MA 341
Homework 2
(Non-homogeneous Linear Differential Equations)

Hoon Hong

Undetermined Coefficient Method: 1st-order, 1 variable

Problem:

$y' + 3y = 5e^{2t}$

$y(0) = 0$

1. Find the general solution of the homogeneous equation:

$\lambda + 3 = 0$

$\lambda = -3$

$y_g = C e^{-3t}$

2. Find a particular soultion of the non-homogeneous equation:

$y = D e^{2t}$

$5D = 5$

$D = 1$

$y_p = e^{2t}$

3. Find the general solution of the non-homogeneous equation:

$y = C e^{-3t} + e^{2t}$

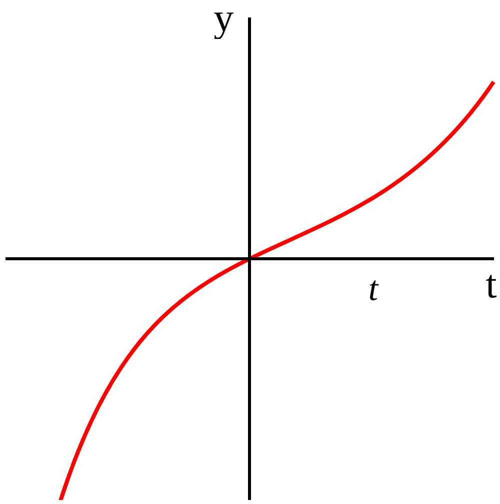
4. Find the particular solution of the non-homogeneous equation:

$0 = C + 1$

$C = -1$

$y = -e^{-3t} + e^{2t}$

5. Sketch the particular solution:


y
t
t

Problem:

$$y' + 4y = 5e^{-2t}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda + 4 = 0$$

$$\lambda = -4$$

$$y_g = C e^{-4t}$$

2. Find a particular soultion of the non-homogeneous equation:

$$y = D e^{-2t}$$

$$2D = 5$$

$$D = \frac{5}{2}$$

$$y_p = \frac{5}{2} e^{-2t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C e^{-4t} + \frac{5}{2} e^{-2t}$$

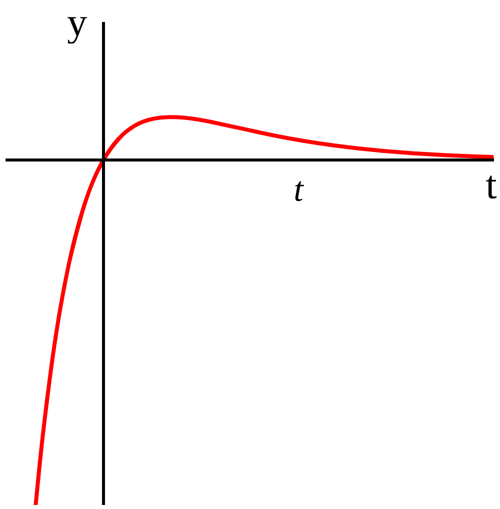
4. Find the particular solution of the non-homogeneous equation:

$$0 = C + \frac{5}{2}$$

$$C = -\frac{5}{2}$$

$$y = -\frac{5}{2} e^{-4t} + \frac{5}{2} e^{-2t}$$

5. Sketch the particular solution:



Problem:

$y' - 4y = 5$

$y(0) = 0$

1. Find the general solution of the homogeneous equation:

$\lambda - 4 = 0$

$\lambda = 4$

$y_g = C e^{4t}$

2. Find a particular soultion of the non-homogeneous equation:

$y = D$

$-4D = 5$

$D = -\frac{5}{4}$

$y_p = -\frac{5}{4}$

3. Find the general solution of the non-homogeneous equation:

$y = C e^{4t} - \frac{5}{4}$

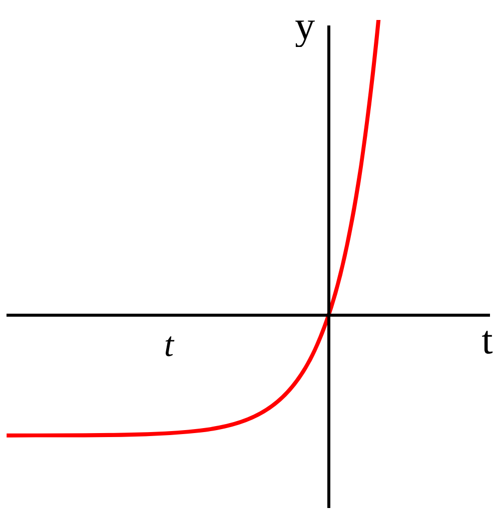
4. Find the particular solution of the non-homogeneous equation:

$0 = C - \frac{5}{4}$

$C = \frac{5}{4}$

$y = \frac{5}{4} e^{4t} - \frac{5}{4}$

5. Sketch the particular solution:



Problem:

$$y' + 2y = -e^{2t}$$
$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda + 2 = 0$$
$$\lambda = -2$$
$$y_g = C e^{-2t}$$

2. Find a particular soultion of the non-homogeneous equation:

$$y = D e^{2t}$$
$$4D = -1$$
$$D = -\frac{1}{4}$$
$$y_p = -\frac{1}{4} e^{2t}$$

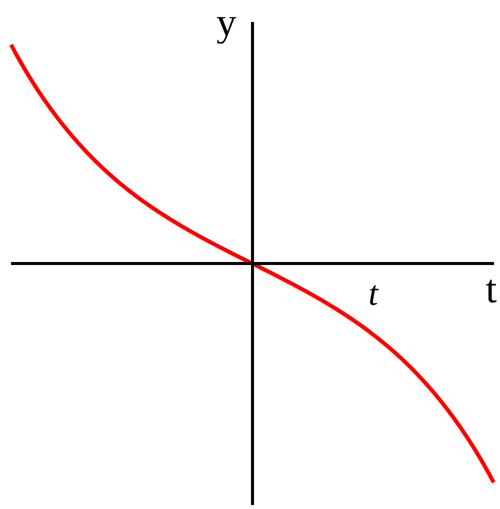
3. Find the general solution of the non-homogeneous equation:

$$y = C e^{-2t} - \frac{1}{4} e^{2t}$$

4. Find the particular solution of the non-homogeneous equation:

$$0 = C - \frac{1}{4}$$
$$C = \frac{1}{4}$$
$$y = \frac{1}{4} e^{-2t} - \frac{1}{4} e^{2t}$$

5. Sketch the particular solution:



Problem:

$y' - 3y = 2$

$y(0) = 0$

1. Find the general solution of the homogeneous equation:

$\lambda - 3 = 0$

$\lambda = 3$

$y_g = C e^{3t}$

2. Find a particular soultion of the non-homogeneous equation:

$y = \mathrm{D}$

$-3\,\mathrm{D} = 2$

$\mathrm{D} = -\frac{2}{3}$

$y_p = -\frac{2}{3}$

3. Find the general solution of the non-homogeneous equation:

$y = C e^{3t} - \frac{2}{3}$

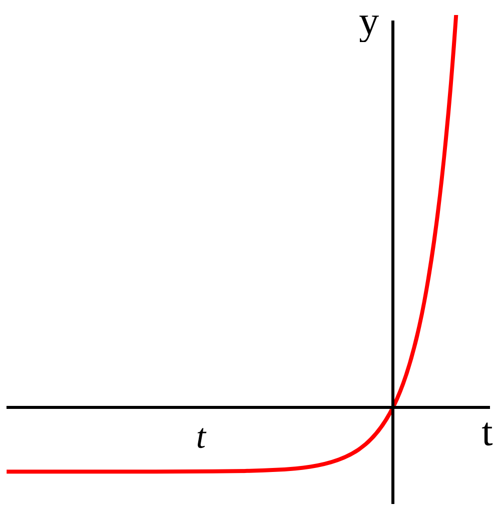
4. Find the particular solution of the non-homogeneous equation:

$0 = C - \frac{2}{3}$

$C = \frac{2}{3}$

$y = \frac{2}{3} e^{3t} - \frac{2}{3}$

5. Sketch the particular solution:



Problem:

$$y' + 4y = 3e^{-2t}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda + 4 = 0$$

$$\lambda = -4$$

$$y_g = C e^{-4t}$$

2. Find a particular soultion of the non-homogeneous equation:

$$y = D e^{-2t}$$

$$2D = 3$$

$$D = \frac{3}{2}$$

$$y_p = \frac{3}{2} e^{-2t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C e^{-4t} + \frac{3}{2} e^{-2t}$$

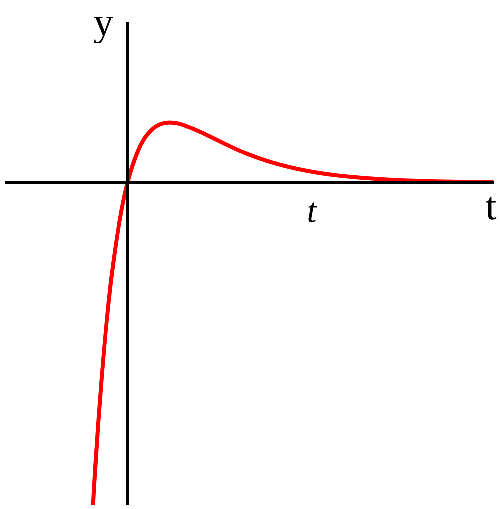
4. Find the particular solution of the non-homogeneous equation:

$$0 = C + \frac{3}{2}$$

$$C = -\frac{3}{2}$$

$$y = -\frac{3}{2} e^{-4t} + \frac{3}{2} e^{-2t}$$

5. Sketch the particular solution:



Undetermined Coefficient Method: 2nd-order, 1 variable

Problem:

$$y'' + 3\,y' + 2\,y = 4\,e^{2\,t}$$

$$y(0) = 0,\ y'(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda^2 + 3\,\lambda + 2 = 0$$

$$\lambda = -1,\ -2$$

$$y_g = C_1\,e^{-t} + C_2\,e^{-2\,t}$$

2. Find a particular soultion of the non-homogeneous equation:

$$y = D\,e^{2\,t}$$

$$12\,D = 4$$

$$D = \frac{1}{3}$$

$$y_p = \frac{1}{3}\,e^{2\,t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1\,e^{-t} + C_2\,e^{-2\,t} + \frac{1}{3}\,e^{2\,t}$$

4. Find the particular solution of the non-homogeneous equation:

$$y' = -C_1\,e^{-t} - 2\,C_2\,e^{-2\,t} + \frac{2}{3}\,e^{2\,t}$$

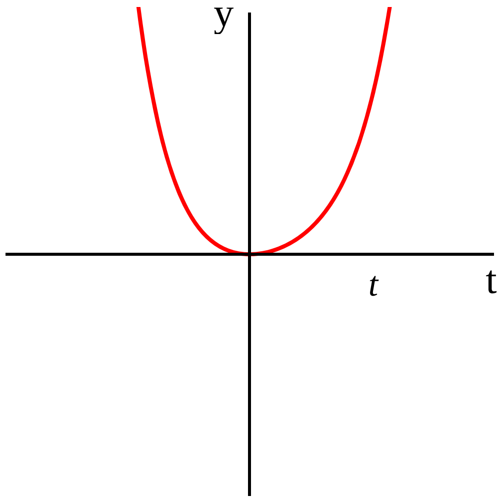
$$0 = C_1 + C_2 + \frac{1}{3}$$

$$0 = -C_1 - 2\,C_2 + \frac{2}{3}$$

$$C_1 = -\frac{4}{3},\ C_2 = 1$$

$$y = -\frac{4}{3}\,e^{-t} + e^{-2\,t} + \frac{1}{3}\,e^{2\,t}$$

5. Sketch the particular solution:


y
t
t

Problem:

$$y'' - 3y' + 2y = 4e^{5t}$$

$$y(0) = 0,\ y'(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\lambda = 2, 1$$

$$y_g = C_1 e^{2t} + C_2 e^{t}$$

2. Find a particular soultion of the non-homogeneous equation:

$$y = D e^{5t}$$

$$12 D = 4$$

$$D = \frac{1}{3}$$

$$y_p = \frac{1}{3} e^{5t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 e^{2t} + C_2 e^{t} + \frac{1}{3} e^{5t}$$

4. Find the particular solution of the non-homogeneous equation:

$$y' = 2C_1 e^{2t} + C_2 e^{t} + \frac{5}{3} e^{5t}$$

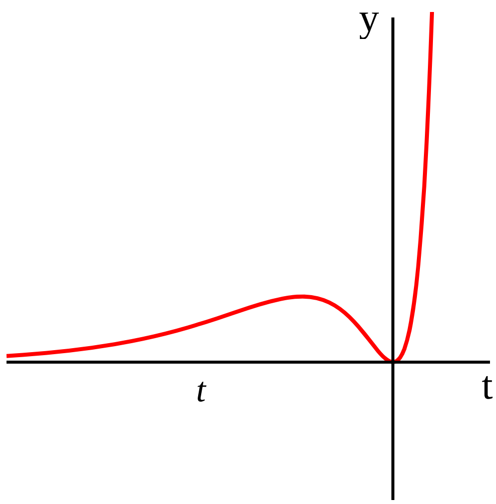
$$0 = C_1 + C_2 + \frac{1}{3}$$

$$0 = 2C_1 + C_2 + \frac{5}{3}$$

$$C_1 = -\frac{4}{3},\ C_2 = 1$$

$$y = -\frac{4}{3} e^{2t} + e^{t} + \frac{1}{3} e^{5t}$$

5. Sketch the particular solution:


y
t
t

Problem:

$$y'' + 5y' + 4y = -4$$

$$y(0) = 0,\ y'(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda^2 + 5\lambda + 4 = 0$$

$$\lambda = -1,\ -4$$

$$y_g = C_1 e^{-t} + C_2 e^{-4t}$$

2. Find a particular soultion of the non-homogeneous equation:

$$y = D$$

$$4D = -4$$

$$D = -1$$

$$y_p = -1$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 e^{-t} + C_2 e^{-4t} - 1$$

