0.057 & 0.614 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 18.454 & 0.000 & 0.000 & 0.000 \\ 0.000 & 7.787 & 0.000 & 0.000 \\ 0.000 & 0.000 & 2.662 & 0.000 \\ 0.000 & 0.000 & 0.000 & 2.178 \end{bmatrix}$$

$$V = \begin{bmatrix} -0.121 & 0.068 & -0.202 & 0.970 \\ -0.640 & 0.459 & -0.571 & -0.231 \\ -0.673 & 0.012 & 0.737 & 0.069 \\ -0.350 & -0.886 & -0.301 & -0.044 \end{bmatrix}$$

   b. Embed the watermark into the singular value matrix

$k = 0.200$

$$\Sigma_w = \begin{bmatrix} 18.454 & 0.400 & 0.800 & 1.200 \\ 0.400 & 8.587 & 1.200 & 0.800 \\ 0.800 & 1.200 & 3.462 & 0.400 \\ 1.200 & 0.800 & 0.400 & 2.178 \end{bmatrix}$$

   c. Construct the watermarked image

$$A_w = \begin{bmatrix} -0.384 & 3.125 & 6.692 & 3.492 \\ 0.114 & 1.818 & 2.056 & 9.487 \\ -0.454 & 8.854 & 6.046 & 4.717 \\ 2.467 & 8.651 & 7.561 & 0.175 \end{bmatrix}$$

2. Bad person distorts the watermarked image
   a. Filter

$$filter = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 100 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

   b. Convolution

$$A_{wd} = \begin{bmatrix} -0.425 & 3.138 & 6.810 & 3.398 \\ 0.107 & 1.747 & 1.891 & 9.675 \\ -0.586 & 9.055 & 6.056 & 4.701 \\ 2.433 & 8.724 & 7.641 & 0.053 \end{bmatrix}$$

3. Detect the watermark
   a. Recover the watermarked singular values

$$\Sigma_{wd} = \begin{bmatrix} 18.546 & 0.279 & 0.793 & 1.336 \\ 0.296 & 8.810 & 1.300 & 0.775 \\ 0.773 & 1.301 & 3.635 & 0.448 \\ 1.283 & 0.745 & 0.435 & 2.282 \end{bmatrix}$$

   b. Recover the watermark

$$W_d = \begin{bmatrix} 0.460 & 1.393 & 3.967 & 6.682 \\ 1.478 & 5.114 & 6.499 & 3.873 \\ 3.866 & 6.503 & 4.867 & 2.242 \\ 6.414 & 3.726 & 2.174 & 0.520 \end{bmatrix}$$

Images: original A and watermark W

$$A = \begin{bmatrix} 5.000 & 5.000 & 2.000 & 4.000 \\ 1.000 & 0.000 & 0.000 & 10.000 \\ 10.000 & 6.000 & 3.000 & 8.000 \\ 4.000 & 9.000 & 6.000 & 7.000 \end{bmatrix}$$

$$W = \begin{bmatrix} 0.000 & 2.000 & 4.000 & 6.000 \\ 2.000 & 4.000 & 6.000 & 4.000 \\ 4.000 & 6.000 & 4.000 & 2.000 \\ 6.000 & 4.000 & 2.000 & 0.000 \end{bmatrix}$$

1. Embed the watermark
   a. SVD of A

$$U = \begin{bmatrix} -0.375 & 0.192 & -0.140 & -0.896 \\ -0.317 & -0.932 & 0.150 & -0.091 \\ -0.645 & 0.076 & -0.654 & 0.388 \\ -0.585 & 0.297 & 0.728 & 0.196 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 21.781 & 0.000 & 0.000 & 0.000 \\ 0.000 & 7.797 & 0.000 & 0.000 \\ 0.000 & 0.000 & 5.133 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.678 \end{bmatrix}$$

$$V = \begin{bmatrix} -0.504 & 0.254 & -0.813 & 0.143 \\ -0.506 & 0.525 & 0.377 & -0.572 \\ -0.285 & 0.307 & 0.415 & 0.808 \\ -0.639 & -0.752 & 0.158 & -0.020 \end{bmatrix}$$

   b. Embed the watermark into the singular value matrix

$k = 0.200$

$$\Sigma_w = \begin{bmatrix} 21.781 & 0.400 & 0.800 & 1.200 \\ 0.400 & 8.597 & 1.200 & 0.800 \\ 0.800 & 1.200 & 5.933 & 0.400 \\ 1.200 & 0.800 & 0.400 & 0.678 \end{bmatrix}$$

   c. Construct the watermarked image

$$A_w = \begin{bmatrix} 5.725 & 5.097 & 1.537 & 5.294 \\ 1.883 & -0.095 & -1.527 & 10.624 \\ 10.348 & 5.920 & 1.685 & 8.416 \\ 3.319 & 9.375 & 6.139 & 5.714 \end{bmatrix}$$

