

Problem:

$$y1' = \frac{1}{(y2 - 1)(y2 - 2)}$$

$$y2' = \frac{1}{(y1 + 1)(y1 + 3)}$$

$$y1(0) = 1$$

$$y2(0) = 0$$

1. Find the general solution:

$$dy1/dy2 = \frac{(y1 + 1)(y1 + 3)}{(y2 - 1)(y2 - 2)}$$

$$\frac{dy1}{(y1 + 1)(y1 + 3)} = \frac{dy2}{(y2 - 1)(y2 - 2)}$$

$$\left(-\frac{1}{2(y1 + 3)} + \frac{1}{2(y1 + 1)} \right) dy1 = \left(-\frac{1}{y2 - 1} + \frac{1}{y2 - 2} \right) dy2$$

$$-\frac{1}{2} \ln(|y1 + 3|) + \frac{1}{2} \ln(|y1 + 1|) = \ln(|y2 - 2|) - \ln(|y2 - 1|) + C$$

$$-\ln(|y1 + 3|) + \ln(|y1 + 1|) = 2 \ln(|y2 - 2|) - 2 \ln(|y2 - 1|) + C$$

$$\ln\left(\frac{|y1 + 1|}{|y1 + 3|}\right) = \ln\left(\frac{|y2 - 2|^2}{|y2 - 1|^2}\right) + C$$

$$\frac{|y1 + 1|}{|y1 + 3|} = \frac{C |y2 - 2|^2}{|y2 - 1|^2}, C > 0$$

2. Find the particular solution:

$$\frac{1}{2} = 4C$$

$$C = \frac{1}{8}$$

$$\frac{|y1 + 1|}{|y1 + 3|} = \frac{1}{8} \frac{|y2 - 2|^2}{|y2 - 1|^2}$$