4. Find the particular solution of the non-homogeneous equation:

$$y' = -C_1 e^{-t} - 4C_2 e^{-4t}$$

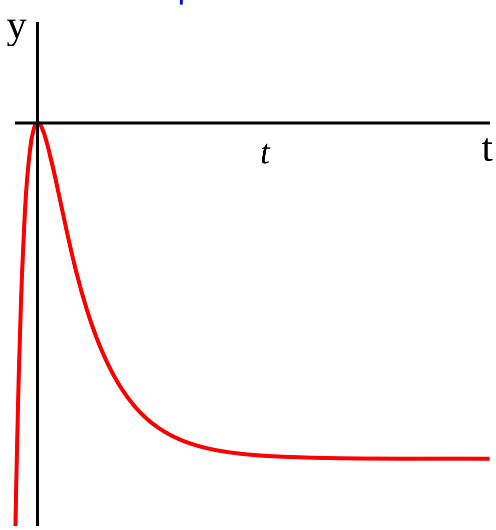
$$0 = C_1 + C_2 - 1$$

$$0 = -C_1 - 4C_2$$

$$C_1 = \frac{4}{3},\ C_2 = -\frac{1}{3}$$

$$y = \frac{4}{3} e^{-t} - \frac{1}{3} e^{-4t} - 1$$

5. Sketch the particular solution:



Problem:

$$y'' + 5y' + 6y = -e^{2t}$$

$$y(0) = 0,\ y'(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda^2 + 5\lambda + 6 = 0$$

$$\lambda = -2,\ -3$$

$$y_g = C_1 e^{-2t} + C_2 e^{-3t}$$

2. Find a particular soultion of the non-homogeneous equation:

$$y = D e^{2t}$$

$$20 D = -1$$

$$D = -\frac{1}{20}$$

$$y_p = -\frac{1}{20} e^{2t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 e^{-2t} + C_2 e^{-3t} - \frac{1}{20} e^{2t}$$

4. Find the particular solution of the non-homogeneous equation:

$$y' = -2 C_1 e^{-2t} - 3 C_2 e^{-3t} - \frac{1}{10} e^{2t}$$

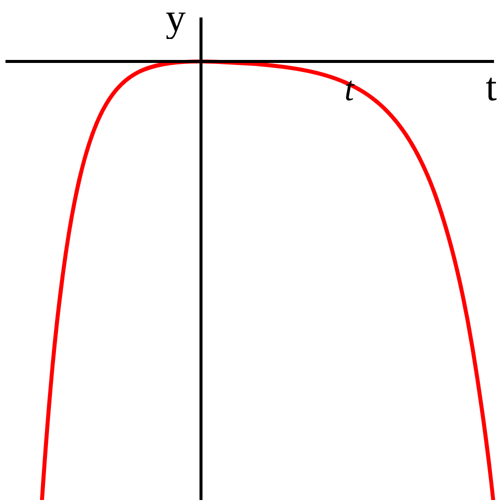
$$0 = C_1 + C_2 - \frac{1}{20}$$

$$0 = -2 C_1 - 3 C_2 - \frac{1}{10}$$

$$C_1 = \frac{1}{4},\ C_2 = -\frac{1}{5}$$

$$y = \frac{1}{4} e^{-2t} - \frac{1}{5} e^{-3t} - \frac{1}{20} e^{2t}$$

5. Sketch the particular solution:


y
t
t

Problem:

$$y'' + 7y' + 10y = -10$$

$$y(0) = 0,\ y'(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda^2 + 7\lambda + 10 = 0$$

$$\lambda = -2,\ -5$$

$$y_g = C_1 e^{-2t} + C_2 e^{-5t}$$

2. Find a particular soultion of the non-homogeneous equation:

$$y = D$$

$$10D = -10$$

$$D = -1$$

$$y_p = -1$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 e^{-2t} + C_2 e^{-5t} - 1$$

4. Find the particular solution of the non-homogeneous equation:

$$y' = -2C_1 e^{-2t} - 5C_2 e^{-5t}$$

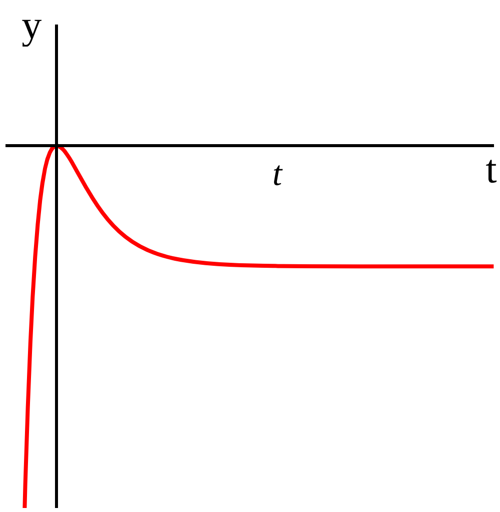
$$0 = C_1 + C_2 - 1$$

$$0 = -2C_1 - 5C_2$$

$$C_1 = \frac{5}{3},\ C_2 = -\frac{2}{3}$$

$$y = \frac{5}{3} e^{-2t} - \frac{2}{3} e^{-5t} - 1$$

5. Sketch the particular solution:



Problem:

$$y'' + 3y' + 2y = -4e^{-3t}$$

$$y(0) = 0,\ y'(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda^2 + 3\lambda + 2 = 0$$

$$\lambda = -1,\ -2$$

$$y_g = C_1 e^{-t} + C_2 e^{-2t}$$

2. Find a particular soultion of the non-homogeneous equation:

$$y = D e^{-3t}$$

$$2D = -4$$

$$D = -2$$

$$y_p = -2e^{-3t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 e^{-t} + C_2 e^{-2t} - 2e^{-3t}$$

4. Find the particular solution of the non-homogeneous equation:

$$y' = -C_1 e^{-t} - 2C_2 e^{-2t} + 6e^{-3t}$$

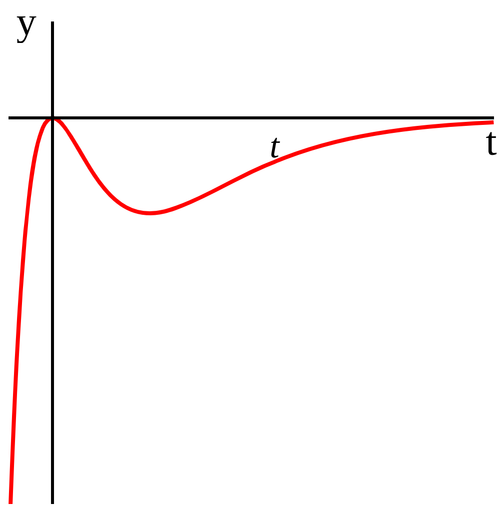
$$0 = C_1 + C_2 - 2$$

$$0 = -C_1 - 2C_2 + 6$$

$$C_1 = -2,\ C_2 = 4$$

$$y = -2e^{-t} + 4e^{-2t} - 2e^{-3t}$$

5. Sketch the particular solution:



Undetermined Coefficient Method: 1st-order, 2 variables

Problem:

$$y' = \begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix} y + \begin{bmatrix} 2 \\ 4 \end{bmatrix} e^{2t}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$Det \begin{bmatrix} 2-\lambda & 1 \\ -3 & -2-\lambda \end{bmatrix} = 0$$

$$\lambda^2 - 1 = 0$$

$$\lambda = 1, v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\lambda = -1, v = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{t} + C_2 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-t}$$

2. Find a particular soultion of the non-homogeneous equation:

$$y = D\, e^{2t}$$

$$2\,D = \begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix} D + \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} D$$

$$D = \begin{bmatrix} 4 \\ -2 \end{bmatrix}, y_p = \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{2t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{t} + C_2 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{2t}$$

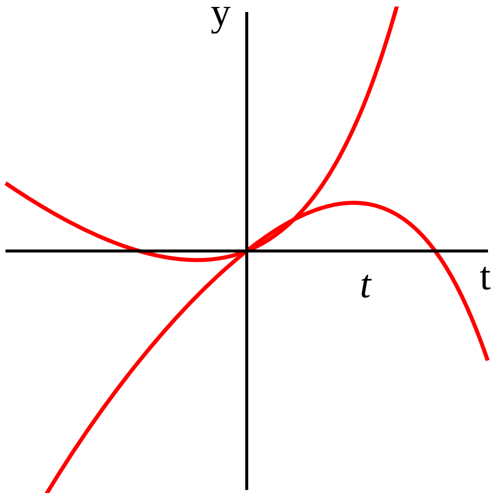
4. Find the particular solution of the non-homogeneous equation:

$$0 = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ -3 \end{bmatrix} + \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$C_1 = -5, C_2 = 1$$

$$y = \begin{bmatrix} -5 \\ 5 \end{bmatrix} e^{t} + \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{2t}$$

5. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 4 & 1 \\ -5 & -2 \end{bmatrix} y + \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{t}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$Det \begin{bmatrix} 4-\lambda & 1 \\ -5 & -2-\lambda \end{bmatrix} = 0$$

$$\lambda^{2} - 2\lambda - 3 = 0$$

$$\lambda = 3,\ v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\lambda = -1,\ v = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} 1 \\ -5 \end{bmatrix} e^{-t}$$

2. Find a particular soultion of the non-homogeneous equation:

$$y = D\, e^{t}$$

$$D = \begin{bmatrix} 4 & 1 \\ -5 & -2 \end{bmatrix} D + \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -5 & -3 \end{bmatrix} D$$

$$D = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ y_p = \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} 1 \\ -5 \end{bmatrix} e^{-t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{t}$$

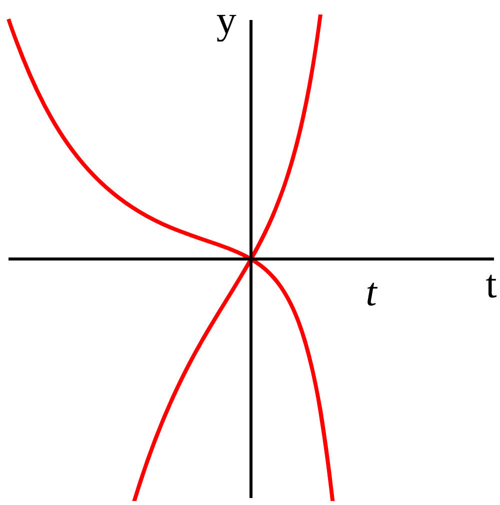
4. Find the particular solution of the non-homogeneous equation:

$$0 = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ -5 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C_1 = -\frac{1}{4},\ C_2 = \frac{1}{4}$$

$$y = \begin{bmatrix} -\frac{1}{4} \\ \frac{1}{4} \end{bmatrix} e^{3t} + \begin{bmatrix} \frac{1}{4} \\ -\frac{5}{4} \end{bmatrix} e^{-t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{t}$$

5. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} y + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$Det \begin{bmatrix} 1-\lambda & 2 \\ -1 & 4-\lambda \end{bmatrix} = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$\lambda = 3,\ v = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\lambda = 2,\ v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 2 \\ 2 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t}$$

2. Find a particular soultion of the non-homogeneous equation:

$$y = D$$

$$0 = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} D + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} D$$

$$D = \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix},\ y_p = \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 \begin{bmatrix} 2 \\ 2 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} + \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}$$

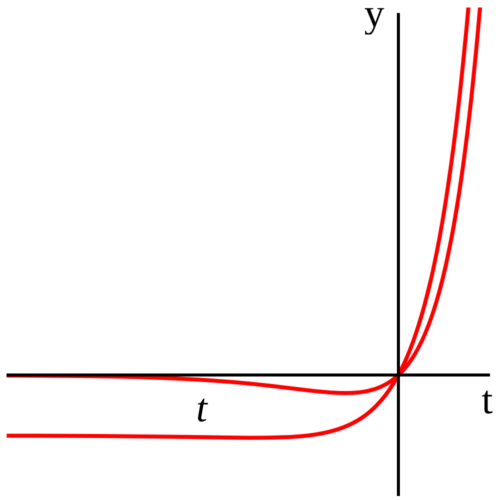
4. Find the particular solution of the non-homogeneous equation:

$$0 = C_1 \begin{bmatrix} 2 \\ 2 \end{bmatrix} + C_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}$$

$C_1 = \frac{1}{2}, C_2 = -\frac{1}{2}$

$$y = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + \begin{bmatrix} -1 \\ -\frac{1}{2} \end{bmatrix} e^{2t} + \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}$$

5. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -2 & -1 \\ 3 & 2 \end{bmatrix} y + \begin{bmatrix} -2 \\ -4 \end{bmatrix} e^{-2t}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$Det \begin{bmatrix} -2-\lambda & -1 \\ 3 & 2-\lambda \end{bmatrix} = 0$$

$$\lambda^2 - 1 = 0$$

$$\lambda = 1, v = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$\lambda = -1, v = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^t + C_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t}$$

2. Find a particular soultion of the non-homogeneous equation:

$$y = D\, e^{-2t}$$

$$-2\, D = \begin{bmatrix} -2 & -1 \\ 3 & 2 \end{bmatrix} D + \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 3 & 4 \end{bmatrix} D$$

$$D = \begin{bmatrix} 4 \\ -2 \end{bmatrix}, y_p = \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{-2t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^t + C_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{-2t}$$