2. Bad person distorts the watermarked image
   a. Filter

$$filter = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 100 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

   b. Convolution

$$A_{wd} = \begin{bmatrix} 5.681 & 5.018 & 1.578 & 5.309 \\ 1.983 & 0.018 & -1.336 & 10.433 \\ 10.154 & 5.897 & 1.802 & 8.346 \\ 3.447 & 9.252 & 6.123 & 5.745 \end{bmatrix}$$

3. Detect the watermark
   a. Recover the watermarked singular values

$$\Sigma_{wd} = \begin{bmatrix} 21.711 & 0.346 & 0.757 & 1.042 \\ 0.449 & 8.303 & 1.164 & 0.742 \\ 0.697 & 1.188 & 5.698 & 0.408 \\ 1.211 & 0.819 & 0.419 & 0.650 \end{bmatrix}$$

   b. Recover the watermark

$$W_d = \begin{bmatrix} -0.351 & 1.729 & 3.783 & 5.210 \\ 2.245 & 2.533 & 5.822 & 3.709 \\ 3.485 & 5.941 & 2.825 & 2.040 \\ 6.054 & 4.095 & 2.094 & -0.139 \end{bmatrix}$$

Images: original A and watermark W

$$A=\begin{bmatrix} 0.000 & 2.000 & 5.000 & 6.000 \\ 3.000 & 8.000 & 8.000 & 5.000 \\ 0.000 & 8.000 & 3.000 & 1.000 \\ 6.000 & 3.000 & 9.000 & 3.000 \end{bmatrix}$$

$$W=\begin{bmatrix} 0.000 & 2.000 & 4.000 & 6.000 \\ 2.000 & 4.000 & 6.000 & 4.000 \\ 4.000 & 6.000 & 4.000 & 2.000 \\ 6.000 & 4.000 & 2.000 & 0.000 \end{bmatrix}$$

1. Embed the watermark
   a. SVD of A

$$U=\begin{bmatrix} -0.358 & 0.186 & 0.854 & -0.327 \\ -0.660 & -0.192 & 0.043 & 0.725 \\ -0.357 & -0.758 & -0.182 & -0.515 \\ -0.556 & 0.595 & -0.485 & -0.320 \end{bmatrix}$$

$$\Sigma=\begin{bmatrix} 19.169 & 0.000 & 0.000 & 0.000 \\ 0.000 & 6.791 & 0.000 & 0.000 \\ 0.000 & 0.000 & 4.720 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.415 \end{bmatrix}$$

$$V=\begin{bmatrix} -0.277 & 0.441 & -0.589 & 0.618 \\ -0.549 & -0.802 & -0.182 & 0.152 \\ -0.686 & 0.364 & -0.062 & -0.627 \\ -0.390 & 0.174 & 0.785 & 0.449 \end{bmatrix}$$

   b. Embed the watermark into the singular value matrix

$k=0.200$

$$\Sigma_w=\begin{bmatrix} 19.169 & 0.400 & 0.800 & 1.200 \\ 0.400 & 7.591 & 1.200 & 0.800 \\ 0.800 & 1.200 & 5.520 & 0.400 \\ 1.200 & 0.800 & 0.400 & 0.415 \end{bmatrix}$$

   c. Construct the watermarked image

$$A_w = \begin{bmatrix} -0.013 & 1.103 & 4.961 & 6.403 \\ 2.547 & 7.313 & 8.120 & 3.956 \\ -0.085 & 9.804 & 4.020 & -0.535 \\ 5.798 & 3.747 & 9.410 & 2.741 \end{bmatrix}$$

2. Bad person distorts the watermarked image
   a. Filter

$$filter = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 100 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

   b. Convolution

$$A_{wd} = \begin{bmatrix} -0.050 & 1.011 & 4.953 & 6.442 \\ 2.551 & 7.393 & 8.248 & 3.934 \\ -0.275 & 10.056 & 3.908 & -0.662 \\ 5.879 & 3.605 & 9.592 & 2.706 \end{bmatrix}$$

3. Detect the watermark
   a. Recover the watermarked singular values

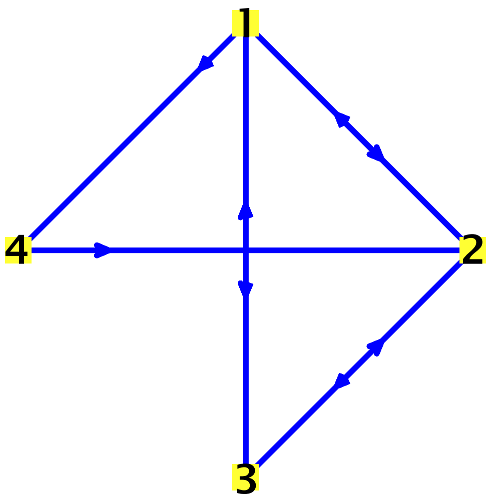
$$\Sigma_{wd} = \begin{bmatrix} 19.249 & 0.399 & 0.846 & 1.335 \\ 0.369 & 7.995 & 1.206 & 0.801 \\ 0.858 & 1.213 & 5.612 & 0.446 \\ 1.091 & 0.878 & 0.379 & 0.432 \end{bmatrix}$$

   b. Recover the watermark

$$W_d = \begin{bmatrix} 0.403 & 1.993 & 4.229 & 6.673 \\ 1.846 & 6.022 & 6.028 & 4.005 \\ 4.291 & 6.067 & 4.463 & 2.228 \\ 5.454 & 4.391 & 1.897 & 0.085 \end{bmatrix}$$

Page Ranking (Google)

Rank the web pages with the following links:



1. Construct the transition matrix

$$T = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} & 1 \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \end{bmatrix}$$

2. Set up a system of equations

$$\begin{bmatrix} -1 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & -1 & \frac{1}{2} & 1 \\ \frac{1}{3} & \frac{1}{2} & -1 & 0 \\ \frac{1}{3} & 0 & 0 & -1 \end{bmatrix} . P = 0$$

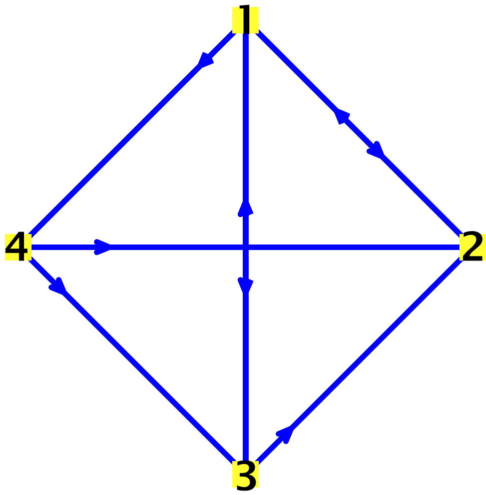
3. Solve the system so that the sum = 1

$$P = \begin{bmatrix} 0.300 \\ 0.333 \\ 0.267 \\ 0.100 \end{bmatrix}$$

4. Rank the web pages

2
1
3
4

Rank the web pages with the following links:



1. Construct the transition matrix

$$T = \begin{bmatrix} 0 & 1 & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \end{bmatrix}$$

2. Set up a system of equations

$$\begin{bmatrix} -1 & 1 & \frac{1}{2} & 0 \\ \frac{1}{3} & -1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & 0 & -1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & -1 \end{bmatrix} . P = 0$$

3. Solve the system so that the sum = 1

$$P = \begin{bmatrix} 0.387 \\ 0.290 \\ 0.194 \\ 0.129 \end{bmatrix}$$

4. Rank the web pages

1

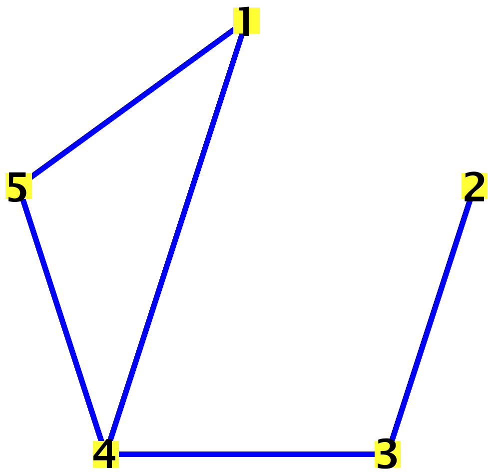
2

3

4

Graph partition (Facebook Community Identification)

Partition the graph into two subgraphs with minimum extra-cut size:



1. Adjacency Matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

2. Degrees

$$d = [2, 1, 2, 3, 2]$$

3. Approximate Probability Matrix

$$P = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{1}{10} & \frac{1}{5} & \frac{3}{10} & \frac{1}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{2}{5} \\ \frac{3}{5} & \frac{3}{10} & \frac{3}{5} & \frac{9}{10} & \frac{3}{5} \\ \frac{2}{5} & \frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{2}{5} \end{bmatrix}$$

4. Modulation Matrix

$$M = \begin{bmatrix} -\frac{2}{5} & -\frac{1}{5} & -\frac{2}{5} & \frac{2}{5} & \frac{3}{5} \\ -\frac{1}{5} & -\frac{1}{10} & \frac{4}{5} & -\frac{3}{10} & -\frac{1}{5} \\ -\frac{2}{5} & \frac{4}{5} & -\frac{2}{5} & \frac{2}{5} & -\frac{2}{5} \\ \frac{2}{5} & -\frac{3}{10} & \frac{2}{5} & -\frac{9}{10} & \frac{2}{5} \\ \frac{3}{5} & -\frac{1}{5} & -\frac{2}{5} & \frac{2}{5} & -\frac{2}{5} \end{bmatrix}$$

5. Eigen decomposition of M

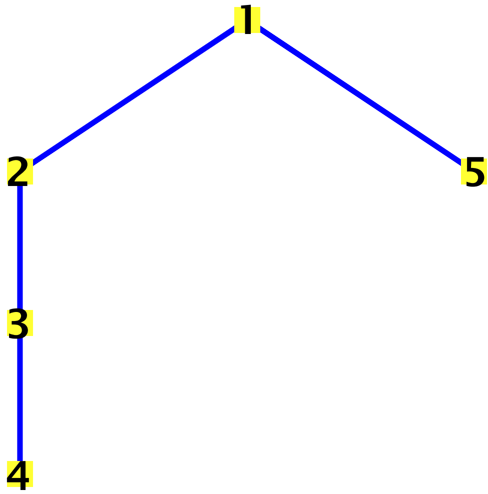
$$\Lambda = \begin{bmatrix} 1.048 & 0 & 0 & 0 & 0 \\ 0 & 6.235\ 10^{-10} & 0 & 0 & 0 \\ 0 & 0 & -0.569 & 0 & 0 \\ 0 & 0 & 0 & -1.000 & 0 \\ 0 & 0 & 0 & 0 & -1.679 \end{bmatrix}$$

$$V = \begin{bmatrix} -0.450 & 0.447 & 0.217 & 0.707 & 0.225 \\ 0.555 & 0.447 & 0.602 & 8.188\ 10^{-10} & -0.360 \\ 0.510 & 0.447 & -0.460 & 3.462\ 10^{-10} & 0.573 \\ -0.166 & 0.447 & -0.576 & 1.111\ 10^{-9} & -0.664 \\ -0.450 & 0.447 & 0.217 & -0.707 & 0.225 \end{bmatrix}$$

6. Partition

$G1 = [2, 3], G2 = [1, 4, 5]$

Partition the graph into two subgraphs with minimum extra-cut size:



1. Adjacency Matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Degrees

$$d = [2, 2, 2, 1, 1]$$

3. Approximate Probability Matrix

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{bmatrix}$$

4. Modulation Matrix

$$M = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{4} & \frac{3}{4} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} & -\frac{1}{8} & -\frac{1}{8} \\ \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{8} & -\frac{1}{8} \end{bmatrix}$$

5. Eigen decomposition of M

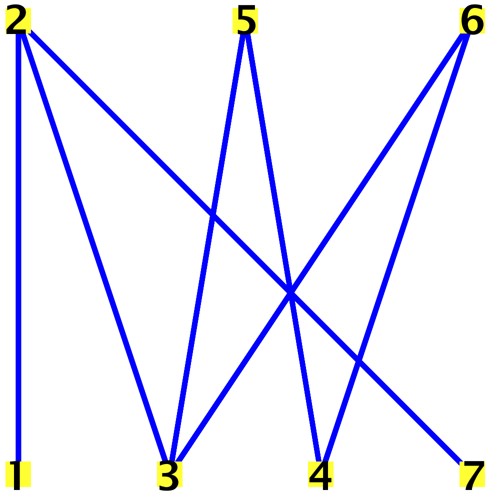
$$\Lambda = \begin{bmatrix} 1.000 & 0 & 0 & 0 & 0 \\ 0 & -7.852\ 10^{-11} & 0 & 0 & 0 \\ 0 & 0 & -1.493\ 10^{-10} & 0 & 0 \\ 0 & 0 & 0 & -1.000 & 0 \\ 0 & 0 & 0 & 0 & -1.750 \end{bmatrix}$$

$$V = \begin{bmatrix} 0.500 & 0.363 & 0.463 & -0.500 & -0.535 \\ 2.960\ 10^{-10} & 0.126 & 0.617 & -9.789\ 10^{-11} & 0.535 \\ -0.500 & 0.363 & 0.463 & 0.500 & -0.535 \\ -0.500 & 0.600 & 0.309 & -0.500 & 0.267 \\ 0.500 & 0.600 & 0.309 & 0.500 & 0.267 \end{bmatrix}$$

6. Partition

$G1 = [1, 2, 5], G2 = [3, 4]$

Partition the graph into two subgraphs with minimum extra-cut size:



1. Adjacency Matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Degrees

$$d = [1, 3, 3, 2, 2, 2, 1]$$

3. Approximate Probability Matrix

$$P = \begin{bmatrix} \frac{1}{14} & \frac{3}{14} & \frac{3}{14} & \frac{1}{7} & \frac{1}{7} & \frac{1}{7} & \frac{1}{14} \\ \frac{3}{14} & \frac{9}{14} & \frac{9}{14} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{14} \\ \frac{3}{14} & \frac{9}{14} & \frac{9}{14} & \frac{3}{7} & \frac{3}{7} & \frac{3}{7} & \frac{3}{14} \\ \frac{1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{2}{7} & \frac{2}{7} & \frac{2}{7} & \frac{1}{7} \\ \frac{1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{2}{7} & \frac{2}{7} & \frac{2}{7} & \frac{1}{7} \\ \frac{1}{7} & \frac{3}{7} & \frac{3}{7} & \frac{2}{7} & \frac{2}{7} & \frac{2}{7} & \frac{1}{7} \\ \frac{1}{14} & \frac{3}{14} & \frac{3}{14} & \frac{1}{7} & \frac{1}{7} & \frac{1}{7} & \frac{1}{14} \end{bmatrix}$$

4. Modulation Matrix

$$M = \begin{bmatrix} -\frac{1}{14} & \frac{11}{14} & -\frac{3}{14} & -\frac{1}{7} & -\frac{1}{7} & -\frac{1}{7} & -\frac{1}{14} \\ \frac{11}{14} & -\frac{9}{14} & \frac{5}{14} & -\frac{3}{7} & -\frac{3}{7} & -\frac{3}{7} & \frac{11}{14} \\ -\frac{3}{14} & \frac{5}{14} & -\frac{9}{14} & -\frac{3}{7} & \frac{4}{7} & \frac{4}{7} & -\frac{3}{14} \\ -\frac{1}{7} & -\frac{3}{7} & -\frac{3}{7} & -\frac{2}{7} & \frac{5}{7} & \frac{5}{7} & -\frac{1}{7} \\ -\frac{1}{7} & -\frac{3}{7} & \frac{4}{7} & \frac{5}{7} & -\frac{2}{7} & -\frac{2}{7} & -\frac{1}{7} \\ -\frac{1}{7} & -\frac{3}{7} & \frac{4}{7} & \frac{5}{7} & -\frac{2}{7} & -\frac{2}{7} & -\frac{1}{7} \\ -\frac{1}{14} & \frac{11}{14} & -\frac{3}{14} & -\frac{1}{7} & -\frac{1}{7} & -\frac{1}{7} & -\frac{1}{14} \end{bmatrix}$$

5. Eigen decomposition of M

$$\Lambda = \begin{bmatrix} 1.428 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.610\ 10^{-20} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -7.800\ 10^{-11} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.088\ 10^{-10} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.562\ 10^{-10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.475 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2.238 \end{bmatrix}$$

$$V = \begin{bmatrix} 0.372 & 2.466\ 10^{-9} & 0.835 & -0.799 & 0.202 & 0.363 & -0.182 \\ 0.507 & 3.795\ 10^{-10} & 0.145 & -0.053 & -0.310 & -0.640 & 0.427 \\ -0.088 & -1.528\ 10^{-10} & -0.036 & 0.273 & -0.491 & -0.095 & -0.535 \\ -0.460 & 9.193\ 10^{-10} & 0.327 & -0.380 & -0.129 & -0.458 & -0.353 \\ -0.351 & -0.707 & 0.145 & -0.053 & -0.310 & 0.233 & 0.413 \\ -0.351 & 0.707 & 0.145 & -0.053 & -0.310 & 0.233 & 0.413 \\ 0.372 & -1.200\ 10^{-9} & -0.363 & 0.366 & -0.642 & 0.363 & -0.182 \end{bmatrix}$$

6. Partition

$G1 = [1, 2, 7], G2 = [3, 4, 5, 6]$

Column space (Image)

Compute the column space of A

$$A = \begin{bmatrix} 1 & -3 & -1 & 1 & -8 & -8 & 1 & -3 \\ 0 & 0 & 1 & 1 & 2 & -3 & 1 & 6 \\ 0 & 0 & -1 & 0 & -3 & -1 & -1 & -7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 3 & 1 & 1 & 6 & 0 & 1 & 9 \\ -1 & 3 & 0 & 0 & 4 & 3 & 0 & 3 \end{bmatrix}$$

1. Compute a rref Rt of the transpose of A

$$Rt = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Construct a basis CS of the column space of A

$$CS = \left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right)$$

Compute the column space of A

$$A = \begin{bmatrix} 1 & -4 & 0 & 4 & 0 & 3 & 0 & 2 \\ -1 & 4 & 1 & -6 & 1 & -3 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & -2 & 0 & 2 & -1 & -3 \\ 1 & -4 & -1 & 6 & 0 & 1 & 0 & 3 \\ -1 & 4 & 0 & -4 & 0 & -3 & 1 & 0 \end{bmatrix}$$

1. Compute a rref Rt of the transpose of A

$$Rt = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Construct a basis CS of the column space of A

$$CS = \left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \right)$$

Compute the column space of A

$$A = \begin{bmatrix} -1 & 1 & 2 & 0 & 0 & -2 & 0 & -3 \\ 0 & 1 & 3 & -1 & -1 & -7 & -1 & -6 \\ 1 & 0 & 1 & -1 & -1 & -5 & 0 & 1 \\ -1 & 1 & 2 & 0 & 0 & -2 & 1 & 1 \\ 0 & -1 & -3 & 1 & 1 & 7 & 1 & 6 \\ 0 & 1 & 3 & 1 & 3 & -1 & 0 & 0 \end{bmatrix}$$

1. Compute a rref Rt of the transpose of A

$$Rt = \begin{bmatrix} 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Construct a basis CS of the column space of A

$$CS = \left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right)$$

Null space (Kernel)

Compute the null space of A

$$A = \begin{bmatrix} 1 & -3 & -1 & 1 & -8 & -8 & 1 & -3 \\ 0 & 0 & 1 & 1 & 2 & -3 & 1 & 6 \\ 0 & 0 & -1 & 0 & -3 & -1 & -1 & -7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 3 & 1 & 1 & 6 & 0 & 1 & 9 \\ -1 & 3 & 0 & 0 & 4 & 3 & 0 & 3 \end{bmatrix}$$

1. Compute a rref R of A

$$R = \begin{bmatrix} 1 & -3 & 0 & 0 & -4 & -3 & 0 & -3 \\ 0 & 0 & 1 & 0 & 3 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 & -4 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Construct a basis NS of the null space of A

$$NS = \left(\begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -3 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -1 \\ 4 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -3 \\ 1 \\ 0 \\ 0 \\ -4 \\ 1 \end{bmatrix} \right)$$

Compute the null space of A

$$A = \begin{bmatrix} 1 & -4 & 0 & 4 & 0 & 3 & 0 & 2 \\ -1 & 4 & 1 & -6 & 1 & -3 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & -2 & 0 & 2 & -1 & -3 \\ 1 & -4 & -1 & 6 & 0 & 1 & 0 & 3 \\ -1 & 4 & 0 & -4 & 0 & -3 & 1 & 0 \end{bmatrix}$$

1. Compute a rref R of A

$$R = \begin{bmatrix} 1 & -4 & 0 & 4 & 0 & 3 & 0 & 2 \\ 0 & 0 & 1 & -2 & 0 & 2 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -2 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Construct a basis NS of the null space of A

$$NS = \left(\begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ -2 \\ 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \\ -4 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right)$$

Compute the null space of A

$$A = \begin{bmatrix} -1 & 1 & 2 & 0 & 0 & -2 & 0 & -3 \\ 0 & 1 & 3 & -1 & -1 & -7 & -1 & -6 \\ 1 & 0 & 1 & -1 & -1 & -5 & 0 & 1 \\ -1 & 1 & 2 & 0 & 0 & -2 & 1 & 1 \\ 0 & -1 & -3 & 1 & 1 & 7 & 1 & 6 \\ 0 & 1 & 3 & 1 & 3 & -1 & 0 & 0 \end{bmatrix}$$

1. Compute a rref R of A

$$R = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & -2 & 0 & 2 \\ 0 & 1 & 3 & 0 & 1 & -4 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Construct a basis NS of the null space of A

$$NS = \left(\begin{bmatrix} -1 \\ -3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \\ -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 0 \\ -3 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ -1 \\ 0 \\ 0 \\ -4 \\ 1 \end{bmatrix} \right)$$

Row space (Factor)

Compute the row space of A

$$A = \begin{bmatrix} 1 & -3 & -1 & 1 & -8 & -8 & 1 & -3 \\ 0 & 0 & 1 & 1 & 2 & -3 & 1 & 6 \\ 0 & 0 & -1 & 0 & -3 & -1 & -1 & -7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 3 & 1 & 1 & 6 & 0 & 1 & 9 \\ -1 & 3 & 0 & 0 & 4 & 3 & 0 & 3 \end{bmatrix}$$

1. Compute a rref R of A

$$R = \begin{bmatrix} 1 & -3 & 0 & 0 & -4 & -3 & 0 & -3 \\ 0 & 0 & 1 & 0 & 3 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 & -4 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Construct a basis RS of the row space of A

$$RS = \left(\begin{bmatrix} 1 & -3 & 0 & 0 & -4 & -3 & 0 & -3 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 0 & 3 & 1 & 0 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 & -1 & -4 & 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix} \right)$$

Compute the row space of A

$$A = \begin{bmatrix} 1 & -4 & 0 & 4 & 0 & 3 & 0 & 2 \\ -1 & 4 & 1 & -6 & 1 & -3 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & -2 & 0 & 2 & -1 & -3 \\ 1 & -4 & -1 & 6 & 0 & 1 & 0 & 3 \\ -1 & 4 & 0 & -4 & 0 & -3 & 1 & 0 \end{bmatrix}$$

1. Compute a rref R of A

$$R = \begin{bmatrix} 1 & -4 & 0 & 4 & 0 & 3 & 0 & 2 \\ 0 & 0 & 1 & -2 & 0 & 2 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -2 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Construct a basis RS of the row space of A

$$RS = \left(\begin{bmatrix} 1 & -4 & 0 & 4 & 0 & 3 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & -2 & 0 & 2 & 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -2 & 0 & 4 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix} \right)$$

Compute the row space of A

$$A = \begin{bmatrix} -1 & 1 & 2 & 0 & 0 & -2 & 0 & -3 \\ 0 & 1 & 3 & -1 & -1 & -7 & -1 & -6 \\ 1 & 0 & 1 & -1 & -1 & -5 & 0 & 1 \\ -1 & 1 & 2 & 0 & 0 & -2 & 1 & 1 \\ 0 & -1 & -3 & 1 & 1 & 7 & 1 & 6 \\ 0 & 1 & 3 & 1 & 3 & -1 & 0 & 0 \end{bmatrix}$$

1. Compute a rref R of A

$$R = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & -2 & 0 & 2 \\ 0 & 1 & 3 & 0 & 1 & -4 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Construct a basis RS of the row space of A

$$RS = \left(\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & -2 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 3 & 0 & 1 & -4 & 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 & 2 & 3 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix} \right)$$