

MA 341
Homework 1
(Homogeneous Linear Differential Equations)

Hoon Hong

▼ **1st-order, 1 variable**

Problem:

$$y' - 2y = 0$$

$$y(0) = -3$$

1. Find the general solution:

$$\lambda - 2 = 0$$

$$\lambda = 2$$

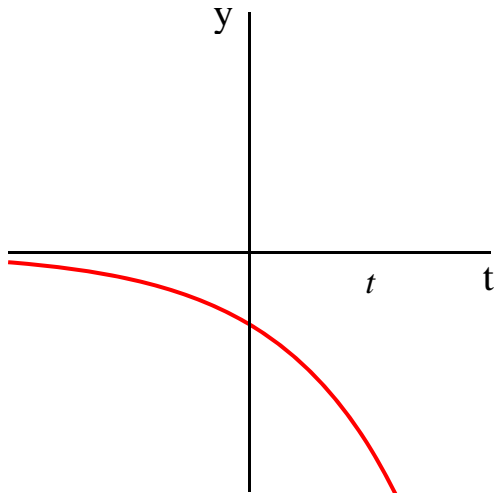
$$y = C e^{2t}$$

2. Find the particular solution:

$$C = -3$$

$$y = -3 e^{2t}$$

3. Sketch the particular solution:



Problem:

$$y' - 2y = 0$$

$$y(0) = 4$$

1. Find the general solution:

$$\lambda - 2 = 0$$

$$\lambda = 2$$

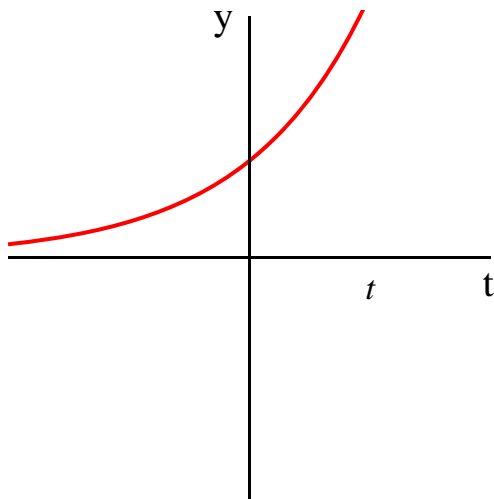
$$y = C e^{2t}$$

2. Find the particular solution:

$$C = 4$$

$$y = 4 e^{2t}$$

3. Sketch the particular solution:



Problem:

$$y' + 3y = 0$$

$$y(0) = 2$$

1. Find the general solution:

$$\lambda + 3 = 0$$

$$\lambda = -3$$

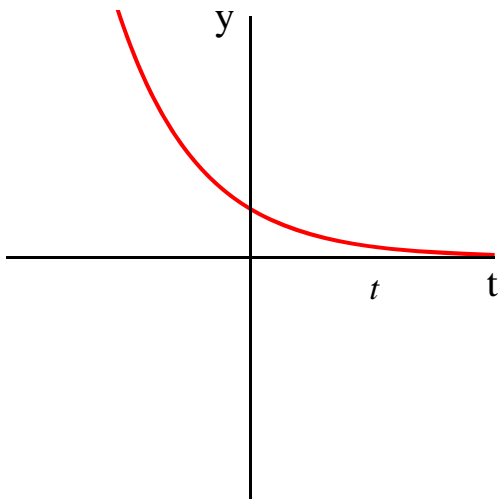
$$y = C e^{-3t}$$

2. Find the particular solution:

$$C = 2$$

$$y = 2 e^{-3t}$$

3. Sketch the particular solution:



Problem:

$$y' + 3y = 0$$

$$y(0) = -2$$

1. Find the general solution:

$$\lambda + 3 = 0$$

$$\lambda = -3$$

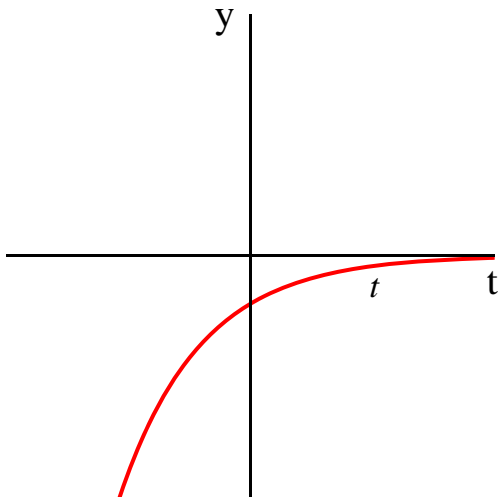
$$y = C e^{-3t}$$

2. Find the particular solution:

$$C = -2$$

$$y = -2 e^{-3t}$$

3. Sketch the particular solution:



▼ 2nd-order, 1 variable: Real eigenvalues

Problem:

$$y'' + 3y' + 2y = 0$$

$$y(0) = -3$$

$$y'(0) = 2$$

1. Find the general solution:

$$\lambda^2 + 3\lambda + 2 = 0$$

$$\lambda = -1, -2$$

$$y = C_1 e^{-t} + C_2 e^{-2t}$$

2. Find the particular solution:

$$y' = -C_1 e^{-t} - 2C_2 e^{-2t}$$

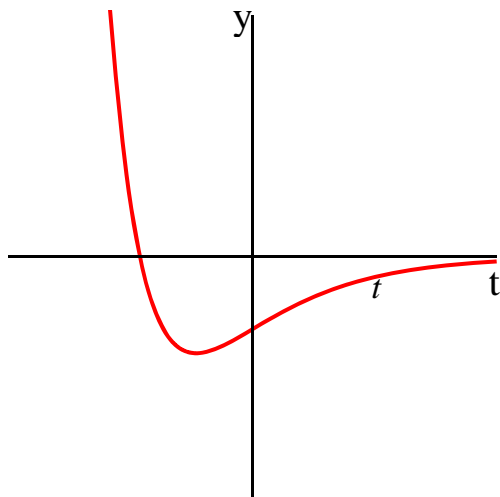
$$C_1 + C_2 = -3$$

$$-C_1 - 2C_2 = 2$$

$$C_1 = -4, C_2 = 1$$

$$y = -4e^{-t} + e^{-2t}$$

3. Sketch the particular solution:



Problem:

$$y'' - 3y' + 2y = 0$$

$$y(0) = 3$$

$$y'(0) = 1$$

1. Find the general solution:

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\lambda = 2, 1$$

$$y = C_1 e^{2t} + C_2 e^t$$

2. Find the particular solution:

$$y' = 2C_1 e^{2t} + C_2 e^t$$

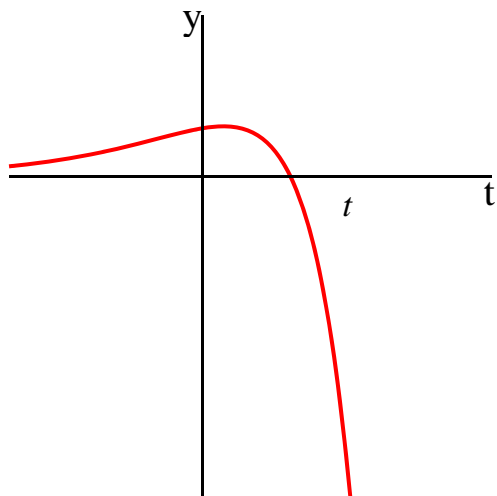
$$C_1 + C_2 = 3$$

$$2C_1 + C_2 = 1$$

$$C_1 = -2, C_2 = 5$$

$$y = -2e^{2t} + 5e^t$$

3. Sketch the particular solution:



Problem:

$$y'' + y' - 6y = 0$$

$$y(0) = -3$$

$$y'(0) = 4$$

1. Find the general solution:

$$\lambda^2 + \lambda - 6 = 0$$

$$\lambda = 2, -3$$

$$y = C_1 e^{2t} + C_2 e^{-3t}$$

2. Find the particular solution:

$$y' = 2C_1 e^{2t} - 3C_2 e^{-3t}$$

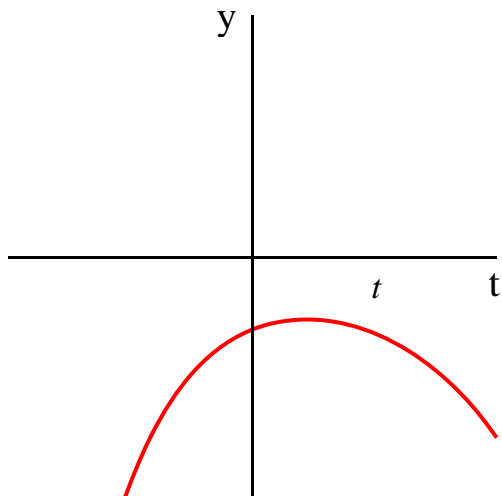
$$C_1 + C_2 = -3$$

$$2C_1 - 3C_2 = 4$$

$$C_1 = -1, C_2 = -2$$

$$y = -e^{2t} - 2e^{-3t}$$

3. Sketch the particular solution:



Problem:

$$y'' + 5y' + 6y = 0$$

$$y(0) = 3$$

$$y'(0) = -8$$

1. Find the general solution:

$$\lambda^2 + 5\lambda + 6 = 0$$

$$\lambda = -2, -3$$

$$y = C_1 e^{-2t} + C_2 e^{-3t}$$

2. Find the particular solution:

$$y' = -2C_1 e^{-2t} - 3C_2 e^{-3t}$$

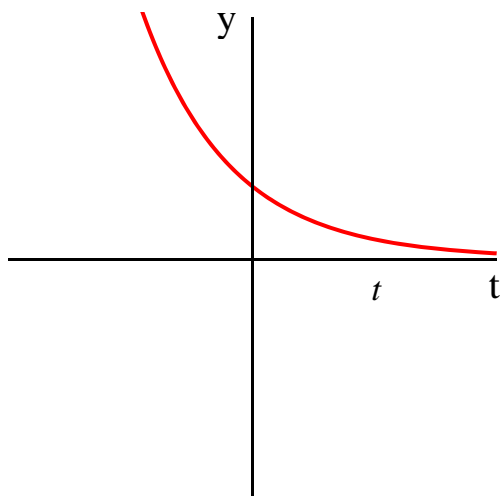
$$C_1 + C_2 = 3$$

$$-2C_1 - 3C_2 = -8$$

$$C_1 = 1, C_2 = 2$$

$$y = e^{-2t} + 2e^{-3t}$$

3. Sketch the particular solution:



Problem:

$$y'' + 7y' + 12y = 0$$

$$y(0) = 6$$

$$y'(0) = -22$$

1. Find the general solution:

$$\lambda^2 + 7\lambda + 12 = 0$$

$$\lambda = -3, -4$$

$$y = C_1 e^{-3t} + C_2 e^{-4t}$$

2. Find the particular solution:

$$y' = -3C_1 e^{-3t} - 4C_2 e^{-4t}$$

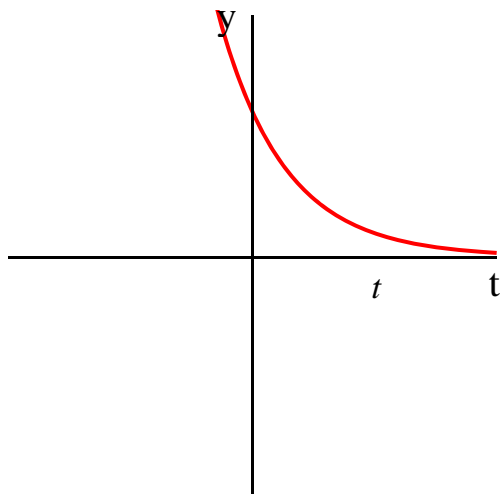
$$C_1 + C_2 = 6$$

$$-3C_1 - 4C_2 = -22$$

$$C_1 = 2, C_2 = 4$$

$$y = 2e^{-3t} + 4e^{-4t}$$

3. Sketch the particular solution:



Problem:

$$y'' - 5y' + 6y = 0$$

$$y(0) = 2$$

$$y'(0) = 1$$

1. Find the general solution:

$$\lambda^2 - 5\lambda + 6 = 0$$

$$\lambda = 3, 2$$

$$y = C_1 e^{3t} + C_2 e^{2t}$$

2. Find the particular solution:

$$y' = 3C_1 e^{3t} + 2C_2 e^{2t}$$

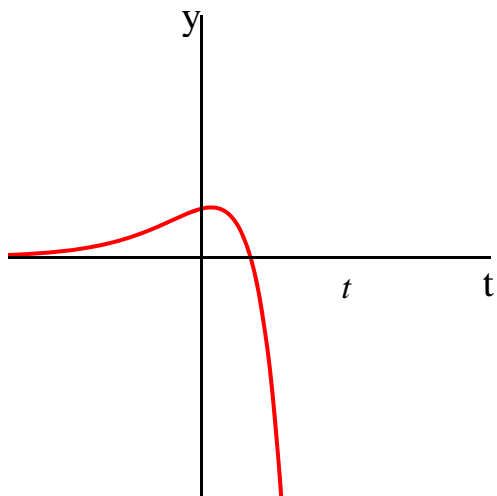
$$C_1 + C_2 = 2$$

$$3C_1 + 2C_2 = 1$$

$$C_1 = -3, C_2 = 5$$

$$y = -3e^{3t} + 5e^{2t}$$

3. Sketch the particular solution:



Problem:

$$y'' + y' - 2y = 0$$

$$y(0) = -4$$

$$y'(0) = 5$$

1. Find the general solution:

$$\lambda^2 + \lambda - 2 = 0$$

$$\lambda = 1, -2$$

$$y = C_1 e^t + C_2 e^{-2t}$$

2. Find the particular solution:

$$y' = C_1 e^t - 2C_2 e^{-2t}$$

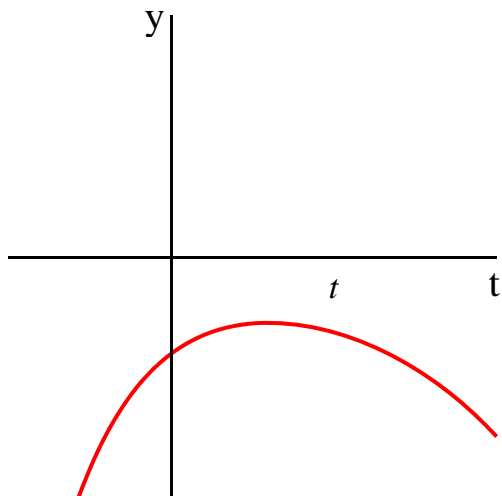
$$C_1 + C_2 = -4$$

$$C_1 - 2C_2 = 5$$

$$C_1 = -1, C_2 = -3$$

$$y = -e^t - 3e^{-2t}$$

3. Sketch the particular solution:



Problem:

$$y'' + 5y' + 6y = 0$$

$$y(0) = -1$$

$$y'(0) = 1$$

1. Find the general solution:

$$\lambda^2 + 5\lambda + 6 = 0$$

$$\lambda = -2, -3$$

$$y = C_1 e^{-2t} + C_2 e^{-3t}$$

2. Find the particular solution:

$$y' = -2C_1 e^{-2t} - 3C_2 e^{-3t}$$

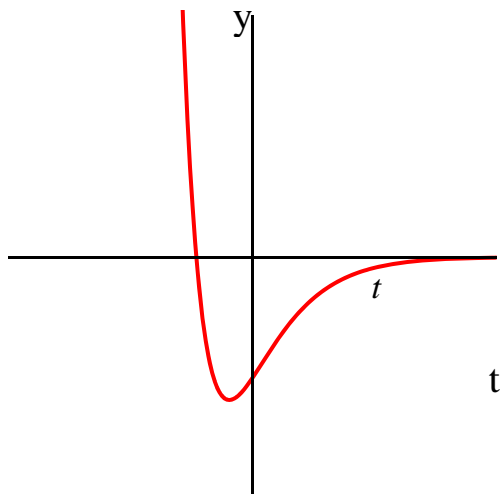
$$C_1 + C_2 = -1$$

$$-2C_1 - 3C_2 = 1$$

$$C_1 = -2, C_2 = 1$$

$$y = -2e^{-2t} + e^{-3t}$$

3. Sketch the particular solution:



▼ 2nd-order, 1 variable: Non-real eigenvalues

Problem:

$$y'' - 4y' + 68y = 0$$

$$y(0) = 1$$

$$y'(0) = 10$$

1. Find the general solution:

$$\lambda^2 - 4\lambda + 68 = 0$$

$$\lambda = 2 + 8i, 2 - 8i$$

$$y = e^{2t} (C_1 \cos(8t) + C_2 \sin(8t))$$

2. Find the particular solution:

$$y' = 2e^{2t} (C_1 \cos(8t) + C_2 \sin(8t)) + e^{2t} (-8C_1 \sin(8t) + 8C_2 \cos(8t))$$