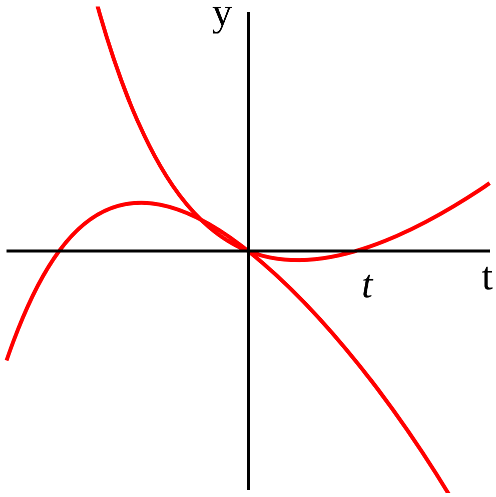
4. Find the particular solution of the non-homogeneous equation:

$$0 = C_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$C_1 = -1, C_2 = 5$$

$$y = \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{t} + \begin{bmatrix} -5 \\ 5 \end{bmatrix} e^{-t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{-2t}$$

5. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix} y + \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{2t}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$Det \begin{bmatrix} 4-\lambda & -1 \\ 5 & -2-\lambda \end{bmatrix} = 0$$

$$\lambda^2 - 2\lambda - 3 = 0$$

$$\lambda = 3, v = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\lambda = -1, v = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} -1 \\ -1 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} -1 \\ -5 \end{bmatrix} e^{-t}$$

2. Find a particular soultion of the non-homogeneous equation:

$$y = \mathrm{D}\, e^{2t}$$

$$2\,\mathrm{D} = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix} \mathrm{D} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 5 & -4 \end{bmatrix} \mathrm{D}$$

$$\mathrm{D} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}, y_p = \begin{bmatrix} -2 \\ -3 \end{bmatrix} e^{2t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 \begin{bmatrix} -1 \\ -1 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} -1 \\ -5 \end{bmatrix} e^{-t} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} e^{2t}$$

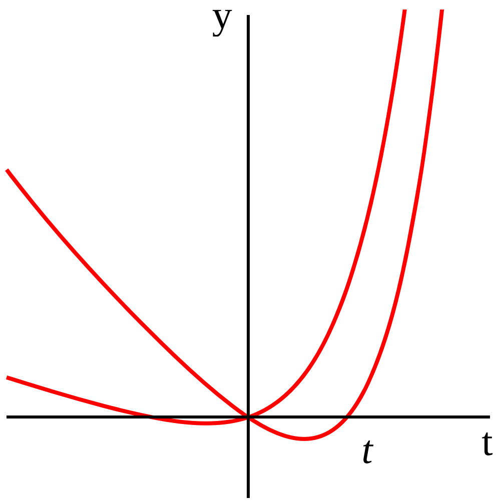
4. Find the particular solution of the non-homogeneous equation:

$$0 = C_1 \begin{bmatrix} -1 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ -5 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$C_1 = -\frac{7}{4}, C_2 = -\frac{1}{4}$$

$$y = \begin{bmatrix} \frac{7}{4} \\ \frac{7}{4} \end{bmatrix} e^{3t} + \begin{bmatrix} \frac{1}{4} \\ \frac{5}{4} \end{bmatrix} e^{-t} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} e^{2t}$$

5. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} y + \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$Det \begin{bmatrix} -1-\lambda & 2 \\ -1 & -4-\lambda \end{bmatrix} = 0$$

$$\lambda^2 + 5\lambda + 6 = 0$$

$$\lambda = -2,\ v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\lambda = -3,\ v = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-2t} + C_2 \begin{bmatrix} 2 \\ -2 \end{bmatrix} e^{-3t}$$

2. Find a particular soultion of the non-homogeneous equation:

$$y = D$$

$$0 = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} D + \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} D$$

$$D = \begin{bmatrix} -\frac{4}{3} \\ -\frac{1}{6} \end{bmatrix},\ y_p = \begin{bmatrix} -\frac{4}{3} \\ -\frac{1}{6} \end{bmatrix}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-2t} + C_2 \begin{bmatrix} 2 \\ -2 \end{bmatrix} e^{-3t} + \begin{bmatrix} -\frac{4}{3} \\ -\frac{1}{6} \end{bmatrix}$$

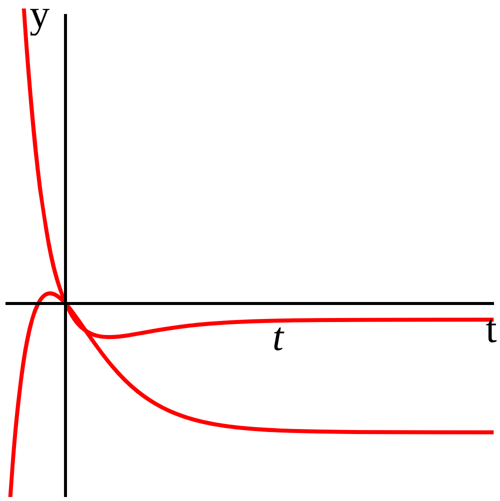
4. Find the particular solution of the non-homogeneous equation:

$$0 = C_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} 2 \\ -2 \end{bmatrix} + \begin{bmatrix} -\frac{4}{3} \\ -\frac{1}{6} \end{bmatrix}$$

$$C_1 = \frac{3}{2}, C_2 = -\frac{5}{6}$$

$$y = \begin{bmatrix} 3 \\ -\frac{3}{2} \end{bmatrix} e^{-2t} + \begin{bmatrix} -\frac{5}{3} \\ \frac{5}{3} \end{bmatrix} e^{-3t} + \begin{bmatrix} -\frac{4}{3} \\ -\frac{1}{6} \end{bmatrix}$$

5. Sketch the particular solution:



Laplace Transform Method: 1st-order, 1 variable

Problem:

$y' + 3y = 5e^{2t}$

$y(0) = 0$

1. Laplace Transform:

$$Ys + 3Y = \frac{5}{s-2}$$

2. Solve for Y:

$$Y = \frac{5}{(s+3)(s-2)}$$

3. Inverse Laplace Transform:

$$\frac{5}{(s+3)(s-2)} = \frac{A}{s+3} + \frac{B}{s-2}$$

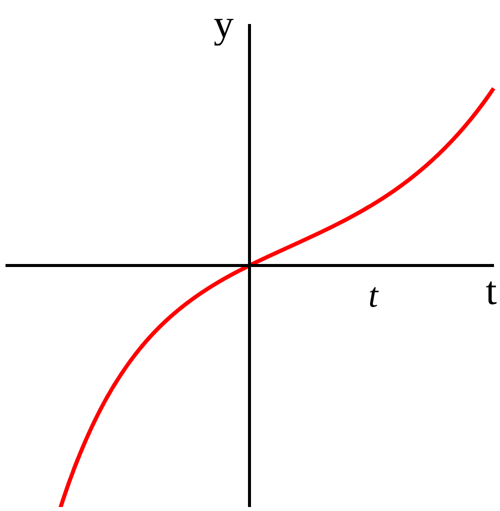
$5 = A(s-2) + B(s+3)$

$s = -3,\ 5 = -5A,\ A = -1$

$s = 2,\ 5 = 5B,\ B = 1$

$y = -e^{-3t} + e^{2t}$

4. Sketch the particular solution:



Problem:

$$y' + 4y = 5e^{-2t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 4Y = \frac{5}{s+2}$$

2. Solve for Y:

$$Y = \frac{5}{(s+4)\,(s+2)}$$

3. Inverse Laplace Transform:

$$\frac{5}{(s+4)\,(s+2)} = \frac{A}{s+4} + \frac{B}{s+2}$$

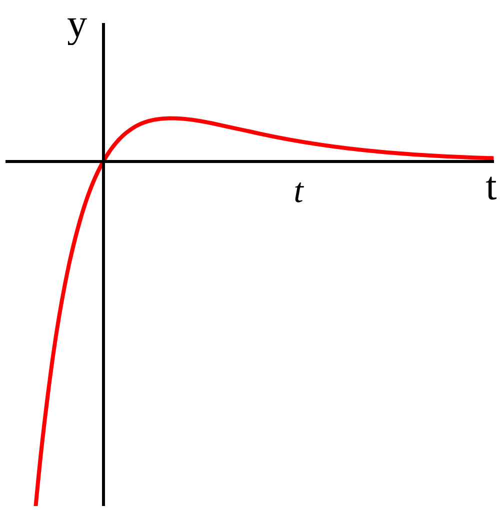
$$5 = A\,(s+2) + B\,(s+4)$$

$$s = -4,\ 5 = -2A,\ A = -\frac{5}{2}$$

$$s = -2,\ 5 = 2B,\ B = \frac{5}{2}$$

$$y = -\frac{5}{2}e^{-4t} + \frac{5}{2}e^{-2t}$$

4. Sketch the particular solution:



Problem:

$$y' - 4\,y = 5$$

$$y(0) = 0$$

1. Laplace Transform:

$$Y s - 4\,Y = \frac{5}{s}$$

2. Solve for Y:

$$Y = \frac{5}{(s-4)\,s}$$

3. Inverse Laplace Transform:

$$\frac{5}{(s-4)\,s} = \frac{A}{s-4} + \frac{B}{s}$$

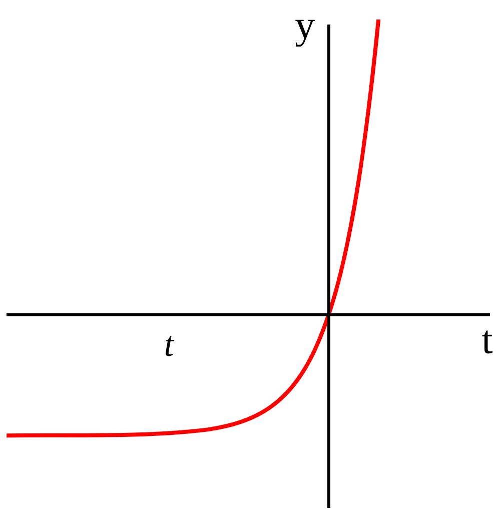
$$5 = A\,s + B\,(s-4)$$

$$s = 4,\ 5 = 4\,A,\ A = \frac{5}{4}$$

$$s = 0,\ 5 = -4\,B,\ B = -\frac{5}{4}$$

$$y = \frac{5}{4}\,e^{4t} - \frac{5}{4}$$

4. Sketch the particular solution:



Problem:

$$y' + 2\,y = -\mathrm{e}^{2\,t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$Y\,s + 2\,Y = -\frac{1}{s-2}$$

2. Solve for Y:

$$Y = -\frac{1}{(s+2)\,(s-2)}$$

3. Inverse Laplace Transform:

$$-\frac{1}{(s+2)\,(s-2)} = \frac{A}{s+2} + \frac{B}{s-2}$$

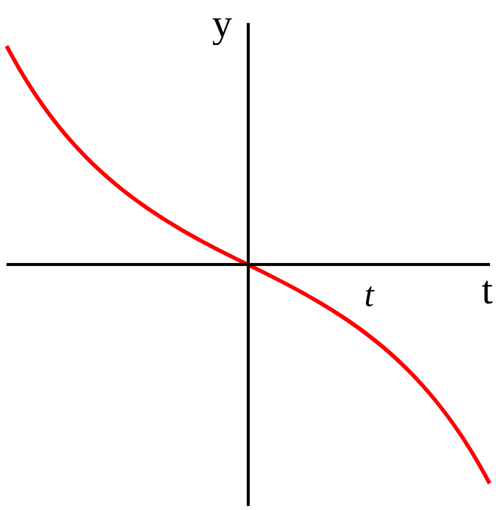
$$-1 = A\,(s-2) + B\,(s+2)$$

$$s = -2,\ -1 = -4\,A,\ A = \frac{1}{4}$$

$$s = 2,\ -1 = 4\,B,\ B = -\frac{1}{4}$$

$$y = \frac{1}{4}\,\mathrm{e}^{-2\,t} - \frac{1}{4}\,\mathrm{e}^{2\,t}$$

4. Sketch the particular solution:



Problem:

$$y' - 3y = 2$$

$$y(0) = 0$$

1. Laplace Transform:

$$Y s - 3 Y = \frac{2}{s}$$

2. Solve for Y:

$$Y = \frac{2}{(s-3)\,s}$$

3. Inverse Laplace Transform:

$$\frac{2}{(s-3)\,s} = \frac{A}{s-3} + \frac{B}{s}$$

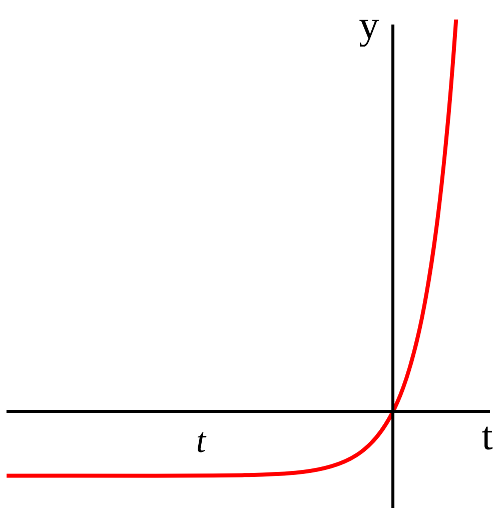
$$2 = A s + B\,(s-3)$$

$$s = 3,\ 2 = 3A,\ A = \frac{2}{3}$$

$$s = 0,\ 2 = -3B,\ B = -\frac{2}{3}$$

$$y = \frac{2}{3}\,e^{3t} - \frac{2}{3}$$

4. Sketch the particular solution:



Problem:

$$y' + 4y = 3e^{-2t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 4Y = \frac{3}{s+2}$$

2. Solve for Y:

$$Y = \frac{3}{(s+4)(s+2)}$$

3. Inverse Laplace Transform:

$$\frac{3}{(s+4)(s+2)} = \frac{A}{s+4} + \frac{B}{s+2}$$

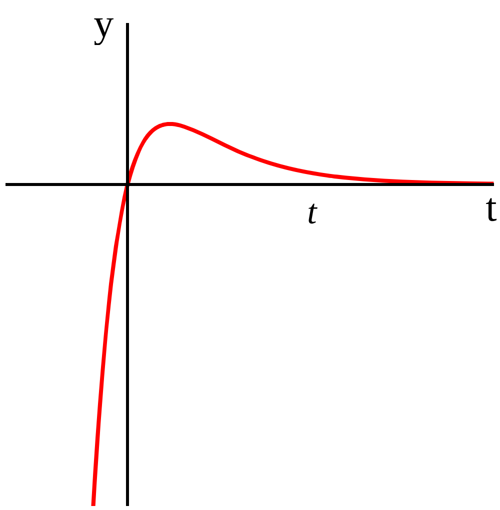
$$3 = A(s+2) + B(s+4)$$

$$s = -4,\ 3 = -2A,\ A = -\frac{3}{2}$$

$$s = -2,\ 3 = 2B,\ B = \frac{3}{2}$$

$$y = -\frac{3}{2}e^{-4t} + \frac{3}{2}e^{-2t}$$

4. Sketch the particular solution:



Laplace Transform Method: 2nd-order, 1 variable

Problem:

$$y'' + 3y' + 2y = 4e^{2t}$$

$$y(0) = 0, y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 + 3Ys + 2Y = \frac{4}{s-2}$$

2. Solve for Y:

$$Y = \frac{4}{(s^2 + 3s + 2)(s-2)}$$

3. Inverse Laplace Transform:

$$\frac{4}{(s+1)(s+2)(s-2)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s-2}$$

$$4 = A(s+2)(s-2) + B(s+1)(s-2) + C(s+1)(s+2)$$

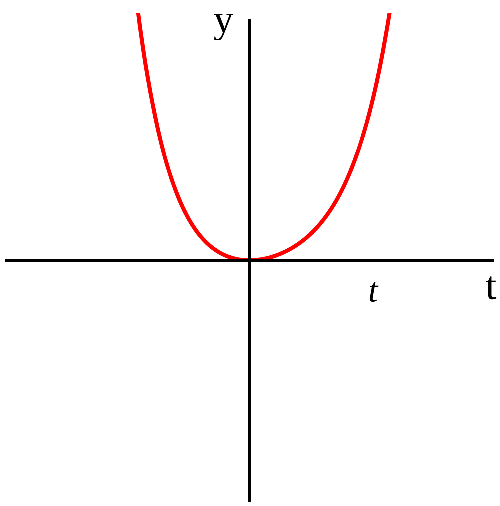
$$s = -1, 4 = -3A, A = -\frac{4}{3}$$

$$s = -2, 4 = 4B, B = 1$$

$$s = 2, 4 = 12C, C = \frac{1}{3}$$

$$y = -\frac{4}{3}e^{-t} + e^{-2t} + \frac{1}{3}e^{2t}$$

4. Sketch the particular solution:



Problem:

$$y'' - 3\,y' + 2\,y = 4\,e^{5\,t}$$

$$y(0) = 0,\ y'(0) = 0$$

1. Laplace Transform:

$$Y s^2 - 3\,Y s + 2\,Y = \frac{4}{s-5}$$

2. Solve for Y:

$$Y = \frac{4}{\left(s^2 - 3\,s + 2\right)\,(s-5)}$$

3. Inverse Laplace Transform:

$$\frac{4}{(s-2)\,(s-1)\,(s-5)} = \frac{A}{s-2} + \frac{B}{s-1} + \frac{C}{s-5}$$

$$4 = A\,(s-1)\,(s-5) + B\,(s-2)\,(s-5) + C\,(s-2)\,(s-1)$$

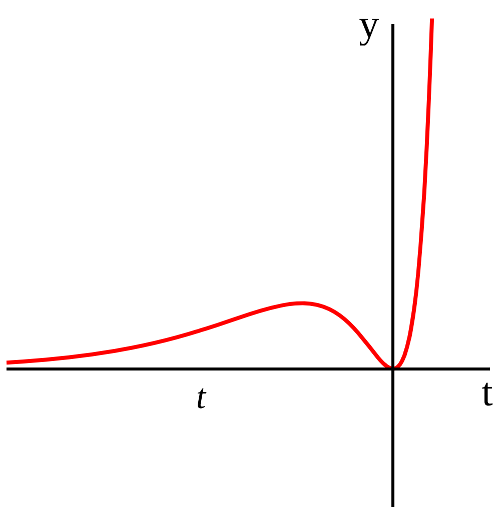
$$s = 2,\ 4 = -3\,A,\ A = -\frac{4}{3}$$

$$s = 1,\ 4 = 4\,B,\ B = 1$$

$$s = 5,\ 4 = 12\,C,\ C = \frac{1}{3}$$

$$y = -\frac{4}{3}\,e^{2\,t} + e^{t} + \frac{1}{3}\,e^{5\,t}$$

4. Sketch the particular solution:



Problem:

$$y'' + 5y' + 4y = -4$$

$$y(0) = 0,\ y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 + 5Ys + 4Y = -\frac{4}{s}$$

2. Solve for Y:

$$Y = -\frac{4}{(s^2 + 5s + 4)\, s}$$

3. Inverse Laplace Transform:

$$-\frac{4}{(s+1)\,(s+4)\, s} = \frac{A}{s+1} + \frac{B}{s+4} + \frac{C}{s}$$

$$-4 = A\,(s+4)\,s + B\,(s+1)\,s + C\,(s+1)\,(s+4)$$

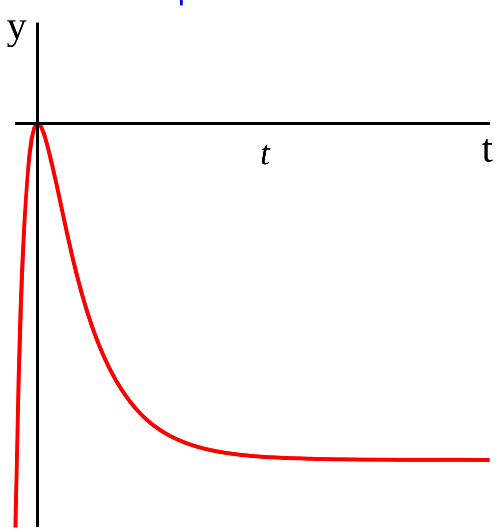
$$s = -1,\ -4 = -3A,\ A = \frac{4}{3}$$

$$s = -4,\ -4 = 12B,\ B = -\frac{1}{3}$$

$$s = 0,\ -4 = 4C,\ C = -1$$

$$y = \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t} - 1$$

4. Sketch the particular solution:



Problem:

$$y'' + 5y' + 6y = -e^{2t}$$

$$y(0) = 0,\ y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 + 5Ys + 6Y = -\frac{1}{s-2}$$

2. Solve for Y:

$$Y = -\frac{1}{(s^2+5s+6)(s-2)}$$

3. Inverse Laplace Transform:

$$-\frac{1}{(s+2)(s+3)(s-2)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s-2}$$

$$-1 = A(s+3)(s-2) + B(s+2)(s-2) + C(s+2)(s+3)$$

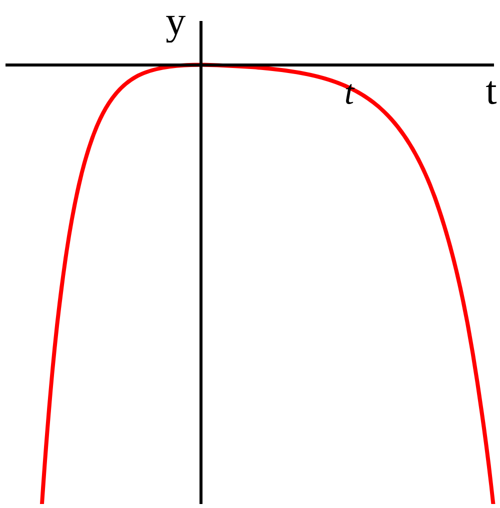
$$s = -2,\ -1 = -4A,\ A = \frac{1}{4}$$

$$s = -3,\ -1 = 5B,\ B = -\frac{1}{5}$$

$$s = 2,\ -1 = 20C,\ C = -\frac{1}{20}$$

$$y = \frac{1}{4}e^{-2t} - \frac{1}{5}e^{-3t} - \frac{1}{20}e^{2t}$$

4. Sketch the particular solution:



Problem:

$$y'' + 7\,y' + 10\,y = -10$$

$$y(0) = 0,\ y'(0) = 0$$

1. Laplace Transform:

$$Y s^2 + 7\,Y s + 10\,Y = -\frac{10}{s}$$

2. Solve for Y:

$$Y = -\frac{10}{\left(s^2 + 7\,s + 10\right)\,s}$$

3. Inverse Laplace Transform:

$$-\frac{10}{(s+2)\,(s+5)\,s} = \frac{A}{s+2} + \frac{B}{s+5} + \frac{C}{s}$$

$$-10 = A\,(s+5)\,s + B\,(s+2)\,s + C\,(s+2)\,(s+5)$$

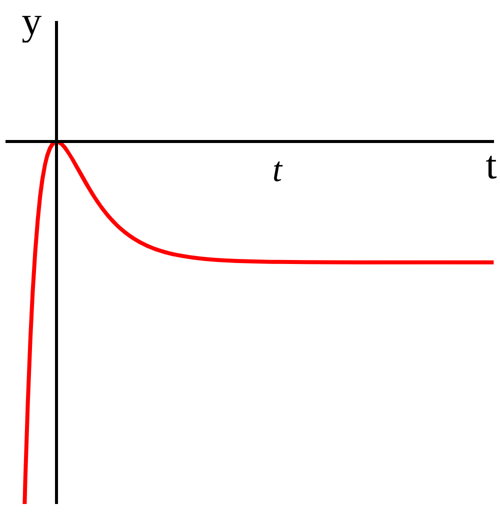
$$s = -2,\ -10 = -6\,A,\ A = \frac{5}{3}$$

$$s = -5,\ -10 = 15\,B,\ B = -\frac{2}{3}$$

$$s = 0,\ -10 = 10\,C,\ C = -1$$

$$y = \frac{5}{3}\,e^{-2\,t} - \frac{2}{3}\,e^{-5\,t} - 1$$

4. Sketch the particular solution:



Problem:

$$y'' + 3y' + 2y = -4e^{-3t}$$

$$y(0) = 0,\ y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 + 3Ys + 2Y = -\frac{4}{s+3}$$

2. Solve for Y:

$$Y = -\frac{4}{(s^2 + 3s + 2)(s+3)}$$

3. Inverse Laplace Transform:

$$-\frac{4}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$-4 = A(s+2)(s+3) + B(s+1)(s+3) + C(s+1)(s+2)$$

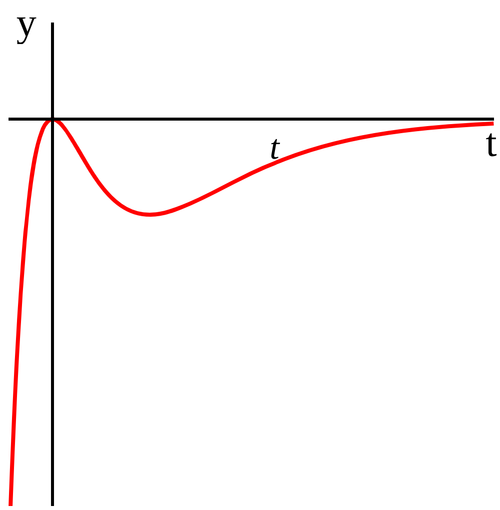
$$s = -1,\ -4 = 2A,\ A = -2$$

$$s = -2,\ -4 = -B,\ B = 4$$

$$s = -3,\ -4 = 2C,\ C = -2$$

$$y = -2e^{-t} + 4e^{-2t} - 2e^{-3t}$$

4. Sketch the particular solution:



Laplace Transform Method: 1st-order, 2 variables

Problem:

$$y' = \begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix} y + \begin{bmatrix} 2 \\ 4 \end{bmatrix} e^{2t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$s\,Y = \begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix} Y + \frac{\begin{bmatrix} 2 \\ 4 \end{bmatrix}}{s-2}$$

2. Solve for Y:

$$\begin{bmatrix} s-2 & -1 \\ 3 & s+2 \end{bmatrix} Y = \frac{\begin{bmatrix} 2 \\ 4 \end{bmatrix}}{s-2}$$

$$Y = \frac{\begin{bmatrix} 2\,s+8 \\ -14+4\,s \end{bmatrix}}{\left(s^2-1\right)\,(s-2)}$$

3. Inverse Laplace Transform:

$$\frac{\begin{bmatrix} 2\,s+8 \\ -14+4\,s \end{bmatrix}}{(s-1)\,(s+1)\,(s-2)} = \frac{A}{s-1} + \frac{B}{s+1} + \frac{C}{s-2}$$

$$\begin{bmatrix} 2\,s+8 \\ -14+4\,s \end{bmatrix} = A\,(s+1)\,(s-2) + B\,(s-1)\,(s-2) + C\,(s-1)\,(s+1)$$

