MA 341 Supplement on Linear Algebra

Adding two vectors

Formula:

$$\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a+c \\ b+d \end{bmatrix}$$

Example:

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1+3 \\ 2+4 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Multiplying a number and a vector

Formula:

$$c\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} ca \\ cb \end{bmatrix}$$

Example:

$$3\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \cdot 2 \\ 3 \cdot 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$$

Adding two matrices

Formula:

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix}$$

Example:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 9 & 7 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 1+9 & 2+7 \\ 3+5 & 4+6 \end{bmatrix} = \begin{bmatrix} 10 & 9 \\ 8 & 10 \end{bmatrix}$$

Subtracting a number from a matrix

Formula:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \lambda = \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}$$

Example:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - 5 = \begin{bmatrix} 1-5 & 2 \\ 3 & 4-5 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$$

Multiplying a number and a matrix

Formula:

$$r\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ra & rb \\ rc & rd \end{bmatrix}$$

Example:

$$5\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$$

Multiplying a matrix and a vector

Formula:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

Example:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} (1)(5) + (2)(6) \\ (3)(5) + (4)(6) \end{bmatrix} = \begin{bmatrix} 17 \\ 39 \end{bmatrix}$$

Determinant of a matrix

Formula:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Example:

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (1)(4) - (2)(3) = -2$$

Writing a system of equation in terms of matrix and vector

Formula:

$$\begin{cases} p = ax + by \\ q = cx + dy \end{cases} \text{ can be rewritten as } \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Example:

$$\begin{cases} 5 = x + 2y \\ 6 = 3x + 4y \end{cases} \text{ can be rewritten as } \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Solving a system of equation using Cramer's Rule

Formula:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix} \text{ can be solved as}$$

$$x = \frac{\begin{vmatrix} p & b \\ q & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} a & p \\ c & q \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

Example:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \text{ can be solved as}$$

$$x = \frac{\begin{vmatrix} 5 & 2 \\ 6 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}} = \frac{8}{-2} = -4$$

$$y = \frac{\begin{vmatrix} 1 & 5 \\ 3 & 6 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}} = \frac{-9}{-2} = \frac{9}{2}$$

Writing a system of diffrerential equations in terms of matrix and vector

Formula:

$$\begin{cases} y_1' = ay_1 + by_2 \\ y_2' = cy_1 + dy_2 \end{cases} \qquad \qquad y' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} y$$

can be rewritten as

$$\begin{cases} y_1(0) = p \\ y_2(0) = q \end{cases} \qquad \qquad y(0) = \begin{bmatrix} p \\ q \end{bmatrix}$$

Example:

$$\begin{cases} y_1' = 5y_1 + 2y_2 \\ y_2' = 3y_1 + 4y_2 \end{cases} \qquad \qquad y' = \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix} y$$

can be rewritten as

$$\begin{cases} y_1(0) = 8 \\ y_2(0) = 6 \end{cases} \qquad \qquad y(0) = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$