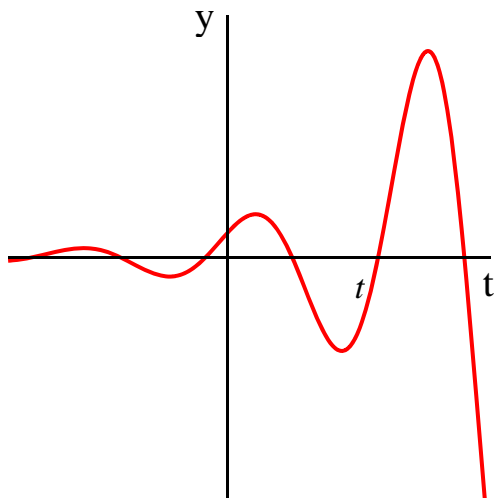
$$C_1 = 1$$

$$2C_1 + 8C_2 = 10$$

$$C_2 = 1$$

$$y = e^{2t} (\cos(8t) + \sin(8t))$$

3. Sketch the particular solution:



Problem:

$$y'' + 2y' + 50y = 0$$

$$y(0) = 2$$

$$y'(0) = 19$$

1. Find the general solution:

$$\lambda^2 + 2\lambda + 50 = 0$$

$$\lambda = -1 + 7i, -1 - 7i$$

$$y = e^{-t} (C_1 \cos(7t) + C_2 \sin(7t))$$

2. Find the particular solution:

$$y' = -e^{-t} (C_1 \cos(7t) + C_2 \sin(7t)) + e^{-t} (-7C_1 \sin(7t) + 7C_2 \cos(7t))$$

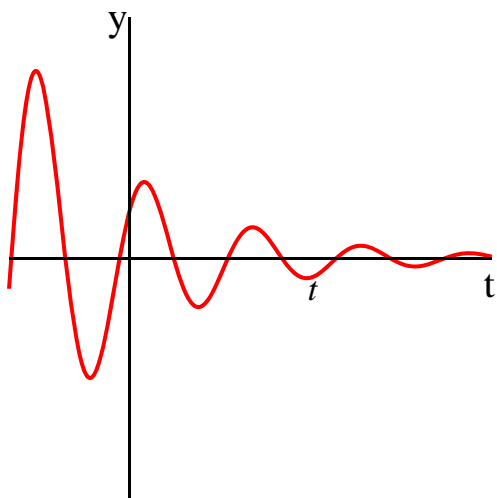
$$C_1 = 2$$

$$-C_1 + 7C_2 = 19$$

$$C_2 = 3$$

$$y = e^{-t} (2 \cos(7t) + 3 \sin(7t))$$

3. Sketch the particular solution:



Problem:

$$y'' + 64y = 0$$

$$y(0) = 2$$

$$y'(0) = 8$$

1. Find the general solution:

$$\lambda^2 + 64 = 0$$

$$\lambda = 8i, -8i$$

$$y = C_1 \cos(8t) + C_2 \sin(8t)$$

2. Find the particular solution:

$$y' = -8C_1 \sin(8t) + 8C_2 \cos(8t)$$

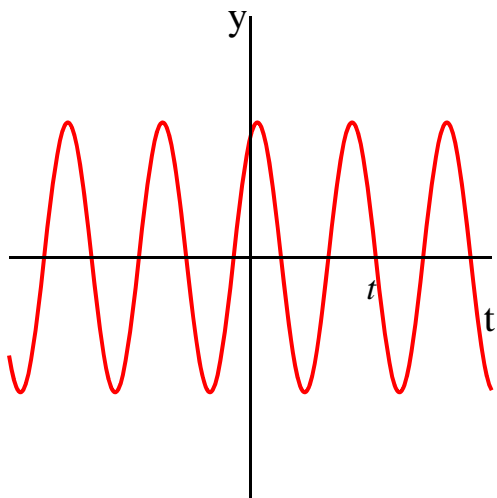
$$C_1 = 2$$

$$8C_2 = 8$$

$$C_2 = 1$$

$$y = 2 \cos(8t) + \sin(8t)$$

3. Sketch the particular solution:



Problem:

$$y'' + 2y' + 82y = 0$$

$$y(0) = 0$$

$$y'(0) = 9$$

1. Find the general solution:

$$\lambda^2 + 2\lambda + 82 = 0$$

$$\lambda = -1 + 9I, -1 - 9I$$

$$y = e^{-t} (C_1 \cos(9t) + C_2 \sin(9t))$$

2. Find the particular solution:

$$y' = -e^{-t} (C_1 \cos(9t) + C_2 \sin(9t)) + e^{-t} (-9C_1 \sin(9t) + 9C_2 \cos(9t))$$

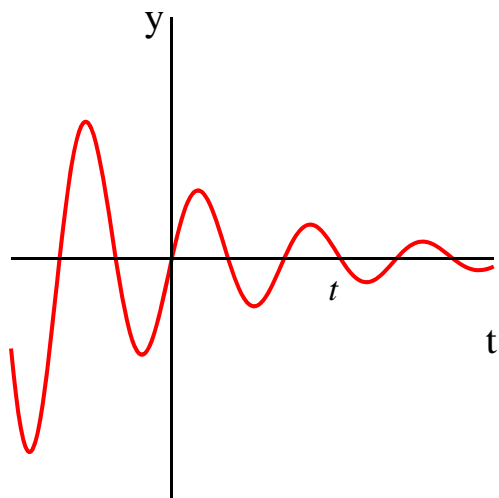
$$C_1 = 0$$

$$-C_1 + 9C_2 = 9$$

$$C_2 = 1$$

$$y = e^{-t} \sin(9t)$$

3. Sketch the particular solution:



Problem:

$$y'' - 4y' + 125y = 0$$

$$y(0) = 1$$

$$y'(0) = 35$$

1. Find the general solution:

$$\lambda^2 - 4\lambda + 125 = 0$$

$$\lambda = 2 + 11i, 2 - 11i$$

$$y = e^{2t} (C_1 \cos(11t) + C_2 \sin(11t))$$

2. Find the particular solution:

$$y' = 2e^{2t} (C_1 \cos(11t) + C_2 \sin(11t)) + e^{2t} (-11C_1 \sin(11t) + 11C_2 \cos(11t))$$