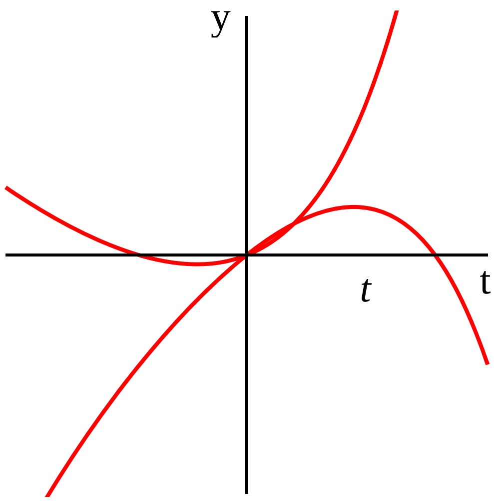
$$s = 1, \begin{bmatrix} 10 \\ -10 \end{bmatrix} = -2\,A,\ A = \begin{bmatrix} -5 \\ 5 \end{bmatrix}$$

$$s = -1, \begin{bmatrix} 6 \\ -18 \end{bmatrix} = 6\,B,\ B = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$s = 2, \begin{bmatrix} 12 \\ -6 \end{bmatrix} = 3\,C,\ C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$y = \begin{bmatrix} -5 \\ 5 \end{bmatrix} e^{t} + \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{2t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 4 & 1 \\ -5 & -2 \end{bmatrix} y + \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$s\,Y = \begin{bmatrix} 4 & 1 \\ -5 & -2 \end{bmatrix} Y + \frac{\begin{bmatrix} -1 \\ 3 \end{bmatrix}}{s-2}$$

2. Solve for Y:

$$\begin{bmatrix} s-4 & -1 \\ 5 & s+2 \end{bmatrix} Y = \frac{\begin{bmatrix} -1 \\ 3 \end{bmatrix}}{s-2}$$

$$Y = \frac{\begin{bmatrix} -s+1 \\ -7+3\,s \end{bmatrix}}{\left(s^2-2\,s-3\right)\,(s-2)}$$

3. Inverse Laplace Transform:

$$\frac{\begin{bmatrix} -s+1 \\ -7+3\,s \end{bmatrix}}{(s+1)\,(s-3)\,(s-2)} = \frac{A}{s-3} + \frac{B}{s+1} + \frac{C}{s-2}$$

$$\begin{bmatrix} -s+1 \\ -7+3\,s \end{bmatrix} = A\,(s+1)\,(s-2) + B\,(s-3)\,(s-2) + C\,(s-3)\,(s+1)$$

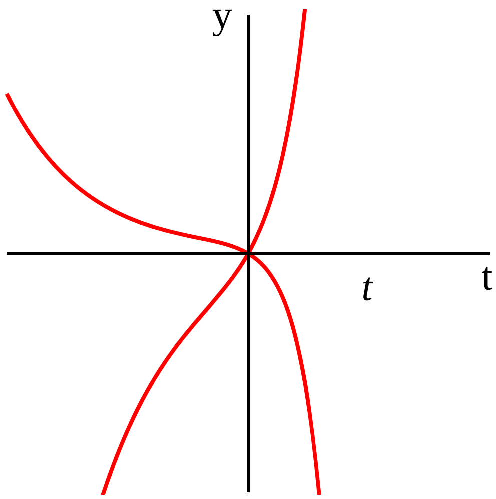
$$s = 3, \begin{bmatrix} -2 \\ 2 \end{bmatrix} = 4\,A,\ A = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$s = -1, \begin{bmatrix} 2 \\ -10 \end{bmatrix} = 12\,B,\ B = \begin{bmatrix} \frac{1}{6} \\ -\frac{5}{6} \end{bmatrix}$$

$$s = 2, \begin{bmatrix} -1 \\ -1 \end{bmatrix} = -3\,C,\ C = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$y = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix} e^{3t} + \begin{bmatrix} \frac{1}{6} \\ -\frac{5}{6} \end{bmatrix} e^{-t} + \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} e^{2t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} y + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$y(0) = 0$$

1. Laplace Transform:

$$s\,Y = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} Y + \frac{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}{s}$$

2. Solve for Y:

$$\begin{bmatrix} s-1 & -2 \\ 1 & s-4 \end{bmatrix} Y = \frac{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}{s}$$

$$Y = \frac{\begin{bmatrix} s \\ -3+2\,s \end{bmatrix}}{(s^2-5\,s+6)\,s}$$

3. Inverse Laplace Transform:

$$\frac{\begin{bmatrix} s \\ -3+2\,s \end{bmatrix}}{(s-2)\,(s-3)\,s} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s}$$

$$\begin{bmatrix} s \\ -3+2\,s \end{bmatrix} = A\,(s-2)\,s + B\,(s-3)\,s + C\,(s-3)\,(s-2)$$

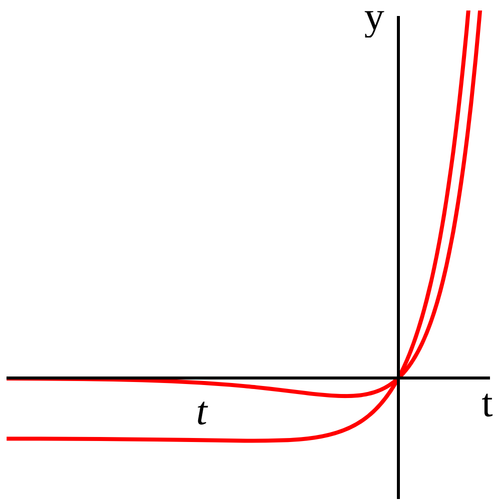
$$s = 3, \begin{bmatrix} 3 \\ 3 \end{bmatrix} = 3\,A,\ A = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$s = 2, \begin{bmatrix} 2 \\ 1 \end{bmatrix} = -2\,B,\ B = \begin{bmatrix} -1 \\ -\frac{1}{2} \end{bmatrix}$$

$$s = 0, \begin{bmatrix} 0 \\ -3 \end{bmatrix} = 6\,C,\ C = \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}$$

$$y = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + \begin{bmatrix} -1 \\ -\frac{1}{2} \end{bmatrix} e^{2t} + \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -2 & -1 \\ 3 & 2 \end{bmatrix} y + \begin{bmatrix} -2 \\ -4 \end{bmatrix} e^{-2t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$s\,Y = \begin{bmatrix} -2 & -1 \\ 3 & 2 \end{bmatrix} Y + \frac{\begin{bmatrix} -2 \\ -4 \end{bmatrix}}{s+2}$$

2. Solve for Y:

$$\begin{bmatrix} s+2 & 1 \\ -3 & s-2 \end{bmatrix} Y = \frac{\begin{bmatrix} -2 \\ -4 \end{bmatrix}}{s+2}$$

$$Y = \frac{\begin{bmatrix} -2\,s+8 \\ -14-4\,s \end{bmatrix}}{\left(s^2-1\right)\,(s+2)}$$

3. Inverse Laplace Transform:

$$\frac{\begin{bmatrix} -2\,s+8 \\ -14-4\,s \end{bmatrix}}{(s-1)\,(s+1)\,(s+2)} = \frac{A}{s-1} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$\begin{bmatrix} -2\,s+8 \\ -14-4\,s \end{bmatrix} = A\,(s+1)\,(s+2) + B\,(s-1)\,(s+2) + C\,(s-1)\,(s+1)$$

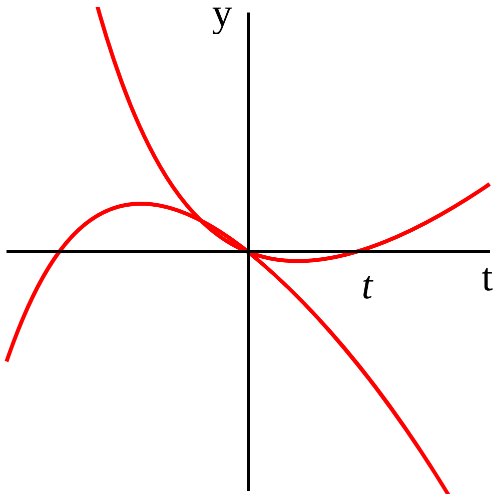
$$s = 1, \begin{bmatrix} 6 \\ -18 \end{bmatrix} = 6\,A, A = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$s = -1, \begin{bmatrix} 10 \\ -10 \end{bmatrix} = -2\,B, B = \begin{bmatrix} -5 \\ 5 \end{bmatrix}$$

$$s = -2, \begin{bmatrix} 12 \\ -6 \end{bmatrix} = 3\,C, C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{t} + \begin{bmatrix} -5 \\ 5 \end{bmatrix} e^{-t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{-2t}$$

4. Sketch the particular solution:


y
t
t

Problem:

$$y' = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix} y + \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{2t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$s\,Y = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix} Y + \frac{\begin{bmatrix} 1 \\ -2 \end{bmatrix}}{s-2}$$

2. Solve for Y:

$$\begin{bmatrix} s-4 & 1 \\ -5 & s+2 \end{bmatrix} Y = \frac{\begin{bmatrix} 1 \\ -2 \end{bmatrix}}{s-2}$$

$$Y = \frac{\begin{bmatrix} s+4 \\ 13-2\,s \end{bmatrix}}{\left(s^2-2\,s-3\right)\,(s-2)}$$

3. Inverse Laplace Transform:

$$\frac{\begin{bmatrix} s+4 \\ 13-2\,s \end{bmatrix}}{(s+1)\,(s-3)\,(s-2)} = \frac{A}{s-3} + \frac{B}{s+1} + \frac{C}{s-2}$$

$$\begin{bmatrix} s+4 \\ 13-2\,s \end{bmatrix} = A\,(s+1)\,(s-2) + B\,(s-3)\,(s-2) + C\,(s-3)\,(s+1)$$

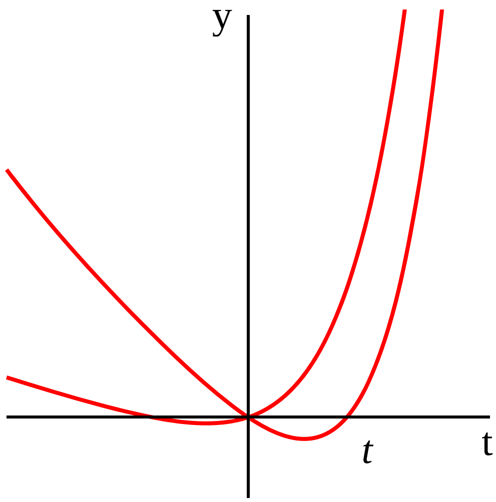
$$s = 3, \begin{bmatrix} 7 \\ 7 \end{bmatrix} = 4\,A, A = \begin{bmatrix} \frac{7}{4} \\ \frac{7}{4} \end{bmatrix}$$

$$s = -1, \begin{bmatrix} 3 \\ 15 \end{bmatrix} = 12\,B, B = \begin{bmatrix} \frac{1}{4} \\ \frac{5}{4} \end{bmatrix}$$

$$s = 2, \begin{bmatrix} 6 \\ 9 \end{bmatrix} = -3\,C, C = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$y = \begin{bmatrix} \frac{7}{4} \\ \frac{7}{4} \end{bmatrix} e^{3t} + \begin{bmatrix} \frac{1}{4} \\ \frac{5}{4} \end{bmatrix} e^{-t} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} e^{2t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} y + \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$y(0) = 0$$

1. Laplace Transform:

$$s\,Y = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} Y + \frac{\begin{bmatrix} -1 \\ -2 \end{bmatrix}}{s}$$

2. Solve for Y:

$$\begin{bmatrix} s+1 & -2 \\ 1 & s+4 \end{bmatrix} Y = \frac{\begin{bmatrix} -1 \\ -2 \end{bmatrix}}{s}$$

$$Y = \frac{\begin{bmatrix} -s-8 \\ -1-2\,s \end{bmatrix}}{\left(s^2+5\,s+6\right)\,s}$$

3. Inverse Laplace Transform:

$$\frac{\begin{bmatrix} -s-8 \\ -1-2\,s \end{bmatrix}}{(s+3)\,(s+2)\,s} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s}$$

$$\begin{bmatrix} -s-8 \\ -1-2\,s \end{bmatrix} = A\,(s+3)\,s + B\,(s+2)\,s + C\,(s+2)\,(s+3)$$

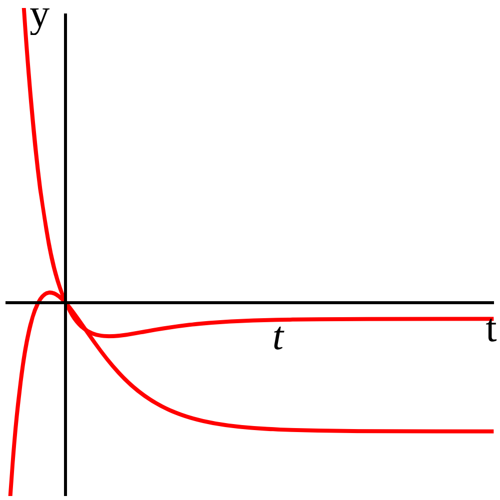
$$s = -2, \begin{bmatrix} -6 \\ 3 \end{bmatrix} = -2\,A, A = \begin{bmatrix} 3 \\ -\frac{3}{2} \end{bmatrix}$$

$$s = -3, \begin{bmatrix} -5 \\ 5 \end{bmatrix} = 3\,B, B = \begin{bmatrix} -\frac{5}{3} \\ \frac{5}{3} \end{bmatrix}$$

$$s = 0, \begin{bmatrix} -8 \\ -1 \end{bmatrix} = 6\,C, C = \begin{bmatrix} -\frac{4}{3} \\ -\frac{1}{6} \end{bmatrix}$$

$$y = \begin{bmatrix} 3 \\ -\frac{3}{2} \end{bmatrix} e^{-2t} + \begin{bmatrix} -\frac{5}{3} \\ \frac{5}{3} \end{bmatrix} e^{-3t} + \begin{bmatrix} -\frac{4}{3} \\ -\frac{1}{6} \end{bmatrix}$$

4. Sketch the particular solution:



Laplace Transform Method: 1st-order, 1 variable, Trigonometric function

Problem:

$$y' + 2y = 3e^{-t}\cos(10t)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 2Y = \frac{3s+3}{(s+1)^2 + 100}$$

2. Solve for Y:

$$Y = \frac{3s+3}{(s+2)\left((s+1)^2+100\right)}$$

3. Inverse Laplace Transform:

$$\frac{3s+3}{(s+2)\left((s+1)^2+100\right)} = \frac{A}{s+2} + \frac{B(s+1)+10C}{(s+1)^2+100}$$

$$3s+3 = A\left((s+1)^2+100\right) + (B(s+1)+10C)(s+2)$$

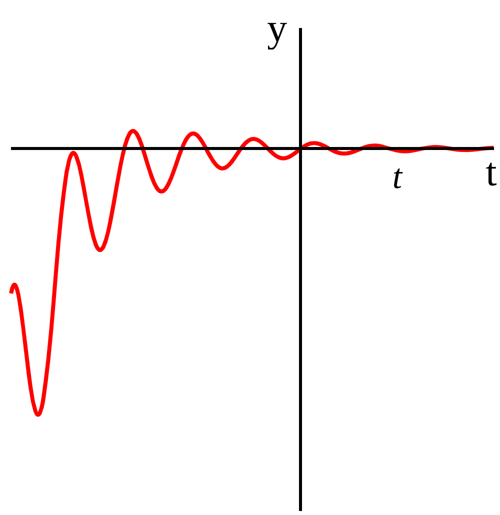
$$s = -2,\ -3 = 101A,\ A = -\frac{3}{101}$$

$$s = -1,\ 0 = 100A + 10C,\ C = \frac{30}{101}$$

$$s = 0,\ 3 = 101A + 2B + 20C,\ B = \frac{3}{101}$$

$$y = -\frac{3}{101}e^{-2t} + e^{-t}\left(\frac{3}{101}\cos(10t) + \frac{30}{101}\sin(10t)\right)$$

4. Sketch the particular solution:



Problem:

$$y' - 2y = 4e^{t}\sin(6t)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys - 2Y = \frac{24}{(s-1)^2 + 36}$$

2. Solve for Y:

$$Y = \frac{24}{(s-2)\left((s-1)^2 + 36\right)}$$

3. Inverse Laplace Transform:

$$\frac{24}{(s-2)\left((s-1)^2 + 36\right)} = \frac{A}{s-2} + \frac{B(s-1) + 6C}{(s-1)^2 + 36}$$

$$24 = A\left((s-1)^2 + 36\right) + (B(s-1) + 6C)(s-2)$$

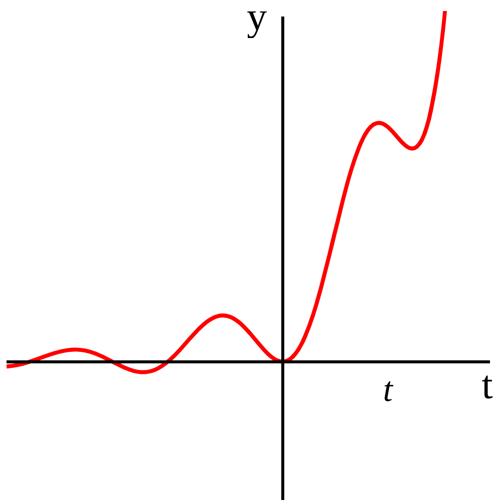
$$s = 2,\ 24 = 37A,\ A = \frac{24}{37}$$

$$s = 1,\ 24 = 36A - 6C,\ C = -\frac{4}{37}$$

$$s = 0,\ 24 = 37A + 2B - 12C,\ B = -\frac{24}{37}$$

$$y = \frac{24}{37}e^{2t} + e^{t}\left(-\frac{24}{37}\cos(6t) - \frac{4}{37}\sin(6t)\right)$$

4. Sketch the particular solution:



Problem:

$$y' + y = e^{2t}\,(2\cos(10\,t) - 3\sin(10\,t))$$

$$y(0) = 0$$

1. Laplace Transform:

$$Y s + Y = \frac{2\,s - 34}{(s-2)^2 + 100}$$

2. Solve for Y:

$$Y = \frac{2\,s - 34}{(s+1)\left((s-2)^2 + 100\right)}$$

3. Inverse Laplace Transform:

$$\frac{2\,s - 34}{(s+1)\left((s-2)^2 + 100\right)} = \frac{A}{s+1} + \frac{B\,(s-2) + 10\,C}{(s-2)^2 + 100}$$

$$2\,s - 34 = A\left((s-2)^2 + 100\right) + (B\,(s-2) + 10\,C)\,(s+1)$$

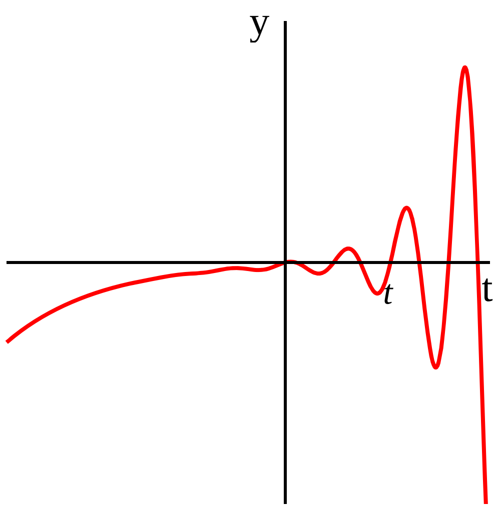
$$s = -1,\ -36 = 109\,A,\ A = -\frac{36}{109}$$

$$s = 2,\ -30 = 100\,A + 30\,C,\ C = \frac{11}{109}$$

$$s = 0,\ -34 = 104\,A - 2\,B + 10\,C,\ B = \frac{36}{109}$$

$$y = -\frac{36}{109}\,e^{-t} + e^{2t}\left(\frac{36}{109}\cos(10\,t) + \frac{11}{109}\sin(10\,t)\right)$$

4. Sketch the particular solution:



Problem:

$$y' + y = -2\cos(2t)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + Y = -\frac{2s}{s^2+4}$$

2. Solve for Y:

$$Y = -\frac{2s}{(s+1)\left(s^2+4\right)}$$

3. Inverse Laplace Transform:

$$-\frac{2s}{(s+1)\left(s^2+4\right)} = \frac{A}{s+1} + \frac{Bs+2C}{s^2+4}$$

$$-2s = A\left(s^2+4\right) + (Bs+2C)(s+1)$$

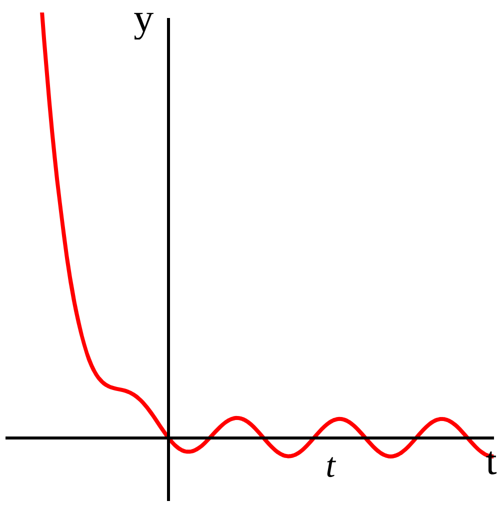
$$s = -1,\ 2 = 5A,\ A = \frac{2}{5}$$

$$s = 0,\ 0 = 4A + 2C,\ C = -\frac{4}{5}$$

$$s = 1,\ -2 = 5A + 2B + 4C,\ B = -\frac{2}{5}$$

$$y = \frac{2}{5}e^{-t} - \frac{2}{5}\cos(2t) - \frac{4}{5}\sin(2t)$$

4. Sketch the particular solution:



Problem:

$$y' - 2y = 4\sin(10t)\,e^{t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys - 2Y = \frac{40}{(s-1)^2 + 100}$$

2. Solve for Y:

$$Y = \frac{40}{(s-2)\left((s-1)^2 + 100\right)}$$

3. Inverse Laplace Transform:

$$\frac{40}{(s-2)\left((s-1)^2 + 100\right)} = \frac{A}{s-2} + \frac{B(s-1) + 10C}{(s-1)^2 + 100}$$

$$40 = A\left((s-1)^2 + 100\right) + (B(s-1) + 10C)(s-2)$$

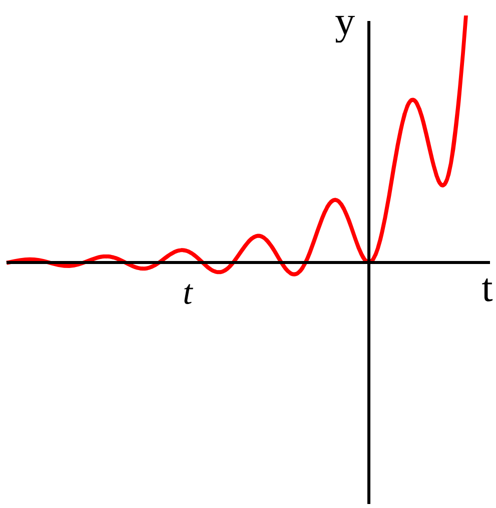
$$s = 2,\ 40 = 101A,\ A = \frac{40}{101}$$

$$s = 1,\ 40 = 100A - 10C,\ C = -\frac{4}{101}$$

$$s = 0,\ 40 = 101A + 2B - 20C,\ B = -\frac{40}{101}$$

$$y = \frac{40}{101}e^{2t} + e^{t}\left(-\frac{40}{101}\cos(10t) - \frac{4}{101}\sin(10t)\right)$$

4. Sketch the particular solution:



Problem:

$$y' + y = e^{-2t}(\cos(10t) - \sin(10t))$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + Y = \frac{s-8}{(s+2)^2+100}$$

2. Solve for Y:

$$Y = \frac{s-8}{(s+1)\left((s+2)^2+100\right)}$$

3. Inverse Laplace Transform:

$$\frac{s-8}{(s+1)\left((s+2)^2+100\right)} = \frac{A}{s+1} + \frac{B(s+2)+10C}{(s+2)^2+100}$$

$$s-8 = A\left((s+2)^2+100\right) + (B(s+2)+10C)(s+1)$$

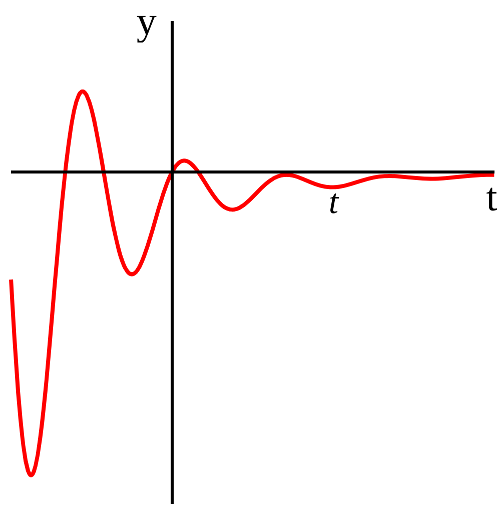
$$s = -1,\ -9 = 101A,\ A = -\frac{9}{101}$$

$$s = -2,\ -10 = 100A - 10C,\ C = \frac{11}{101}$$

$$s = 0,\ -8 = 104A + 2B + 10C,\ B = \frac{9}{101}$$

$$y = -\frac{9}{101}e^{-t} + e^{-2t}\left(\frac{9}{101}\cos(10t) + \frac{11}{101}\sin(10t)\right)$$

4. Sketch the particular solution:



Laplace Transform Method: 2nd-order, 1 variable, Exponential function, Non-real eigenvalues

Problem:

$$y'' + 2y' + 26y = 2e^{-5t}$$

$$y(0) = 0, y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 + 2Ys + 26Y = \frac{2}{s+5}$$

2. Solve for Y:

$$Y = \frac{2}{(s^2 + 2s + 26)(s+5)}$$

3. Inverse Laplace Transform:

$$\alpha = -1, \beta = 5$$

$$\frac{2}{((s+1)^2 + 25)(s+5)} = \frac{A(s+1) + 5B}{(s+1)^2 + 25} + \frac{C}{s+5}$$

$$2 = (A(s+1) + 5B)(s+5) + C((s+1)^2 + 25)$$

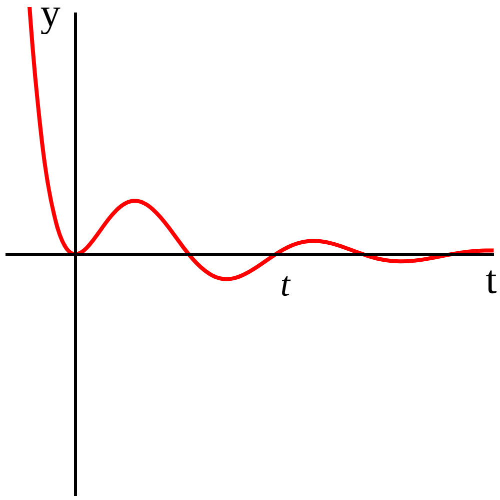
$$s = -5, 2 = 41C, C = \frac{2}{41}$$

$$s = -1, 2 = 20B + 25C, B = \frac{8}{205}$$

$$s = 0, 2 = 5A + 25B + 26C, A = -\frac{2}{41}$$

$$y = e^{-t}\left(-\frac{2}{41}\cos(5t) + \frac{8}{205}\sin(5t)\right) + \frac{2}{41}e^{-5t}$$

4. Sketch the particular solution:


y
t
t

Problem:

$$y'' - 4y' + 53y = -e^{-t}$$

$$y(0) = 0,\ y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 - 4Ys + 53Y = -\frac{1}{s+1}$$

2. Solve for Y:

$$Y = -\frac{1}{(s^2 - 4s + 53)(s+1)}$$

3. Inverse Laplace Transform:

$$\alpha = 2,\ \beta = 7$$

$$-\frac{1}{((s-2)^2 + 49)(s+1)} = \frac{A(s-2) + 7B}{(s-2)^2 + 49} + \frac{C}{s+1}$$

$$-1 = (A(s-2) + 7B)(s+1) + C((s-2)^2 + 49)$$

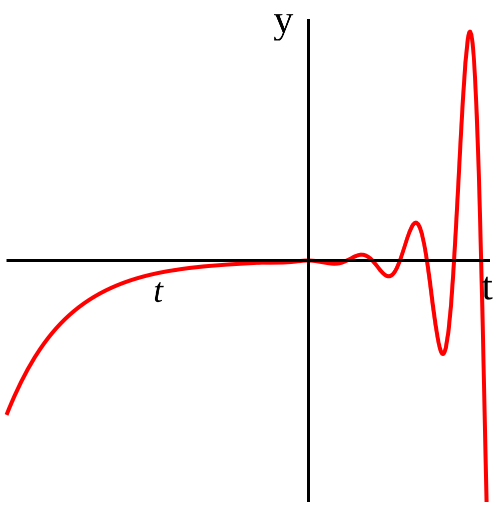
$$s = -1,\ -1 = 58C,\ C = -\frac{1}{58}$$

$$s = 2,\ -1 = 21B + 49C,\ B = -\frac{3}{406}$$

$$s = 0,\ -1 = -2A + 7B + 53C,\ A = \frac{1}{58}$$

$$y = e^{2t}\left(\frac{1}{58}\cos(7t) - \frac{3}{406}\sin(7t)\right) - \frac{1}{58}e^{-t}$$

4. Sketch the particular solution:



Problem:

$$y'' + 2\,y' + 26\,y = 2$$

$$y(0) = 0,\; y'(0) = 0$$

1. Laplace Transform:

$$Y s^2 + 2\,Y s + 26\,Y = \frac{2}{s}$$

2. Solve for Y:

$$Y = \frac{2}{\left(s^2 + 2\,s + 26\right)\,s}$$

3. Inverse Laplace Transform:

$$\alpha = -1,\; \beta = 5$$

$$\frac{2}{\left((s+1)^2 + 25\right)\,s} = \frac{A\,(s+1) + 5\,B}{(s+1)^2 + 25} + \frac{C}{s}$$

$$2 = (A\,(s+1) + 5\,B)\,s + C\,\left((s+1)^2 + 25\right)$$

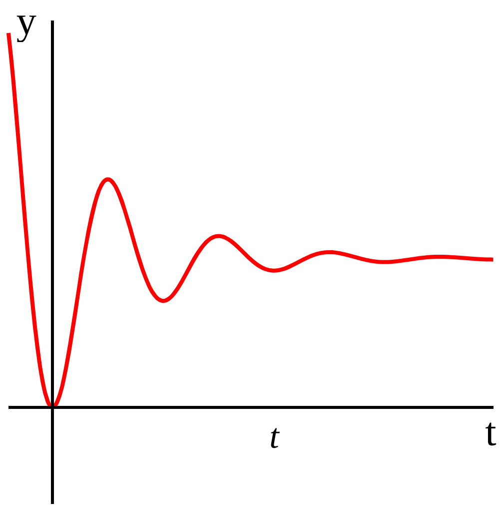
$$s = 0,\; 2 = 26\,C,\; C = \frac{1}{13}$$

$$s = -1,\; 2 = -5\,B + 25\,C,\; B = -\frac{1}{65}$$

$$s = 1,\; 2 = 2\,A + 5\,B + 29\,C,\; A = -\frac{1}{13}$$

$$y = e^{-t}\left(-\frac{1}{13}\cos(5\,t) - \frac{1}{65}\sin(5\,t)\right) + \frac{1}{13}$$

4. Sketch the particular solution:



Problem:

$$y'' - 2\,y' + 26\,y = 2\,e^{2\,t}$$

$$y(0) = 0,\ y'(0) = 0$$

1. Laplace Transform:

$$Y s^2 - 2\,Y s + 26\,Y = \frac{2}{s-2}$$

2. Solve for Y:

$$Y = \frac{2}{\left(s^2 - 2\,s + 26\right)\,(s-2)}$$

3. Inverse Laplace Transform:

$$\alpha = 1,\ \beta = 5$$

$$\frac{2}{\left((s-1)^2 + 25\right)\,(s-2)} = \frac{A\,(s-1) + 5\,B}{(s-1)^2 + 25} + \frac{C}{s-2}$$

$$2 = (A\,(s-1) + 5\,B)\,(s-2) + C\left((s-1)^2 + 25\right)$$

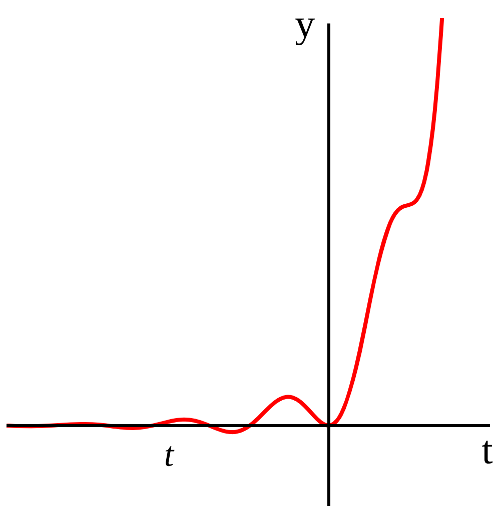
$$s = 2,\ 2 = 26\,C,\ C = \frac{1}{13}$$

$$s = 1,\ 2 = -5\,B + 25\,C,\ B = -\frac{1}{65}$$

$$s = 0,\ 2 = 2\,A - 10\,B + 26\,C,\ A = -\frac{1}{13}$$

$$y = e^{t}\left(-\frac{1}{13}\cos(5\,t) - \frac{1}{65}\sin(5\,t)\right) + \frac{1}{13}\,e^{2\,t}$$

4. Sketch the particular solution:



Problem:

$$y'' - 2y' + 26y = 2$$

$$y(0) = 0,\ y'(0) = 0$$

1. Laplace Transform:

$$Y s^2 - 2 Y s + 26 Y = \frac{2}{s}$$

2. Solve for Y:

$$Y = \frac{2}{\left(s^2 - 2s + 26\right) s}$$

3. Inverse Laplace Transform:

$$\alpha = 1,\ \beta = 5$$

$$\frac{2}{\left((s-1)^2 + 25\right) s} = \frac{A\,(s-1) + 5B}{(s-1)^2 + 25} + \frac{C}{s}$$

$$2 = (A\,(s-1) + 5B)\,s + C\left((s-1)^2 + 25\right)$$

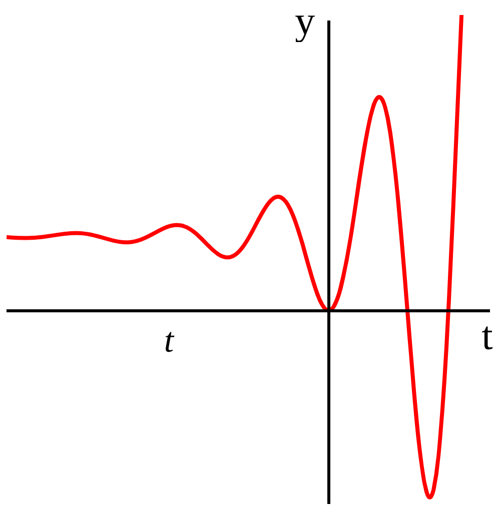
$$s = 0,\ 2 = 26C,\ C = \frac{1}{13}$$

$$s = 1,\ 2 = 5B + 25C,\ B = \frac{1}{65}$$

$$s = 2,\ 2 = 2A + 10B + 26C,\ A = -\frac{1}{13}$$

$$y = e^t\left(-\frac{1}{13}\cos(5t) + \frac{1}{65}\sin(5t)\right) + \frac{1}{13}$$

4. Sketch the particular solution:



Problem:

$$y'' + 2y' + 37y = -e^{-2t}$$

$$y(0) = 0,\ y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 + 2Ys + 37Y = -\frac{1}{s+2}$$

2. Solve for Y:

$$Y = -\frac{1}{(s^2 + 2s + 37)(s+2)}$$

3. Inverse Laplace Transform:

$$\alpha = -1,\ \beta = 6$$

$$-\frac{1}{((s+1)^2 + 36)(s+2)} = \frac{A(s+1) + 6B}{(s+1)^2 + 36} + \frac{C}{s+2}$$

$$-1 = (A(s+1) + 6B)(s+2) + C((s+1)^2 + 36)$$

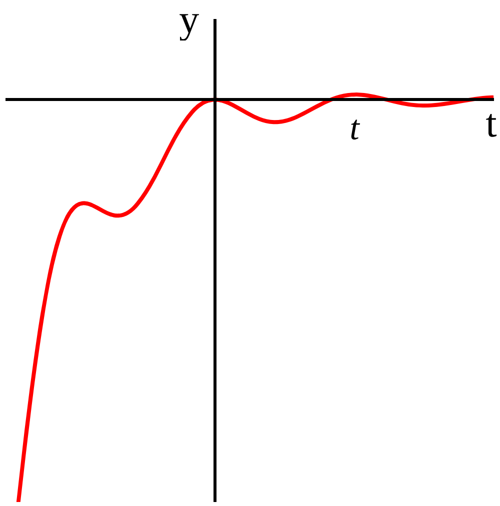
$$s = -2,\ -1 = 37C,\ C = -\frac{1}{37}$$

$$s = -1,\ -1 = 6B + 36C,\ B = -\frac{1}{222}$$

$$s = 0,\ -1 = 2A + 12B + 37C,\ A = \frac{1}{37}$$

$$y = e^{-t}\left(\frac{1}{37}\cos(6t) - \frac{1}{222}\sin(6t)\right) - \frac{1}{37}e^{-2t}$$

4. Sketch the particular solution:



Laplace Transform Method: 1st-order, 1 variable, Step function

Problem:

$$y' + 2y = 3u(t-5)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 2Y = \frac{3e^{-5s}}{s}$$

2. Solve for Y:

$$Y = \frac{3e^{-5s}}{(s+2)s}$$

3. Inverse Laplace Transform:

$$\frac{3}{(s+2)s} = \frac{A}{s+2} + \frac{B}{s}$$

$$3 = As + B(s+2)$$

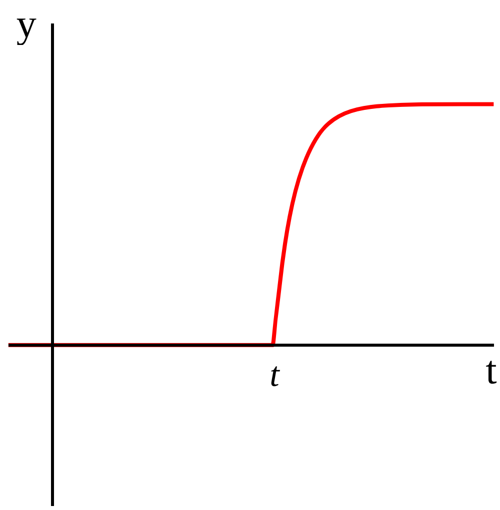
$$s = -2, 3 = -2A, A = -\frac{3}{2}$$

$$s = 0, 3 = 2B, B = \frac{3}{2}$$

$$f = -\frac{3}{2}e^{-2t} + \frac{3}{2}$$

$$y = \left(-\frac{3}{2}e^{-2t+10} + \frac{3}{2}\right)u(t-5)$$

4. Sketch the particular solution:



Problem:

$$y' + 4y = -3e^{-2t+2}u(t-1)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 4Y = -\frac{3e^{-s}}{s+2}$$

2. Solve for Y:

$$Y = -\frac{3e^{-s}}{(s+4)(s+2)}$$

3. Inverse Laplace Transform:

$$-\frac{3}{(s+4)(s+2)} = \frac{A}{s+4} + \frac{B}{s+2}$$

$$-3 = A(s+2) + B(s+4)$$

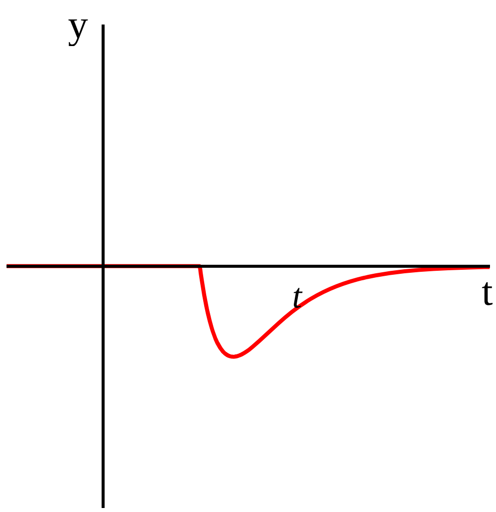
$$s = -4,\ -3 = -2A,\ A = \frac{3}{2}$$

$$s = -2,\ -3 = 2B,\ B = -\frac{3}{2}$$

$$f = \frac{3}{2}e^{-4t} - \frac{3}{2}e^{-2t}$$

$$y = \left(\frac{3}{2}e^{-4t+4} - \frac{3}{2}e^{-2t+2}\right)u(t-1)$$

4. Sketch the particular solution:



Problem:

$$y' - y = -e^{-2t+2}\,u(t-1)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys - Y = -\frac{e^{-s}}{s+2}$$

2. Solve for Y:

$$Y = -\frac{e^{-s}}{(s-1)\,(s+2)}$$

3. Inverse Laplace Transform:

$$-\frac{1}{(s-1)\,(s+2)} = \frac{A}{s-1} + \frac{B}{s+2}$$

$$-1 = A\,(s+2) + B\,(s-1)$$

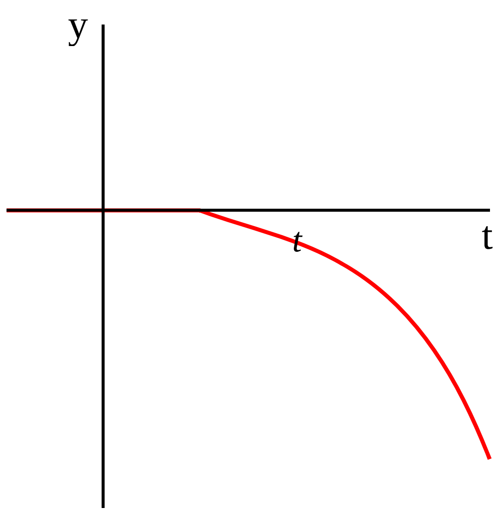
$$s = 1,\ -1 = 3\,A,\ A = -\frac{1}{3}$$

$$s = -2,\ -1 = -3\,B,\ B = \frac{1}{3}$$

$$f = -\frac{1}{3}\,e^{t} + \frac{1}{3}\,e^{-2t}$$

$$y = \left(-\frac{1}{3}\,e^{t-1} + \frac{1}{3}\,e^{-2t+2}\right)u(t-1)$$

4. Sketch the particular solution:



Problem:

$$y' + y = 3\,u(t-2)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Y s + Y = \frac{3\,e^{-2s}}{s}$$

2. Solve for Y:

$$Y = \frac{3\,e^{-2s}}{(s+1)\,s}$$

3. Inverse Laplace Transform:

$$\frac{3}{(s+1)\,s} = \frac{A}{s+1} + \frac{B}{s}$$

$$3 = A\,s + B\,(s+1)$$

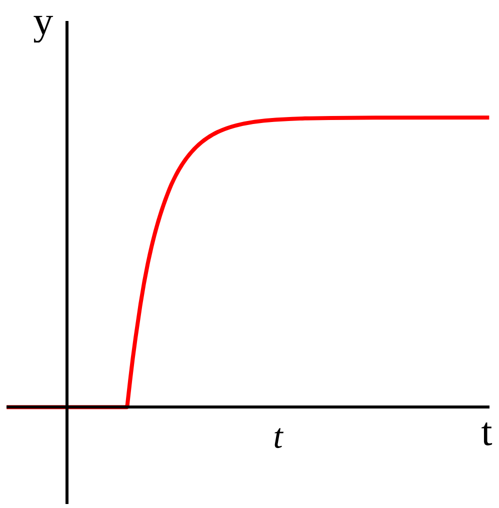
$$s = -1,\ 3 = -A,\ A = -3$$

$$s = 0,\ 3 = B,\ B = 3$$

$$f = -3\,e^{-t} + 3$$

$$y = \left(-3\,e^{-t+2} + 3\right)\,u(t-2)$$

4. Sketch the particular solution:



Problem:

$$y' - 2y = -3e^{-2t+2}u(t-1)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys - 2Y = -\frac{3e^{-s}}{s+2}$$

2. Solve for Y:

$$Y = -\frac{3e^{-s}}{(s-2)(s+2)}$$

3. Inverse Laplace Transform:

$$-\frac{3}{(s-2)(s+2)} = \frac{A}{s-2} + \frac{B}{s+2}$$

$$-3 = A(s+2) + B(s-2)$$

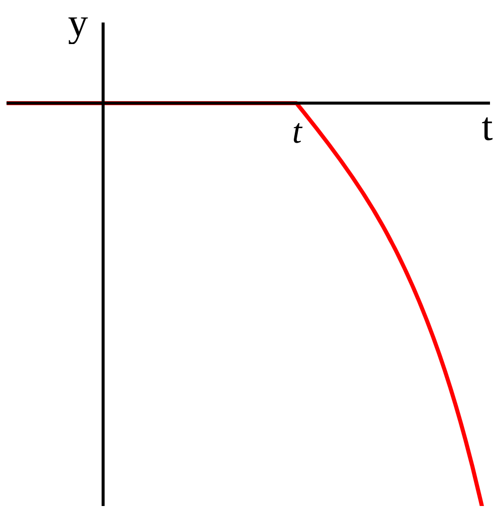
$$s = 2,\ -3 = 4A,\ A = -\frac{3}{4}$$

$$s = -2,\ -3 = -4B,\ B = \frac{3}{4}$$

$$f = -\frac{3}{4}e^{2t} + \frac{3}{4}e^{-2t}$$

$$y = \left(-\frac{3}{4}e^{2t-2} + \frac{3}{4}e^{-2t+2}\right)u(t-1)$$

4. Sketch the particular solution:



Problem:

$$y' + y = 5\,e^{-2t+2}\,u(t-1)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Y s + Y = \frac{5\,e^{-s}}{s+2}$$

2. Solve for Y:

$$Y = \frac{5\,e^{-s}}{(s+1)\,(s+2)}$$

3. Inverse Laplace Transform:

$$\frac{5}{(s+1)\,(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$5 = A\,(s+2) + B\,(s+1)$$

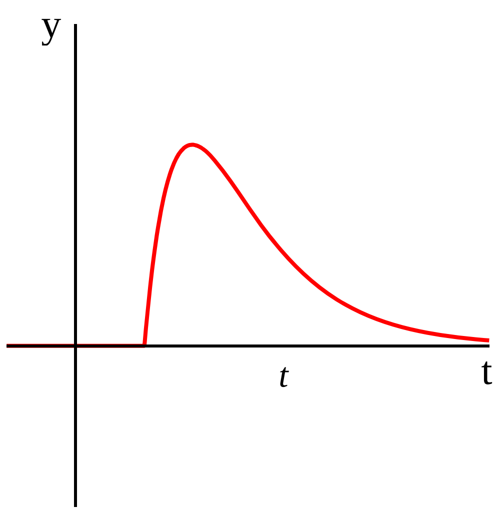
$$s = -1,\ 5 = A,\ A = 5$$

$$s = -2,\ 5 = -B,\ B = -5$$

$$f = 5\,e^{-t} - 5\,e^{-2t}$$

$$y = \left(5\,e^{-t+1} - 5\,e^{-2t+2}\right)\,u(t-1)$$

4. Sketch the particular solution:



Laplace Transform Method: 1st-order, 1 variable, Impulse function

Problem:

$$y' + 2y = 2\,\delta(t-1)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Y s + 2Y = 2\,e^{-s}$$

2. Solve for Y:

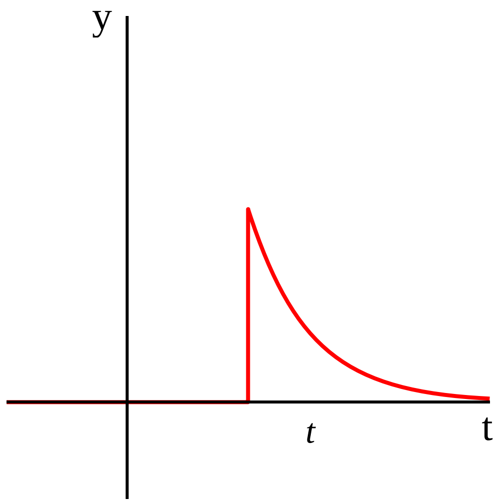
$$Y = \frac{2\,e^{-s}}{s+2}$$

3. Inverse Laplace Transform:

$$f = 2\,e^{-2t}$$

$$y = 2\,e^{-2t+2}\,u(t-1)$$

4. Sketch the particular solution:



Problem:

$$y' - 2y = -3\,\delta(t-3)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys - 2Y = -3\,e^{-3s}$$

2. Solve for Y:

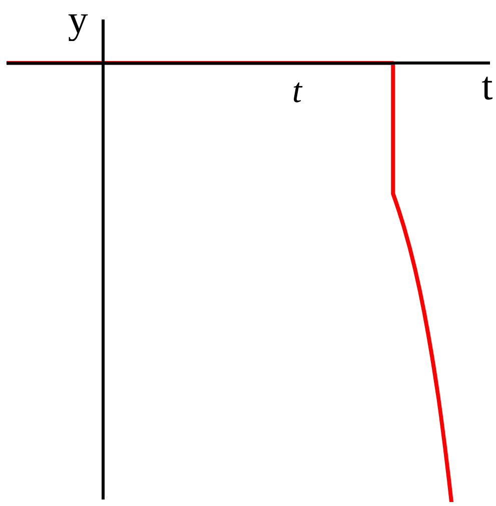
$$Y = -\frac{3\,e^{-3s}}{s-2}$$

3. Inverse Laplace Transform:

$$f = -3\,e^{2t}$$

$$y = -3\,e^{2t-6}\,u(t-3)$$

4. Sketch the particular solution:



Problem:

$$y' + 2y = -2\,\delta(t-3)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 2Y = -2\,e^{-3s}$$

2. Solve for Y:

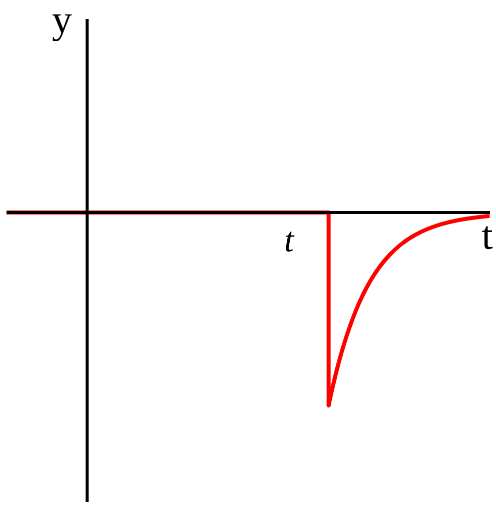
$$Y = -\frac{2\,e^{-3s}}{s+2}$$

3. Inverse Laplace Transform:

$$f = -2\,e^{-2t}$$

$$y = -2\,e^{-2t+6}\,u(t-3)$$

4. Sketch the particular solution:



Problem:

$$y' - 2y = 3\,\delta(t-1)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys - 2Y = 3\,e^{-s}$$

2. Solve for Y:

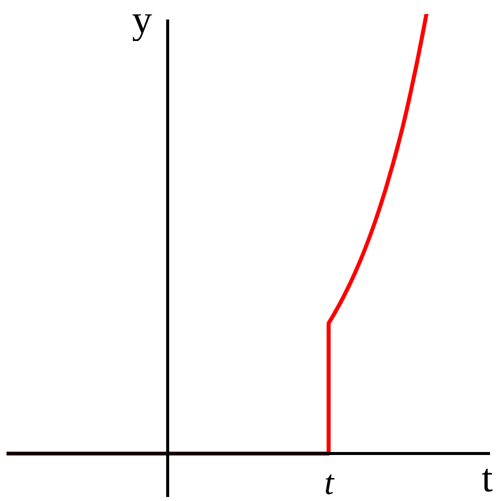
$$Y = \frac{3\,e^{-s}}{s-2}$$

3. Inverse Laplace Transform:

$$f = 3\,e^{2t}$$

$$y = 3\,e^{2t-2}\,u(t-1)$$

4. Sketch the particular solution:



Problem:

$$y' + y = 2\,\delta(t - 2)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Y s + Y = 2\,e^{-2s}$$

2. Solve for Y:

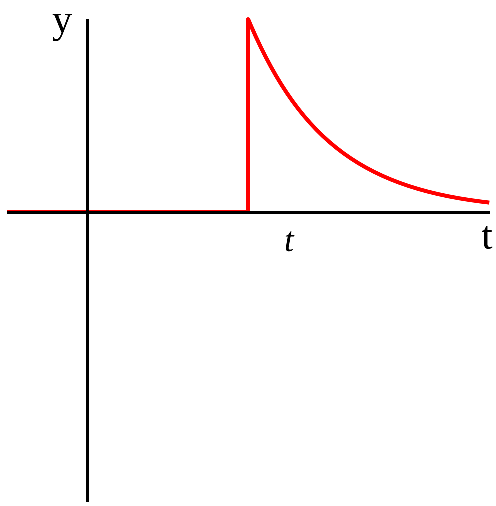
$$Y = \frac{2\,e^{-2s}}{s + 1}$$

3. Inverse Laplace Transform:

$$f = 2\,e^{-t}$$

$$y = 2\,e^{-t+2}\,u(t - 2)$$

4. Sketch the particular solution:



Problem:

$$y' + 2y = -2\,\delta(t-1)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 2Y = -2\,e^{-s}$$

2. Solve for Y:

$$Y = -\frac{2\,e^{-s}}{s+2}$$

3. Inverse Laplace Transform:

$$f = -2\,e^{-2t}$$

$$y = -2\,e^{-2t+2}\,u(t-1)$$

4. Sketch the particular solution: