Linear Algebra Homework 2

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LU Decomposition (LUD)

LU decomposition of A

$$A = \begin{bmatrix} 2 & -1 \\ -4 & -1 \end{bmatrix}$$

1. Forward eliminate

$$\begin{bmatrix} 2 & -1 \\ 0 & -3 \end{bmatrix}, \begin{bmatrix} \\ R2 + 2\,R1 \end{bmatrix}$$

2. Build L and U

$$L = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}, U = \begin{bmatrix} 2 & -1 \\ 0 & -3 \end{bmatrix}$$

LU decomposition of A

$$A = \begin{bmatrix} 3 & 4 \\ -6 & -10 \end{bmatrix}$$

1. Forward eliminate

$$\begin{bmatrix} 3 & 4 \\ 0 & -2 \end{bmatrix}, \begin{bmatrix} \\ R2 + 2\,R1 \end{bmatrix}$$

2. Build L and U

$$L = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}, U = \begin{bmatrix} 3 & 4 \\ 0 & -2 \end{bmatrix}$$

LU decomposition of A

$$A = \begin{bmatrix} 2 & 4 & -1 \\ -6 & -15 & 5 \\ 8 & 22 & -10 \end{bmatrix}$$

1. Forward eliminate

$$\begin{bmatrix} 2 & 4 & -1 \\ 0 & -3 & 2 \\ 0 & 6 & -6 \end{bmatrix}, \begin{bmatrix} \\ R2 + 3\,R1 \\ R3 - 4\,R1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & -1 \\ 0 & -3 & 2 \\ 0 & 0 & -2 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 + 2\,R2 \end{bmatrix}$$

2. Build L and U

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -2 & 1 \end{bmatrix}, U = \begin{bmatrix} 2 & 4 & -1 \\ 0 & -3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$$

LU decomposition of A

$$A = \begin{bmatrix} 3 & 1 & -2 \\ -6 & -3 & 5 \\ 9 & 7 & -8 \end{bmatrix}$$

1. Forward eliminate

$$\begin{bmatrix} 3 & 1 & -2 \\ 0 & -1 & 1 \\ 0 & 4 & -2 \end{bmatrix}, \begin{bmatrix} \\ R2 + 2\,R1 \\ R3 - 3\,R1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & -2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 + 4\,R2 \end{bmatrix}$$

2. Build L and U

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix}, U = \begin{bmatrix} 3 & 1 & -2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

LU decomposition of A

$$A = \begin{bmatrix} 2 & -1 & 2 & -4 \\ 4 & -5 & 5 & -12 \\ 2 & 8 & -5 & 5 \\ -8 & 16 & 4 & 40 \end{bmatrix}$$

1. Forward eliminate

$$\begin{bmatrix} 2 & -1 & 2 & -4 \\ 0 & -3 & 1 & -4 \\ 0 & 9 & -7 & 9 \\ 0 & 12 & 12 & 24 \end{bmatrix}, \begin{bmatrix} \\ R2 - 2\,R1 \\ R3 - R1 \\ R4 + 4\,R1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 2 & -4 \\ 0 & -3 & 1 & -4 \\ 0 & 0 & -4 & -3 \\ 0 & 0 & 16 & 8 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 + 3\,R2 \\ R4 + 4\,R2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 2 & -4 \\ 0 & -3 & 1 & -4 \\ 0 & 0 & -4 & -3 \\ 0 & 0 & 0 & -4 \end{bmatrix}, \begin{bmatrix} \\ \\ \\ R4 + 4\,R3 \end{bmatrix}$$

2. Build L and U

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & -3 & 1 & 0 \\ -4 & -4 & -4 & 1 \end{bmatrix}, U = \begin{bmatrix} 2 & -1 & 2 & -4 \\ 0 & -3 & 1 & -4 \\ 0 & 0 & -4 & -3 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

LU decomposition of A

$$A = \begin{bmatrix} -1 & 4 & -3 & -4 \\ -4 & 20 & -16 & -12 \\ -2 & 24 & -23 & 7 \\ 1 & 4 & -8 & 13 \end{bmatrix}$$

1. Forward eliminate

$$\begin{bmatrix} -1 & 4 & -3 & -4 \\ 0 & 4 & -4 & 4 \\ 0 & 16 & -17 & 15 \\ 0 & 8 & -11 & 9 \end{bmatrix}, \begin{bmatrix} \\ R2 - 4\,R1 \\ R3 - 2\,R1 \\ R4 + R1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 4 & -3 & -4 \\ 0 & 4 & -4 & 4 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -3 & 1 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 - 4\,R2 \\ R4 - 2\,R2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 4 & -3 & -4 \\ 0 & 4 & -4 & 4 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 4 \end{bmatrix}, \begin{bmatrix} \\ \\ \\ R4 - 3\,R3 \end{bmatrix}$$

2. Build L and U

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 \\ -1 & 2 & 3 & 1 \end{bmatrix}, U = \begin{bmatrix} -1 & 4 & -3 & -4 \\ 0 & 4 & -4 & 4 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Cholesky Decomposition (CD)

Cholesky decomposition of S

$$S = \begin{bmatrix} 16 & 8 \\ 8 & 20 \end{bmatrix}$$

1. Iterate

$$S = \begin{bmatrix} 16 & 8 \\ 8 & 20 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$S = \begin{bmatrix} 16 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 4 \end{bmatrix}$$

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2. Build L

$$L = \begin{bmatrix} 4 & 0 \\ 2 & 4 \end{bmatrix}$$

Cholesky decomposition of S

$$S = \begin{bmatrix} 4 & -2 \\ -2 & 10 \end{bmatrix}$$

1. Iterate

$$S = \begin{bmatrix} 4 & -2 \\ -2 & 10 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$S = \begin{bmatrix} 9 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 3 \end{bmatrix}$$

----

2. Build L

$$L = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$$

Cholesky decomposition of S

$$S = \begin{bmatrix} 4 & 4 & -6 \\ 4 & 20 & 10 \\ -6 & 10 & 29 \end{bmatrix}$$

1. Iterate

$$S = \begin{bmatrix} 4 & 4 & -6 \\ 4 & 20 & 10 \\ -6 & 10 & 29 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 2 \\ 2 \\ -3 \end{bmatrix}$$

$$S = \begin{bmatrix} 16 & 16 \\ 16 & 20 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$S = \begin{bmatrix} 4 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 2 \end{bmatrix}$$

----

2. Build L

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 4 & 0 \\ -3 & 4 & 2 \end{bmatrix}$$

Cholesky decomposition of S

$$S = \begin{bmatrix} 4 & -6 & 8 \\ -6 & 13 & -16 \\ 8 & -16 & 36 \end{bmatrix}$$

1. Iterate

$$S = \begin{bmatrix} 4 & -6 & 8 \\ -6 & 13 & -16 \\ 8 & -16 & 36 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$$

$$S = \begin{bmatrix} 4 & -4 \\ -4 & 20 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$S = \begin{bmatrix} 16 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 4 \end{bmatrix}$$

----

2. Build L

$$L = \begin{bmatrix} 2 & 0 & 0 \\ -3 & 2 & 0 \\ 4 & -2 & 4 \end{bmatrix}$$

Cholesky decomposition of S

$$S = \begin{bmatrix} 4 & 8 & -6 & 6 \\ 8 & 17 & -14 & 14 \\ -6 & -14 & 17 & -5 \\ 6 & 14 & -5 & 38 \end{bmatrix}$$

1. Iterate

$$S = \begin{bmatrix} 4 & 8 & -6 & 6 \\ 8 & 17 & -14 & 14 \\ -6 & -14 & 17 & -5 \\ 6 & 14 & -5 & 38 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 2 \\ 4 \\ -3 \\ 3 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 8 & 4 \\ 2 & 4 & 29 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

$$S = \begin{bmatrix} 4 & 8 \\ 8 & 25 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$S = \begin{bmatrix} 9 \end{bmatrix}$$

$$L_4 = \begin{bmatrix} 3 \end{bmatrix}$$

----

2. Build L

$$L = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ -3 & -2 & 2 & 0 \\ 3 & 2 & 4 & 3 \end{bmatrix}$$

Cholesky decomposition of S

$$S = \begin{bmatrix} 16 & -8 & -4 & -4 \\ -8 & 13 & -4 & 8 \\ -4 & -4 & 6 & -7 \\ -4 & 8 & -7 & 25 \end{bmatrix}$$

1. Iterate

$$S = \begin{bmatrix} 16 & -8 & -4 & -4 \\ -8 & 13 & -4 & 8 \\ -4 & -4 & 6 & -7 \\ -4 & 8 & -7 & 25 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 4 \\ -2 \\ -1 \\ -1 \end{bmatrix}$$

$$S = \begin{bmatrix} 9 & -6 & 6 \\ -6 & 5 & -8 \\ 6 & -8 & 24 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & -4 \\ -4 & 20 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

$$S = \begin{bmatrix} 4 \end{bmatrix}$$

$$L_4 = \begin{bmatrix} 2 \end{bmatrix}$$

----

2. Build L

$$L = \begin{bmatrix} 4 & 0 & 0 & 0 \\ -2 & 3 & 0 & 0 \\ -1 & -2 & 1 & 0 \\ -1 & 2 & -4 & 2 \end{bmatrix}$$

QR Decomposition (QRD)

QR decomposition of A

$$A = \begin{bmatrix} 14 & -6 \\ 14 & -9 \\ 7 & -6 \end{bmatrix}$$

1. Iterate

$$w_1 = \begin{bmatrix} 14 \\ 14 \\ 7 \end{bmatrix}, mag(w_1) = 21, q_1 = \begin{bmatrix} \frac{2}{3} \\ \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$q_1^t\, a_2 = -12$$

$$w_2 = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}, mag(w_2) = 3, q_2 = \begin{bmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ -\frac{2}{3} \end{bmatrix}$$

----

2. Build Q and R

$$Q = \begin{bmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix}, R = \begin{bmatrix} 21 & -12 \\ 0 & 3 \end{bmatrix}$$

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QR decomposition of A
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$$A = \begin{bmatrix} -10 & -6 & 29 \\ -10 & -21 & 5 \\ 5 & 18 & 5 \end{bmatrix}$$

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1. Iterate
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$$w_1 = \begin{bmatrix} -10 \\ -10 \\ 5 \end{bmatrix}, mag(w_1) = 15, q_1 = \begin{bmatrix} -\frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$q_1^t a_2 = 24$$

$$w_2 = \begin{bmatrix} 10 \\ -5 \\ 10 \end{bmatrix}, mag(w_2) = 15, q_2 = \begin{bmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$$

$$q_1^t a_3 = -21$$
$$q_2^t a_3 = 21$$

$$w_3 = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}, mag(w_3) = 3, q_3 = \begin{bmatrix} \frac{1}{3} \\ -\frac{2}{3} \\ -\frac{2}{3} \end{bmatrix}$$

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2. Build Q and R
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$$Q = \begin{bmatrix} -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \end{bmatrix}, R = \begin{bmatrix} 15 & 24 & -21 \\ 0 & 15 & 21 \\ 0 & 0 & 3 \end{bmatrix}$$

QR decomposition of A

$$A = \begin{bmatrix} -9 & 9 & 4 \\ -9 & -3 & 12 \\ -9 & -3 & 10 \\ -9 & 9 & 6 \end{bmatrix}$$

1. Iterate

$$w_1 = \begin{bmatrix} -9 \\ -9 \\ -9 \\ -9 \end{bmatrix}, mag(w_1) = 18, q_1 = \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

$$q_1^t\, a_2 = -6$$

$$w_2 = \begin{bmatrix} 6 \\ -6 \\ -6 \\ 6 \end{bmatrix}, mag(w_2) = 12, q_2 = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$q_1^t\, a_3 = -16$$

$$q_2^t\, a_3 = -6$$

$$w_3 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, mag(w_3) = 2, q_3 = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

----

2. Build Q and R

$$Q = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}, R = \begin{bmatrix} 18 & -6 & -16 \\ 0 & 12 & -6 \\ 0 & 0 & 2 \end{bmatrix}$$

QR decomposition of A

$$A = \begin{bmatrix} -3 & -8 & -9 \\ -3 & -10 & 15 \\ -3 & -8 & 7 \\ -3 & -10 & -1 \end{bmatrix}$$

1. Iterate

$$w_1 = \begin{bmatrix} -3 \\ -3 \\ -3 \\ -3 \end{bmatrix}, mag(w_1) = 6, q_1 = \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

$$q_1^t\, a_2 = 18$$

$$w_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, mag(w_2) = 2, q_2 = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

$$q_1^t\, a_3 = -6$$
$$q_2^t\, a_3 = -8$$

$$w_3 = \begin{bmatrix} -8 \\ 8 \\ 8 \\ -8 \end{bmatrix}, mag(w_3) = 16, q_3 = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

----

2. Build Q and R

$$Q = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}, R = \begin{bmatrix} 6 & 18 & -6 \\ 0 & 2 & -8 \\ 0 & 0 & 16 \end{bmatrix}$$

QR decomposition of A

$$A = \begin{bmatrix} -2 & -18 & -2 & -2 \\ 0 & 0 & 0 & 10 \\ 0 & 0 & 14 & 10 \\ 0 & 14 & -12 & -14 \end{bmatrix}$$

1. Iterate

$$w_1 = \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, mag(w_1) = 2, q_1 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$q_1^t a_2 = 18$$

$$w_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 14 \end{bmatrix}, mag(w_2) = 14, q_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$q_1^t a_3 = 2$$

$$q_2^t a_3 = -12$$

$$w_3 = \begin{bmatrix} 0 \\ 0 \\ 14 \\ 0 \end{bmatrix}, mag(w_3) = 14, q_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$q_1^t a_4 = 2$$

$$q_2^t a_4 = -14$$

$$q_3^t a_4 = 10$$

$$w_4 = \begin{bmatrix} 0 \\ 10 \\ 0 \\ 0 \end{bmatrix}, mag(w_4) = 10, q_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

----

2. Build Q and R

$$Q = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, R = \begin{bmatrix} 2 & 18 & 2 & 2 \\ 0 & 14 & -12 & -14 \\ 0 & 0 & 14 & 10 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

QR decomposition of A

$$A = \begin{bmatrix} 8 & -18 & -12 & -12 \\ 0 & 0 & 0 & 16 \\ 0 & 0 & -8 & 8 \\ 0 & 12 & 6 & -14 \end{bmatrix}$$

1. Iterate

$$w_1 = \begin{bmatrix} 8 \\ 0 \\ 0 \\ 0 \end{bmatrix}, mag(w_1) = 8, q_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$q_1^t a_2 = -18$$

$$w_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 12 \end{bmatrix}, mag(w_2) = 12, q_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$q_1^t a_3 = -12$$

$$q_2^t a_3 = 6$$

$$w_3 = \begin{bmatrix} 0 \\ 0 \\ -8 \\ 0 \end{bmatrix}, mag(w_3) = 8, q_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$$q_1^t a_4 = -12$$

$$q_2^t a_4 = -14$$

$$q_3^t a_4 = -8$$

$$w_4 = \begin{bmatrix} 0 \\ 16 \\ 0 \\ 0 \end{bmatrix}, mag(w_4) = 16, q_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

----

2. Build Q and R

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, R = \begin{bmatrix} 8 & -18 & -12 & -12 \\ 0 & 12 & 6 & -14 \\ 0 & 0 & 8 & -8 \\ 0 & 0 & 0 & 16 \end{bmatrix}$$

Eigen Decomposition (ED)

Eigen decomposition of A

$$A = \begin{bmatrix} -1 & 6 \\ -2 & 6 \end{bmatrix}$$

1. Find the eigenvalues

$$0 = \lambda^2 - 5\lambda + 6$$

$$\lambda = (3, 2)$$

2. Find the eigenvectors

$$\lambda = 3, C = \begin{bmatrix} -4 & 6 \\ -2 & 3 \end{bmatrix}, RREF = \begin{bmatrix} 1 & -\frac{3}{2} \\ 0 & 0 \end{bmatrix}, v = \begin{bmatrix} \frac{3}{2} \\ 1 \end{bmatrix}$$

$$\lambda = 2, C = \begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix}, RREF = \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}, v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

3. Build Lambda

$$\Lambda = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

4. Build V

$$V = \begin{bmatrix} \frac{3}{13}\sqrt{13} & \frac{2}{5}\sqrt{5} \\ \frac{2}{13}\sqrt{13} & \frac{1}{5}\sqrt{5} \end{bmatrix}$$

Eigen decomposition of A

$$A = \begin{bmatrix} -2 & -2 \\ 2 & 3 \end{bmatrix}$$

1. Find the eigenvalues

$$0 = \lambda^2 - \lambda - 2$$

$$\lambda = (2, -1)$$

2. Find the eigenvectors

$$\lambda = 2, C = \begin{bmatrix} -4 & -2 \\ 2 & 1 \end{bmatrix}, RREF = \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 0 \end{bmatrix}, v = \begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}$$

$$\lambda = -1, C = \begin{bmatrix} -1 & -2 \\ 2 & 4 \end{bmatrix}, RREF = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, v = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

3. Build Lambda

$$\Lambda = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$$

4. Build V

$$V = \begin{bmatrix} -\frac{1}{5}\sqrt{5} & -\frac{2}{5}\sqrt{5} \\ \frac{2}{5}\sqrt{5} & \frac{1}{5}\sqrt{5} \end{bmatrix}$$

Eigen decomposition of A

$$A = \begin{bmatrix} 2 & 3 & -3 \\ -2 & -1 & 2 \\ 2 & 4 & -3 \end{bmatrix}$$

1. Find the eigenvalues

$$0 = -\lambda^3 - 2\lambda^2 + \lambda + 2$$

$$\lambda = (1, -1, -2)$$

2. Find the eigenvectors

$$\lambda = 1, C = \begin{bmatrix} 1 & 3 & -3 \\ -2 & -2 & 2 \\ 2 & 4 & -4 \end{bmatrix}, RREF = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda = -1, C = \begin{bmatrix} 3 & 3 & -3 \\ -2 & 0 & 2 \\ 2 & 4 & -2 \end{bmatrix}, RREF = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, v = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = -2, C = \begin{bmatrix} 4 & 3 & -3 \\ -2 & 1 & 2 \\ 2 & 4 & -1 \end{bmatrix}, RREF = \begin{bmatrix} 1 & 0 & -\frac{9}{10} \\ 0 & 1 & \frac{1}{5} \\ 0 & 0 & 0 \end{bmatrix}, v = \begin{bmatrix} \frac{9}{10} \\ -\frac{1}{5} \\ 1 \end{bmatrix}$$

3. Build Lambda

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

4. Build V

$$V = \begin{bmatrix} 0 & \frac{1}{2}\sqrt{2} & \frac{9}{185}\sqrt{185} \\ \frac{1}{2}\sqrt{2} & 0 & -\frac{2}{185}\sqrt{185} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} & \frac{2}{37}\sqrt{185} \end{bmatrix}$$

Eigen decomposition of A

$$A = \begin{bmatrix} -1 & 4 & -2 \\ -1 & 3 & -1 \\ -3 & 3 & -1 \end{bmatrix}$$

1. Find the eigenvalues

$$0 = -\lambda^3 + \lambda^2 + 4\lambda - 4$$

$$\lambda = (2, 1, -2)$$

2. Find the eigenvectors

$$\lambda = 2, C = \begin{bmatrix} -3 & 4 & -2 \\ -1 & 1 & -1 \\ -3 & 3 & -3 \end{bmatrix}, RREF = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}, v = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$

$$\lambda = 1, C = \begin{bmatrix} -2 & 4 & -2 \\ -1 & 2 & -1 \\ -3 & 3 & -2 \end{bmatrix}, RREF = \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & 0 \end{bmatrix}, v = \begin{bmatrix} -\frac{1}{3} \\ \frac{1}{3} \\ 1 \end{bmatrix}$$

$$\lambda = -2, C = \begin{bmatrix} 1 & 4 & -2 \\ -1 & 5 & -1 \\ -3 & 3 & 1 \end{bmatrix}, RREF = \begin{bmatrix} 1 & 0 & -\frac{2}{3} \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & 0 \end{bmatrix}, v = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ 1 \end{bmatrix}$$

3. Build Lambda

$$\Lambda = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

4. Build V

$$V = \begin{bmatrix} -\frac{1}{3}\sqrt{6} & -\frac{1}{11}\sqrt{11} & \frac{1}{7}\sqrt{14} \\ -\frac{1}{6}\sqrt{6} & \frac{1}{11}\sqrt{11} & \frac{1}{14}\sqrt{14} \\ \frac{1}{6}\sqrt{6} & \frac{3}{11}\sqrt{11} & \frac{3}{14}\sqrt{14} \end{bmatrix}$$

Eigen decomposition of A

$$A = \begin{bmatrix} 3 & 2 & 2 \\ -1 & -4 & 2 \\ -2 & -2 & -1 \end{bmatrix}$$

1. Find the eigenvalues

$$0 = -\lambda^3 - 2\lambda^2 + \lambda + 2$$

$$\lambda = (1, -1, -2)$$

2. Find the eigenvectors

$$\lambda = 1, C = \begin{bmatrix} 2 & 2 & 2 \\ -1 & -5 & 2 \\ -2 & -2 & -2 \end{bmatrix}, RREF = \begin{bmatrix} 1 & 0 & \frac{7}{4} \\ 0 & 1 & -\frac{3}{4} \\ 0 & 0 & 0 \end{bmatrix}, v = \begin{bmatrix} -\frac{7}{4} \\ \frac{3}{4} \\ 1 \end{bmatrix}$$

$$\lambda = -1, C = \begin{bmatrix} 4 & 2 & 2 \\ -1 & -3 & 2 \\ -2 & -2 & 0 \end{bmatrix}, RREF = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}, v = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda = -2, C = \begin{bmatrix} 5 & 2 & 2 \\ -1 & -2 & 2 \\ -2 & -2 & 1 \end{bmatrix}, RREF = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -\frac{3}{2} \\ 0 & 0 & 0 \end{bmatrix}, v = \begin{bmatrix} -1 \\ \frac{3}{2} \\ 1 \end{bmatrix}$$

3. Build Lambda

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

4. Build V

$$V = \begin{bmatrix} -\frac{7}{74}\sqrt{74} & -\frac{1}{3}\sqrt{3} & -\frac{2}{17}\sqrt{17} \\ \frac{3}{74}\sqrt{74} & \frac{1}{3}\sqrt{3} & \frac{3}{17}\sqrt{17} \\ \frac{2}{37}\sqrt{74} & \frac{1}{3}\sqrt{3} & \frac{2}{17}\sqrt{17} \end{bmatrix}$$

Singular Value Decomposition (SVD)

Singular value decomposition of A

$$A = \begin{bmatrix} 8 & 6 \\ 3 & -4 \end{bmatrix}$$

1. Compute S = At A

$$S = \begin{bmatrix} 73 & 36 \\ 36 & 52 \end{bmatrix}$$

2. Compute the eigen decomposition of S

$$\Lambda = \begin{bmatrix} 100 & 0 \\ 0 & 25 \end{bmatrix}, V = \begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$$

3. Build Sigma

$$\Sigma = \begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix}$$

4. Build U

$$U = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Singular value decomposition of A

$$A = \begin{bmatrix} 18 & 26 \\ 18 & 1 \\ -27 & -14 \end{bmatrix}$$

1. Compute S = At A

$$S = \begin{bmatrix} 1377 & 864 \\ 864 & 873 \end{bmatrix}$$

2. Compute the eigen decomposition of S

$$\Lambda = \begin{bmatrix} 2025 & 0 \\ 0 & 225 \end{bmatrix}, V = \begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$$

3. Build Sigma

$$\Sigma = \begin{bmatrix} 45 & 0 \\ 0 & 15 \end{bmatrix}$$

4. Build U

$$U = \begin{bmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

Singular value decomposition of A

$$A = \begin{bmatrix} 6 & 6 & 3 \\ -1 & 2 & -2 \\ 4 & -2 & -4 \end{bmatrix}$$

1. Compute S = At A

$$S = \begin{bmatrix} 53 & 26 & 4 \\ 26 & 44 & 22 \\ 4 & 22 & 29 \end{bmatrix}$$

2. Compute the eigen decomposition of S

$$\Lambda = \begin{bmatrix} 81 & 0 & 0 \\ 0 & 36 & 0 \\ 0 & 0 & 9 \end{bmatrix}, V = \begin{bmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

3. Build Sigma

$$\Sigma = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

4. Build U

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Singular value decomposition of A

$$A = \begin{bmatrix} 18 & 1 \\ 18 & 1 \\ 6 & 17 \\ -6 & -17 \end{bmatrix}$$

1. Compute S = At A

$$S = \begin{bmatrix} 720 & 240 \\ 240 & 580 \end{bmatrix}$$

2. Compute the eigen decomposition of S

$$\Lambda = \begin{bmatrix} 900 & 0 \\ 0 & 400 \end{bmatrix}, V = \begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$$

3. Build Sigma

$$\Sigma = \begin{bmatrix} 30 & 0 \\ 0 & 20 \end{bmatrix}$$

4. Build U

$$U = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

Singular value decomposition of A

$$A = \begin{bmatrix} 11 & 8 & 2 \\ 13 & 4 & -2 \\ 3 & 12 & -6 \\ -5 & -8 & 10 \end{bmatrix}$$

1. Compute S = At A

$$S = \begin{bmatrix} 324 & 216 & -72 \\ 216 & 288 & -144 \\ -72 & -144 & 144 \end{bmatrix}$$

2. Compute the eigen decomposition of S

$$\Lambda = \begin{bmatrix} 576 & 0 & 0 \\ 0 & 144 & 0 \\ 0 & 0 & 36 \end{bmatrix}, V = \begin{bmatrix} -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

3. Build Sigma

$$\Sigma = \begin{bmatrix} 24 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

4. Build U

$$U = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Singular value decomposition of A

$$A = \begin{bmatrix} 5 & 5 & 5 & 5 \\ -3 & 3 & 3 & -3 \\ -2 & 2 & -2 & 2 \\ 1 & 1 & -1 & -1 \end{bmatrix}$$

1. Compute S = At A

$$S = \begin{bmatrix} 39 & 13 & 19 & 29 \\ 13 & 39 & 29 & 19 \\ 19 & 29 & 39 & 13 \\ 29 & 19 & 13 & 39 \end{bmatrix}$$

2. Compute the eigen decomposition of S

$$\Lambda = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 36 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}, V = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

3. Build Sigma

$$\Sigma = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

4. Build U

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$