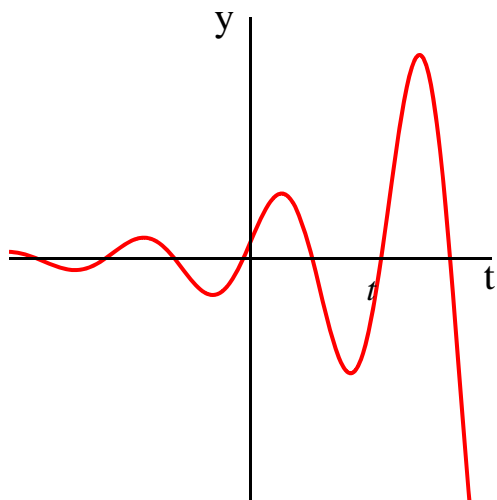
$$C_1 = 1$$

$$2C_1 + 11C_2 = 35$$

$$C_2 = 3$$

$$y = e^{2t} (\cos(11t) + 3 \sin(11t))$$

3. Sketch the particular solution:



Problem:

$$y'' + 9y = 0$$

$$y(0) = 1$$

$$y'(0) = -3$$

1. Find the general solution:

$$\lambda^2 + 9 = 0$$

$$\lambda = 3i, -3i$$

$$y = C_1 \cos(3t) + C_2 \sin(3t)$$

2. Find the particular solution:

$$y' = -3C_1 \sin(3t) + 3C_2 \cos(3t)$$

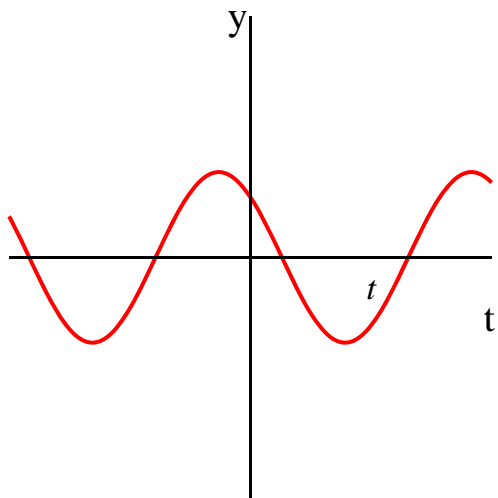
$$C_1 = 1$$

$$3C_2 = -3$$

$$C_2 = -1$$

$$y = \cos(3t) - \sin(3t)$$

3. Sketch the particular solution:



▼ 1st-order, 2 variables: Introduction

Problem:

$$y1' = -y1 + 2 y2$$

$$y2' = 2 y1 + 2 y2$$

$$y1(0) = 3$$

$$y2(0) = 1$$

1. Find the general solution:

$$(-1 - \lambda) v_1 + 2 v_2 = 0$$

$$2 v_1 + (2 - \lambda) v_2 = 0$$

$$\lambda^2 - \lambda - 6 = 0$$

$$\lambda = 3$$

$$v_1 = 2 C_1$$

$$v_2 = 4 C_1$$

$$\lambda = -2$$

$$v_1 = 2 C_2$$

$$v_2 = -C_2$$

$$y1 = 2 C_1 e^{3t} + 2 C_2 e^{-2t}$$

$$y2 = 4 C_1 e^{3t} - C_2 e^{-2t}$$

2. Find the particular solution:

$$2 C_1 + 2 C_2 = 3$$

$$4 C_1 - C_2 = 1$$

$$C_1 = \frac{1}{2}, C_2 = 1$$

$$y1 = e^{3t} + 2 e^{-2t}$$

$$y2 = 2 e^{3t} - e^{-2t}$$

Problem:

$$y1' = y1 - 2y2$$

$$y2' = -2y1 - 2y2$$

$$y1(0) = 3$$

$$y2(0) = 1$$

1. Find the general solution:

$$(1 - \lambda) v_1 - 2 v_2 = 0$$

$$-2 v_1 + (-2 - \lambda) v_2 = 0$$

$$\lambda^2 + \lambda - 6 = 0$$

$$\lambda = -3$$

$$v_1 = -2 C_1$$

$$v_2 = -4 C_1$$

$$\lambda = 2$$

$$v_1 = -2 C_2$$

$$v_2 = C_2$$

$$y1 = -2 C_1 e^{-3t} - 2 C_2 e^{2t}$$

$$y2 = -4 C_1 e^{-3t} + C_2 e^{2t}$$

2. Find the particular solution:

$$-2 C_1 - 2 C_2 = 3$$

$$-4 C_1 + C_2 = 1$$

$$C_1 = -\frac{1}{2}, C_2 = -1$$

$$y1 = e^{-3t} + 2 e^{2t}$$

$$y2 = 2 e^{-3t} - e^{2t}$$

Problem:

$$y_1' = -5y_1 - 2y_2$$

$$y_2' = 2y_1$$

$$y_1(0) = 1$$

$$y_2(0) = 1$$

1. Find the general solution:

$$(-5 - \lambda)v_1 - 2v_2 = 0$$

$$-\lambda v_2 + 2v_1 = 0$$

$$\lambda^2 + 5\lambda + 4 = 0$$

$$\lambda = -1$$

$$v_1 = -2C_1$$

$$v_2 = 4C_1$$

$$\lambda = -4$$

$$v_1 = -2C_2$$

$$v_2 = C_2$$

$$y_1 = -2C_1 e^{-t} - 2C_2 e^{-4t}$$

$$y_2 = 4C_1 e^{-t} + C_2 e^{-4t}$$

2. Find the particular solution:

$$-2C_1 - 2C_2 = 1$$

$$4C_1 + C_2 = 1$$

$$C_1 = \frac{1}{2}, C_2 = -1$$

$$y_1 = -e^{-t} + 2e^{-4t}$$

$$y_2 = 2e^{-t} - e^{-4t}$$

Problem:

$$y1' = 3y1 - y2$$

$$y2' = -2y1 + 4y2$$

$$y1(0) = 1$$

$$y2(0) = 4$$

1. Find the general solution:

$$(3 - \lambda)v_1 - v_2 = 0$$

$$-2v_1 + (4 - \lambda)v_2 = 0$$

$$\lambda^2 - 7\lambda + 10 = 0$$

$$\lambda = 2$$

$$v_1 = -C_1$$

$$v_2 = -C_1$$

$$\lambda = 5$$

$$v_1 = -C_2$$

$$v_2 = 2C_2$$

$$y1 = -C_1 e^{2t} - C_2 e^{5t}$$

$$y2 = -C_1 e^{2t} + 2C_2 e^{5t}$$

2. Find the particular solution:

$$-C_1 - C_2 = 1$$

$$-C_1 + 2C_2 = 4$$

$$C_1 = -2, C_2 = 1$$

$$y1 = 2e^{2t} - e^{5t}$$

$$y2 = 2e^{2t} + 2e^{5t}$$

▼ 1st-order, 2 variables: Real eigenvalues. Opposite Signs

Problem:

$$y' = \begin{bmatrix} -1 & 2 \\ 2 & 2 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

1. Find the general solution:

$$\text{Det} \begin{bmatrix} -1 - \lambda & 2 \\ 2 & 2 - \lambda \end{bmatrix} = 0$$

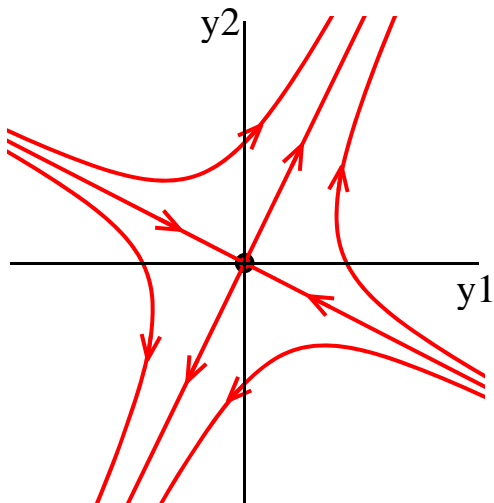
$$\lambda^2 - \lambda - 6 = 0$$

$$\lambda = 3, v = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\lambda = -2, v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 2 \\ 4 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-2t}$$

2. Sketch the general solution:



Saddle

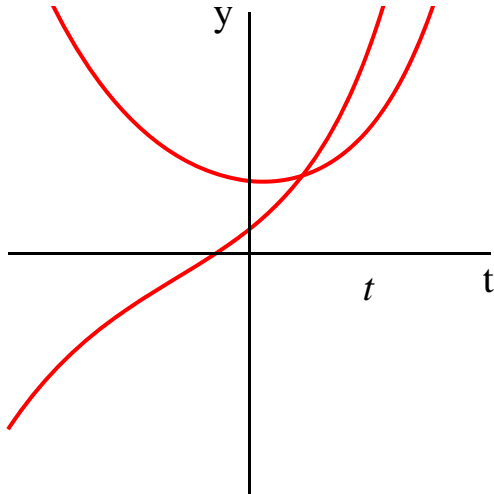
3. Find the particular solution:

$$C_1 \begin{bmatrix} 2 \\ 4 \end{bmatrix} + C_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$C_1 = \frac{1}{2}, C_2 = 1$$

$$y = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-2t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 1 & -2 \\ -2 & -2 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

1. Find the general solution:

$$\text{Det} \begin{bmatrix} 1 - \lambda & -2 \\ -2 & -2 - \lambda \end{bmatrix} = 0$$

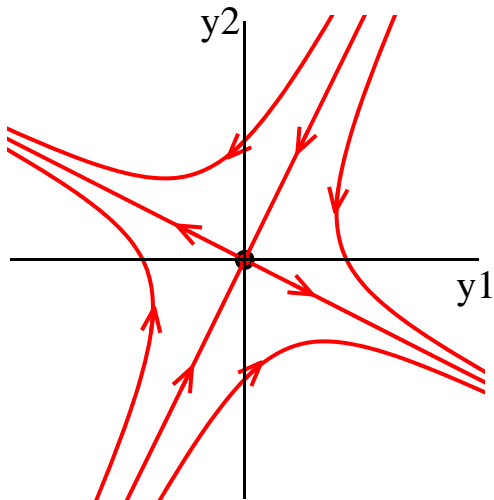
$$\lambda^2 + \lambda - 6 = 0$$

$$\lambda = -3, v = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

$$\lambda = 2, v = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} -2 \\ -4 \end{bmatrix} e^{-3t} + C_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix} e^{2t}$$

2. Sketch the general solution:



Saddle

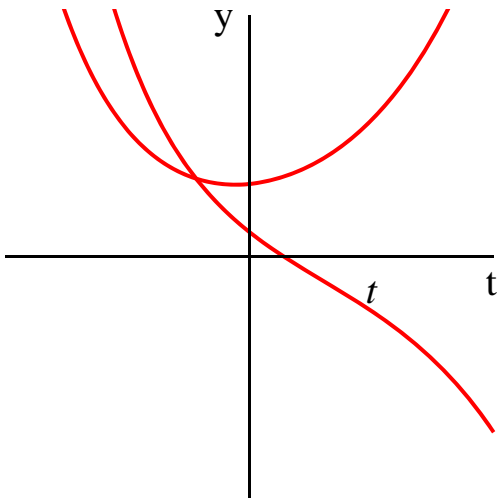
3. Find the particular solution:

$$C_1 \begin{bmatrix} -2 \\ -4 \end{bmatrix} + C_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$C_1 = -\frac{1}{2}, C_2 = -1$$

$$y = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-3t} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{2t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

1. Find the general solution:

$$\text{Det} \begin{bmatrix} -2 - \lambda & 2 \\ 2 & 1 - \lambda \end{bmatrix} = 0$$

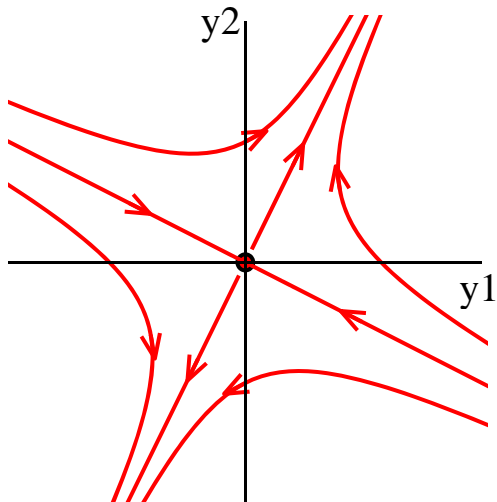
$$\lambda^2 + \lambda - 6 = 0$$

$$\lambda = 2, v = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\lambda = -3, v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 2 \\ 4 \end{bmatrix} e^{2t} + C_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-3t}$$

2. Sketch the general solution:



Saddle

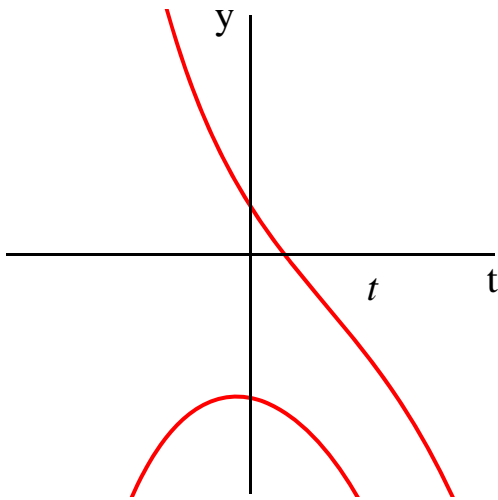
3. Find the particular solution:

$$C_1 \begin{bmatrix} 2 \\ 4 \end{bmatrix} + C_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

$$C_1 = -1, C_2 = 2$$

$$y = \begin{bmatrix} -2 \\ -4 \end{bmatrix} e^{2t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{-3t}$$

4. Sketch the particular solution:



▼ 1st-order, 2 variables: Real eigenvalues. Same Signs

Problem:

$$y' = \begin{bmatrix} 3 & -1 \\ -2 & 4 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

1. Find the general solution:

$$\text{Det} \begin{bmatrix} 3 - \lambda & -1 \\ -2 & 4 - \lambda \end{bmatrix} = 0$$

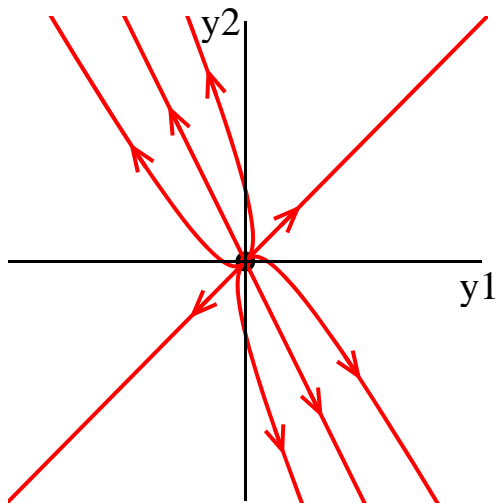
$$\lambda^2 - 7\lambda + 10 = 0$$

$$\lambda = 2, v = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\lambda = 5, v = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} -1 \\ -1 \end{bmatrix} e^{2t} + C_2 \begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{5t}$$

2. Sketch the general solution:



Unstable

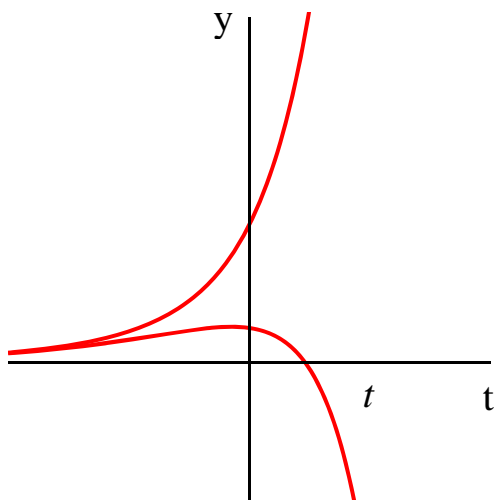
3. Find the particular solution:

$$C_1 \begin{bmatrix} -1 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$C_1 = -2, C_2 = 1$$

$$y = \begin{bmatrix} 2 \\ 2 \end{bmatrix} e^{2t} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{5t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -3 & 2 \\ 1 & -2 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$$

1. Find the general solution:

$$\text{Det} \begin{bmatrix} -3 - \lambda & 2 \\ 1 & -2 - \lambda \end{bmatrix} = 0$$

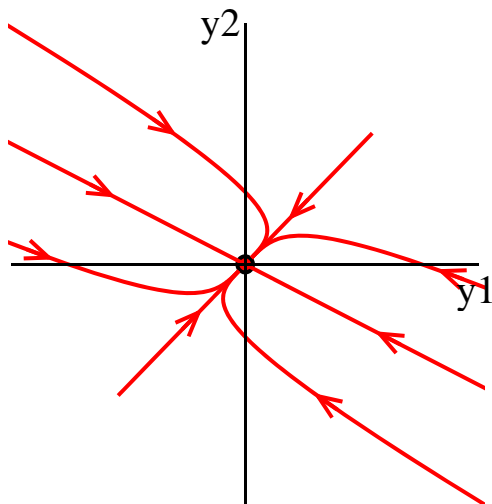
$$\lambda^2 + 5\lambda + 4 = 0$$

$$\lambda = -1, v = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\lambda = -4, v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 2 \\ 2 \end{bmatrix} e^{-t} + C_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-4t}$$

2. Sketch the general solution:



Stable

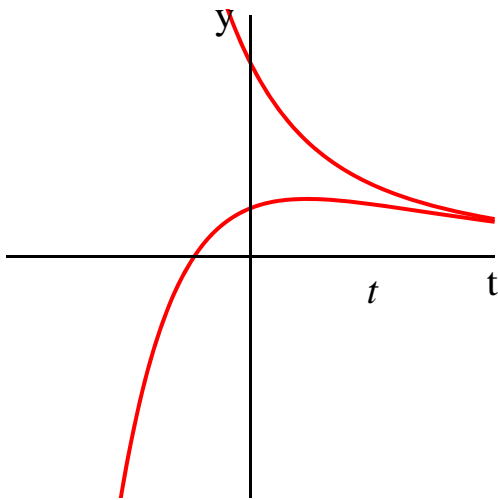
3. Find the particular solution:

$$C_1 \begin{bmatrix} 2 \\ 2 \end{bmatrix} + C_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$$

$$C_1 = 2, C_2 = 2$$

$$y = \begin{bmatrix} 4 \\ 4 \end{bmatrix} e^{-t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{-4t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

1. Find the general solution:

$$\text{Det} \begin{bmatrix} 2-\lambda & 1 \\ 2 & 3-\lambda \end{bmatrix} = 0$$

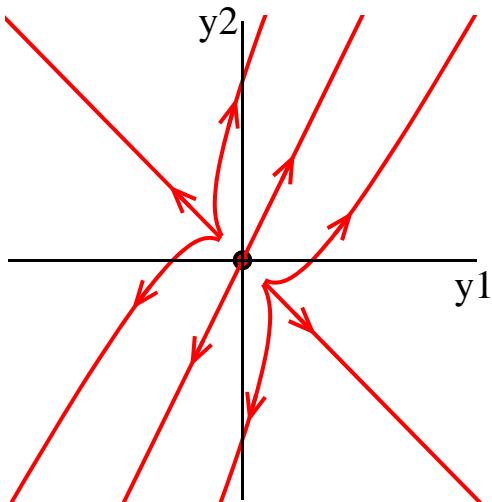
$$\lambda^2 - 5\lambda + 4 = 0$$

$$\lambda = 1, v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\lambda = 4, v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + C_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{4t}$$

2. Sketch the general solution:



Unstable

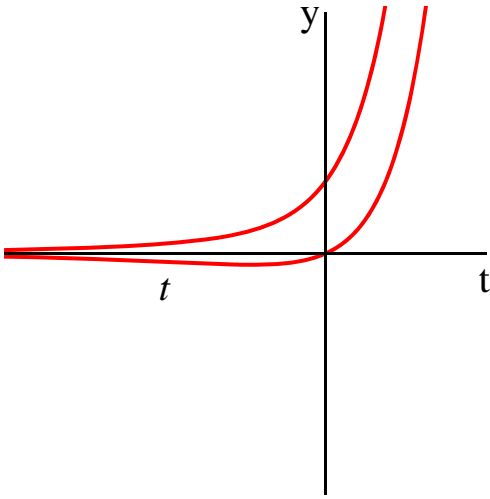
3. Find the particular solution:

$$C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$C_1 = -1, C_2 = 1$$

$$y = \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^t + \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{4t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

1. Find the general solution:

$$\text{Det} \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0$$

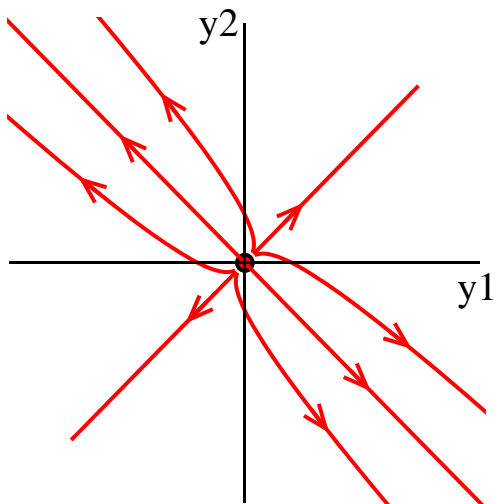
$$\lambda^2 - 4\lambda + 3 = 0$$

$$\lambda = 1, v = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\lambda = 3, v = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} -1 \\ -1 \end{bmatrix} e^t + C_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{3t}$$

2. Sketch the general solution:



Unstable

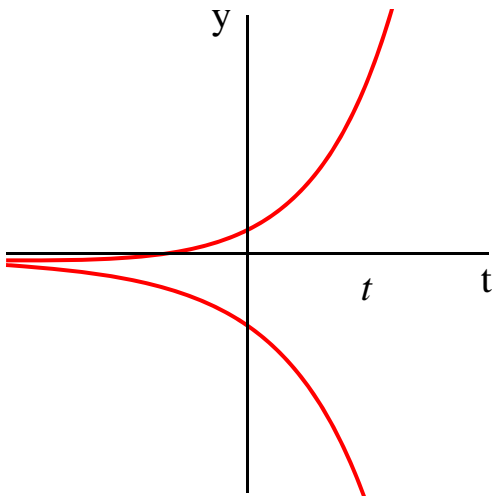
3. Find the particular solution:

$$C_1 \begin{bmatrix} -1 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$C_1 = 1, C_2 = 2$$

$$y = \begin{bmatrix} -1 \\ -1 \end{bmatrix} e^t + \begin{bmatrix} -2 \\ 2 \end{bmatrix} e^{3t}$$

4. Sketch the particular solution:



▼ 1st-order 2 variables: Non-real eigenvalues

Problem:

$$y' = \begin{bmatrix} -2 & -10 \\ 10 & -2 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

1. Find the general solution:

$$\text{Det} \begin{bmatrix} -2 - \lambda & -10 \\ 10 & -2 - \lambda \end{bmatrix} = 0$$

$$\lambda^2 + 4\lambda + 104 = 0$$

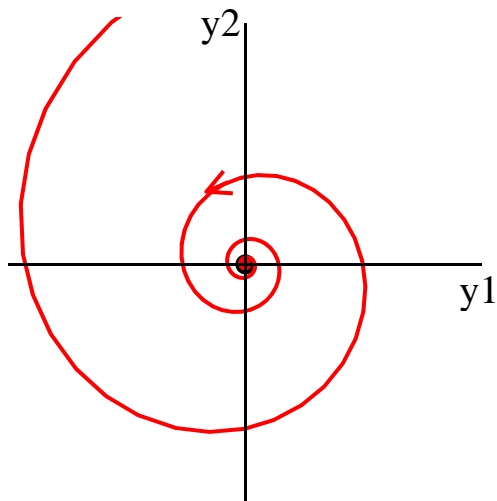
$$\lambda = -2 + 10i, v = \begin{bmatrix} -10 \\ 10i \end{bmatrix}$$

$$\lambda = -2 - 10i, v = \begin{bmatrix} -10 \\ -10i \end{bmatrix}$$

$$\alpha = -2, \beta = 10, p = \begin{bmatrix} -10 \\ 0 \end{bmatrix}, q = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$$y = e^{-2t} \left(\left(C_1 \begin{bmatrix} -10 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 10 \end{bmatrix} \right) \cos(10t) + \left(C_2 \begin{bmatrix} -10 \\ 0 \end{bmatrix} - C_1 \begin{bmatrix} 0 \\ 10 \end{bmatrix} \right) \sin(10t) \right)$$

2. Sketch the general solution:



Stable

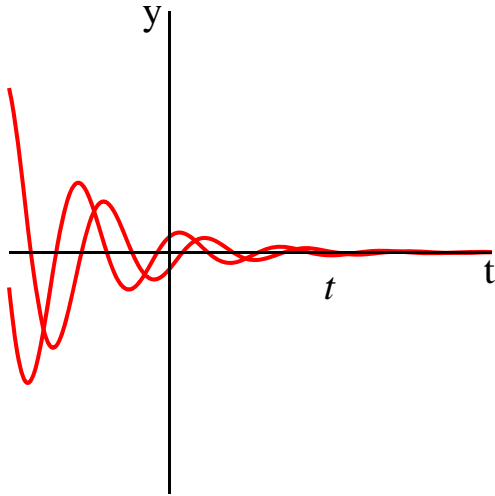
3. Find the particular solution:

$$C_1 \begin{bmatrix} -10 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 10 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$C_1 = -\frac{1}{10}, C_2 = -\frac{1}{10}$$

$$y = e^{-2t} \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \cos(10t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \sin(10t) \right)$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -7 & 16 \\ -8 & 9 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

1. Find the general solution:

$$\text{Det} \begin{bmatrix} -7 - \lambda & 16 \\ -8 & 9 - \lambda \end{bmatrix} = 0$$

$$\lambda^2 - 2\lambda + 65 = 0$$

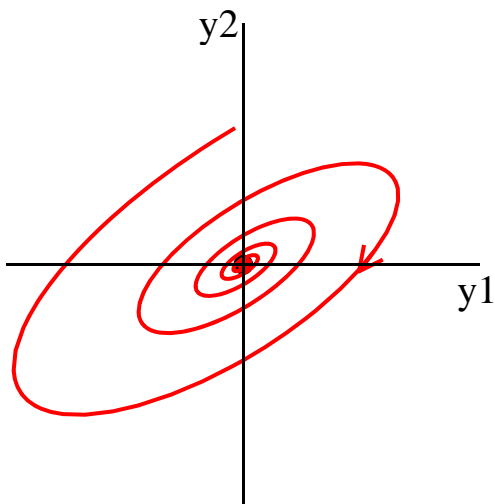
$$\lambda = 1 + 8i, v = \begin{bmatrix} 16 \\ 8 + 8i \end{bmatrix}$$

$$\lambda = 1 - 8i, v = \begin{bmatrix} 16 \\ 8 - 8i \end{bmatrix}$$

$$\alpha = 1, \beta = 8, p = \begin{bmatrix} 16 \\ 8 \end{bmatrix}, q = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$$

$$y = e^t \left(\left(C_1 \begin{bmatrix} 16 \\ 8 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 8 \end{bmatrix} \right) \cos(8t) + \left(C_2 \begin{bmatrix} 16 \\ 8 \end{bmatrix} - C_1 \begin{bmatrix} 0 \\ 8 \end{bmatrix} \right) \sin(8t) \right)$$

2. Sketch the general solution:



Unstable

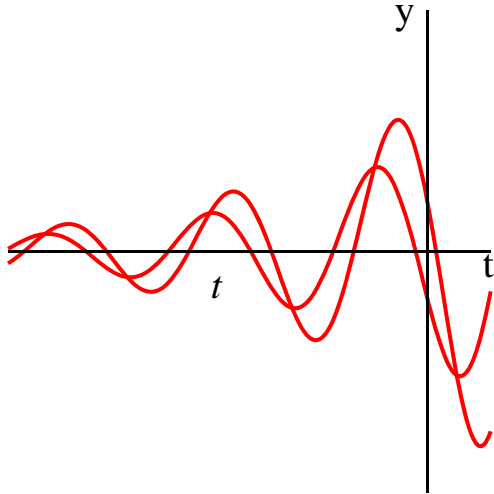
3. Find the particular solution:

$$C_1 \begin{bmatrix} 16 \\ 8 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 8 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

$$C_1 = \frac{1}{4}, C_2 = -\frac{3}{4}$$

$$y = e^t \left(\begin{bmatrix} 4 \\ -4 \end{bmatrix} \cos(8t) + \begin{bmatrix} -12 \\ -8 \end{bmatrix} \sin(8t) \right)$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 2 & 10 \\ -4 & -2 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

1. Find the general solution:

$$\text{Det} \begin{bmatrix} 2 - \lambda & 10 \\ -4 & -2 - \lambda \end{bmatrix} = 0$$

$$\lambda^2 + 36 = 0$$

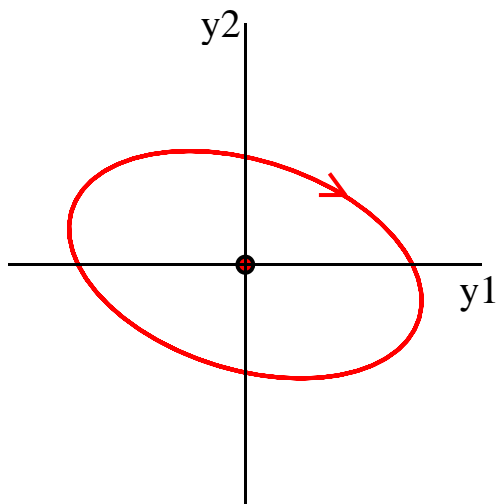
$$\lambda = 6i, v = \begin{bmatrix} 10 \\ -2 + 6i \end{bmatrix}$$

$$\lambda = -6i, v = \begin{bmatrix} 10 \\ -2 - 6i \end{bmatrix}$$

$$\alpha = 0, \beta = 6, p = \begin{bmatrix} 10 \\ -2 \end{bmatrix}, q = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$$

$$y = \left(C_1 \begin{bmatrix} 10 \\ -2 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 6 \end{bmatrix} \right) \cos(6t) + \left(C_2 \begin{bmatrix} 10 \\ -2 \end{bmatrix} - C_1 \begin{bmatrix} 0 \\ 6 \end{bmatrix} \right) \sin(6t)$$

2. Sketch the general solution:



center

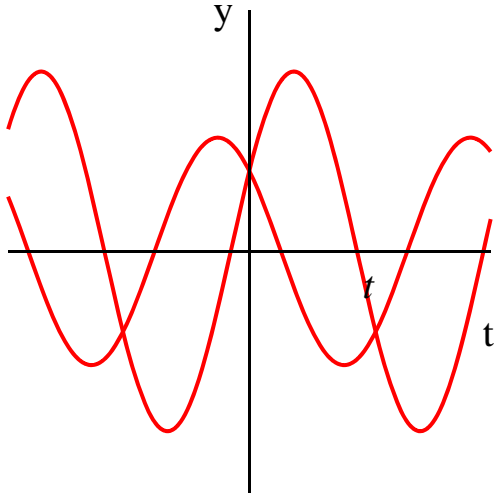
3. Find the particular solution:

$$C_1 \begin{bmatrix} 10 \\ -2 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C_1 = \frac{1}{10}, C_2 = \frac{1}{5}$$

$$y = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cos(6t) + \begin{bmatrix} 2 \\ -1 \end{bmatrix} \sin(6t)$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 1 & -4 \\ 4 & 1 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

1. Find the general solution:

$$\text{Det} \begin{bmatrix} 1 - \lambda & -4 \\ 4 & 1 - \lambda \end{bmatrix} = 0$$

$$\lambda^2 - 2\lambda + 17 = 0$$

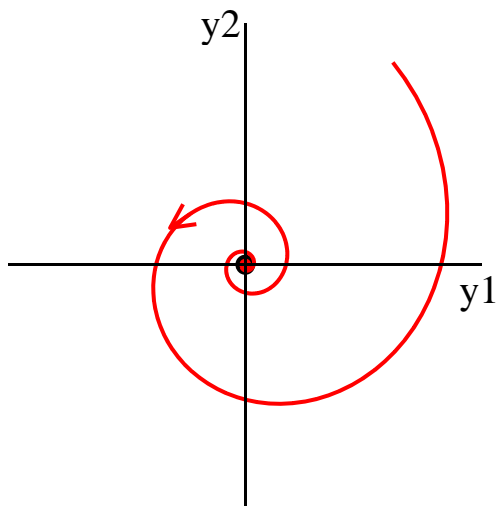
$$\lambda = 1 + 4i, v = \begin{bmatrix} -4 \\ 4i \end{bmatrix}$$

$$\lambda = 1 - 4i, v = \begin{bmatrix} -4 \\ -4i \end{bmatrix}$$

$$\alpha = 1, \beta = 4, p = \begin{bmatrix} -4 \\ 0 \end{bmatrix}, q = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$y = e^t \left(\left(C_1 \begin{bmatrix} -4 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 4 \end{bmatrix} \right) \cos(4t) + \left(C_2 \begin{bmatrix} -4 \\ 0 \end{bmatrix} - C_1 \begin{bmatrix} 0 \\ 4 \end{bmatrix} \right) \sin(4t) \right)$$

2. Sketch the general solution:



Unstable

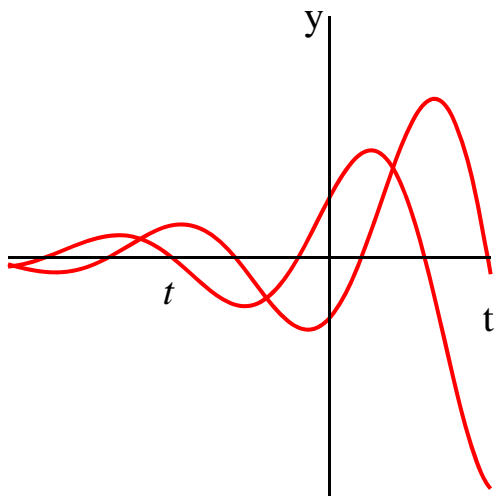
3. Find the particular solution:

$$C_1 \begin{bmatrix} -4 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$C_1 = -\frac{1}{4}, C_2 = -\frac{1}{4}$$

$$y = e^t \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \cos(4t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \sin(4t) \right)$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -14 & 24 \\ -12 & 10 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

1. Find the general solution:

$$\text{Det} \begin{bmatrix} -14 - \lambda & 24 \\ -12 & 10 - \lambda \end{bmatrix} = 0$$

$$\lambda^2 + 4\lambda + 148 = 0$$

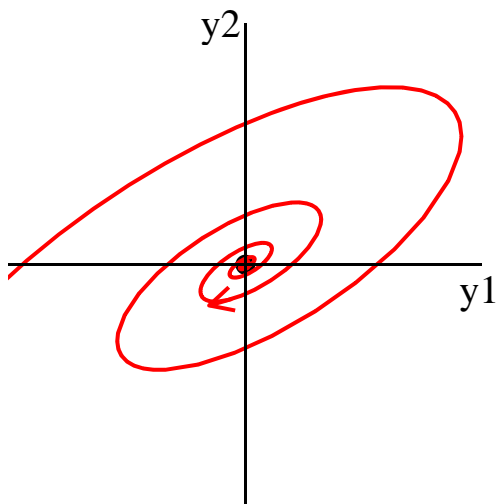
$$\lambda = -2 + 12I, v = \begin{bmatrix} 24 \\ 12 + 12I \end{bmatrix}$$

$$\lambda = -2 - 12I, v = \begin{bmatrix} 24 \\ 12 - 12I \end{bmatrix}$$

$$\alpha = -2, \beta = 12, p = \begin{bmatrix} 24 \\ 12 \end{bmatrix}, q = \begin{bmatrix} 0 \\ 12 \end{bmatrix}$$

$$y = e^{-2t} \left(\left(C_1 \begin{bmatrix} 24 \\ 12 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 12 \end{bmatrix} \right) \cos(12t) + \left(C_2 \begin{bmatrix} 24 \\ 12 \end{bmatrix} - C_1 \begin{bmatrix} 0 \\ 12 \end{bmatrix} \right) \sin(12t) \right)$$

2. Sketch the general solution:



Stable

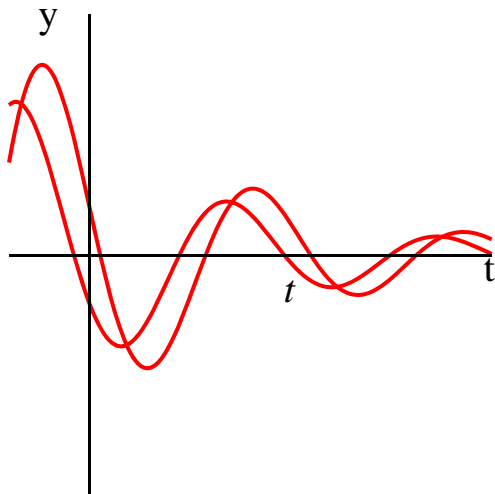
3. Find the particular solution:

$$C_1 \begin{bmatrix} 24 \\ 12 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 12 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

$$C_1 = \frac{1}{6}, C_2 = -\frac{1}{2}$$

$$y = e^{-2t} \left(\begin{bmatrix} 4 \\ -4 \end{bmatrix} \cos(12t) + \begin{bmatrix} -12 \\ -8 \end{bmatrix} \sin(12t) \right)$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 6 & -10 \\ 4 & -6 \end{bmatrix} y$$

$$y(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

1. Find the general solution:

$$\text{Det} \begin{bmatrix} 6 - \lambda & -10 \\ 4 & -6 - \lambda \end{bmatrix} = 0$$

$$\lambda^2 + 4 = 0$$

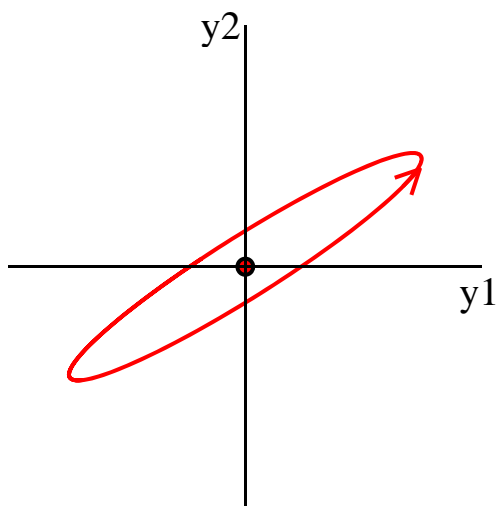
$$\lambda = 2i, v = \begin{bmatrix} -10 \\ -6 + 2i \end{bmatrix}$$

$$\lambda = -2i, v = \begin{bmatrix} -10 \\ -6 - 2i \end{bmatrix}$$

$$\alpha = 0, \beta = 2, p = \begin{bmatrix} -10 \\ -6 \end{bmatrix}, q = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$y = \left(C_1 \begin{bmatrix} -10 \\ -6 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix} \right) \cos(2t) + \left(C_2 \begin{bmatrix} -10 \\ -6 \end{bmatrix} - C_1 \begin{bmatrix} 0 \\ 2 \end{bmatrix} \right) \sin(2t)$$

2. Sketch the general solution:



center

3. Find the particular solution:

$$C_1 \begin{bmatrix} -10 \\ -6 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$C_1 = -\frac{1}{5}, C_2 = -\frac{1}{10}$$

$$y = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \cos(2t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \sin(2t)$$

4. Sketch the particular solution:

