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Linear Algebra Homework 1

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▼ Solve using Gauss-Jordan elimination

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} -3 & -2 \\ 3 & 1 \end{bmatrix}, b = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -3 & -2 & -5 \\ 3 & 1 & 4 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} -3 & -2 & -5 \\ 0 & -1 & -1 \end{bmatrix}, \begin{bmatrix} R2 + R1 \end{bmatrix}$$

3. Backward eliminate

$$\begin{bmatrix} -3 & 0 & -3 \\ 0 & -1 & -1 \end{bmatrix}, \begin{bmatrix} R1 - 2R2 \end{bmatrix}$$

4. Scale

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} 2 & -4 \\ 6 & -8 \end{bmatrix}, b = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 2 & -4 & 0 \\ 6 & -8 & -4 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} 2 & -4 & 0 \\ 0 & 4 & -4 \end{bmatrix}, \begin{bmatrix} R2 - 3R1 \end{bmatrix}$$

3. Backward eliminate

$$\begin{bmatrix} 2 & 0 & -4 \\ 0 & 4 & -4 \end{bmatrix}, \begin{bmatrix} R2 + R1 \end{bmatrix}$$

4. Scale

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} -1 & 4 \\ 1 & -5 \end{bmatrix}, b = \begin{bmatrix} 7 \\ -9 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -1 & 4 & 7 \\ 1 & -5 & -9 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} -1 & 4 & 7 \\ 0 & -1 & -2 \end{bmatrix}, \begin{bmatrix} R2 + R1 \end{bmatrix}$$

3. Backward eliminate

$$\begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -2 \end{bmatrix}, \begin{bmatrix} R1 + 4R2 \end{bmatrix}$$

4. Scale

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} -2 & 4 \\ 2 & 0 \end{bmatrix}, b = \begin{bmatrix} -8 \\ -4 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -2 & 4 & -8 \\ 2 & 0 & -4 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} -2 & 4 & -8 \\ 0 & 4 & -12 \end{bmatrix}, \begin{bmatrix} R2 + R1 \end{bmatrix}$$

3. Backward eliminate

$$\begin{bmatrix} -2 & 0 & 4 \\ 0 & 4 & -12 \end{bmatrix}, \begin{bmatrix} R1 - R2 \end{bmatrix}$$

4. Scale

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} -1 & 4 & -4 \\ -1 & 0 & -2 \\ -3 & 0 & -4 \end{bmatrix}, b = \begin{bmatrix} -4 \\ 0 \\ -4 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -1 & 4 & -4 & -4 \\ -1 & 0 & -2 & 0 \\ -3 & 0 & -4 & -4 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} -1 & 4 & -4 & -4 \\ 0 & -4 & 2 & 4 \\ 0 & -12 & 8 & 8 \end{bmatrix}, \begin{bmatrix} R2 - R1 \\ R3 - 3R1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 4 & -4 & -4 \\ 0 & -4 & 2 & 4 \\ 0 & 0 & 2 & -4 \end{bmatrix}, \begin{bmatrix} R3 - 3R2 \end{bmatrix}$$

3. Backward eliminate

$$\begin{bmatrix} -1 & 4 & 0 & -12 \\ 0 & -4 & 0 & 8 \\ 0 & 0 & 2 & -4 \end{bmatrix}, \begin{bmatrix} R1 + 2R3 \\ R2 - R3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 & -4 \\ 0 & -4 & 0 & 8 \\ 0 & 0 & 2 & -4 \end{bmatrix}, \begin{bmatrix} R2 + R1 \end{bmatrix}$$

4. Scale

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix}$$

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} 3 & -6 & 6 \\ -3 & 9 & -8 \\ 3 & -9 & 6 \end{bmatrix}, b = \begin{bmatrix} -6 \\ 0 \\ 6 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 3 & -6 & 6 & -6 \\ -3 & 9 & -8 & 0 \\ 3 & -9 & 6 & 6 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} 3 & -6 & 6 & -6 \\ 0 & 3 & -2 & -6 \\ 0 & -3 & 0 & 12 \end{bmatrix}, \begin{bmatrix} R2 + R1 \\ R3 - R1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -6 & 6 & -6 \\ 0 & 3 & -2 & -6 \\ 0 & 0 & -2 & 6 \end{bmatrix}, \begin{bmatrix} R3 + R2 \end{bmatrix}$$

3. Backward eliminate

$$\begin{bmatrix} 3 & -6 & 0 & 12 \\ 0 & 3 & 0 & -12 \\ 0 & 0 & -2 & 6 \end{bmatrix}, \begin{bmatrix} R1 + 3R3 \\ R2 - R3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & -12 \\ 0 & 3 & 0 & -12 \\ 0 & 0 & -2 & 6 \end{bmatrix}, \begin{bmatrix} R1 + 2R2 \end{bmatrix}$$

4. Scale

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} -4 \\ -4 \\ -3 \end{bmatrix}$$

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} 4 & -6 & 8 \\ -4 & 3 & -2 \\ 4 & 0 & -2 \end{bmatrix}, b = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 4 & -6 & 8 & -2 \\ -4 & 3 & -2 & -1 \\ 4 & 0 & -2 & 0 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} 4 & -6 & 8 & -2 \\ 0 & -3 & 6 & -3 \\ 0 & 6 & -10 & 2 \end{bmatrix}, \begin{bmatrix} R2 + R1 \\ R3 - R1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -6 & 8 & -2 \\ 0 & -3 & 6 & -3 \\ 0 & 0 & 2 & -4 \end{bmatrix}, \begin{bmatrix} R3 + 2R2 \end{bmatrix}$$

3. Backward eliminate

$$\begin{bmatrix} 4 & -6 & 0 & 14 \\ 0 & -3 & 0 & 9 \\ 0 & 0 & 2 & -4 \end{bmatrix}, \begin{bmatrix} R1 - 4R3 \\ R2 - 3R3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 & 0 & -4 \\ 0 & -3 & 0 & 9 \\ 0 & 0 & 2 & -4 \end{bmatrix}, \begin{bmatrix} R1 - 2R2 \end{bmatrix}$$

4. Scale

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} -1 \\ -3 \\ -2 \end{bmatrix}$$

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} -3 & 1 & 2 \\ 9 & -4 & -8 \\ -9 & 5 & 8 \end{bmatrix}, b = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -3 & 1 & 2 & -3 \\ 9 & -4 & -8 & 3 \\ -9 & 5 & 8 & 1 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} -3 & 1 & 2 & -3 \\ 0 & -1 & -2 & -6 \\ 0 & 2 & 2 & 10 \end{bmatrix}, \begin{bmatrix} R2 + 3R1 \\ R3 - 3R1 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 1 & 2 & -3 \\ 0 & -1 & -2 & -6 \\ 0 & 0 & -2 & -2 \end{bmatrix}, \begin{bmatrix} R3 + 2R2 \end{bmatrix}$$

3. Backward eliminate

$$\begin{bmatrix} -3 & 1 & 0 & -5 \\ 0 & -1 & 0 & -4 \\ 0 & 0 & -2 & -2 \end{bmatrix}, \begin{bmatrix} R1 + R3 \\ R2 - R3 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 0 & 0 & -9 \\ 0 & -1 & 0 & -4 \\ 0 & 0 & -2 & -2 \end{bmatrix}, \begin{bmatrix} R2 + R1 \end{bmatrix}$$

4. Scale

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

5. Solution

$$x = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

▼ Solve using Gauss-Jordan elimination (None/Many Solutions)

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} -2 & 6 & -4 \\ 2 & -4 & 2 \\ 2 & -4 & 2 \end{bmatrix}, b = \begin{bmatrix} -8 \\ -6 \\ -2 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -2 & 6 & -4 & -8 \\ 2 & -4 & 2 & -6 \\ 2 & -4 & 2 & -2 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} -2 & 6 & -4 & -8 \\ 0 & 2 & -2 & -14 \\ 0 & 2 & -2 & -10 \end{bmatrix}, \begin{bmatrix} R2 + R1 \\ R1 + R3 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 6 & -4 & -8 \\ 0 & 2 & -2 & -14 \\ 0 & 0 & 0 & 4 \end{bmatrix}, \begin{bmatrix} R3 - R2 \end{bmatrix}$$

3. Solution: None!

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} -3 & -9 & 12 \\ 3 & 9 & -12 \\ 3 & 9 & -12 \end{bmatrix}, b = \begin{bmatrix} -6 \\ 15 \\ 9 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -3 & -9 & 12 & -6 \\ 3 & 9 & -12 & 15 \\ 3 & 9 & -12 & 9 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} -3 & -9 & 12 & -6 \\ 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \begin{bmatrix} \\ R2 + R1 \\ R1 + R3 \end{bmatrix}$$

3. Solution: None!

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} 4 & -2 & 2 \\ 4 & 0 & 4 \\ 16 & -16 & 0 \end{bmatrix}, b = \begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 4 & -2 & 2 & 4 \\ 4 & 0 & 4 & 8 \\ 16 & -16 & 0 & 0 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} 4 & -2 & 2 & 4 \\ 0 & 2 & 2 & 4 \\ 0 & -8 & -8 & -16 \end{bmatrix}, \begin{bmatrix} R2 - R1 \\ R3 - 4R1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 & 2 & 4 \\ 0 & 2 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} R3 + 4R2 \end{bmatrix}$$

3. Backward eliminate

$$\begin{bmatrix} 4 & 0 & 4 & 8 \\ 0 & 2 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} R2 + R1 \end{bmatrix}$$

4. Scale

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Solution

$$x = t_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} -2 & -6 & -4 \\ 6 & 18 & 12 \\ 2 & 6 & 4 \end{bmatrix}, b = \begin{bmatrix} -4 \\ 12 \\ 4 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -2 & -6 & -4 & -4 \\ 6 & 18 & 12 & 12 \\ 2 & 6 & 4 & 4 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} -2 & -6 & -4 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} R2 + 3R1 \\ R1 + R3 \end{bmatrix}$$

3. Backward eliminate

4. Scale

$$\begin{bmatrix} 1 & 3 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Solution

$$x = t_2 \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} + t_3 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} 3 & 2 & -2 & 8 \\ 12 & 7 & -12 & 19 \\ -12 & -6 & 18 & 0 \\ -9 & -4 & 18 & 14 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 13 \\ -6 \\ 2 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 3 & 2 & -2 & 8 & 3 \\ 12 & 7 & -12 & 19 & 13 \\ -12 & -6 & 18 & 0 & -6 \\ -9 & -4 & 18 & 14 & 2 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} 3 & 2 & -2 & 8 & 3 \\ 0 & -1 & -4 & -13 & 1 \\ 0 & 2 & 10 & 32 & 6 \\ 0 & 2 & 12 & 38 & 11 \end{bmatrix}, \begin{bmatrix} \\ R2 - 4R1 \\ R3 + 4R1 \\ R4 + 3R1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & -2 & 8 & 3 \\ 0 & -1 & -4 & -13 & 1 \\ 0 & 0 & 2 & 6 & 8 \\ 0 & 0 & 4 & 12 & 13 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 + 2R2 \\ R4 + 2R2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & -2 & 8 & 3 \\ 0 & -1 & -4 & -13 & 1 \\ 0 & 0 & 2 & 6 & 8 \\ 0 & 0 & 0 & 0 & -3 \end{bmatrix}, \begin{bmatrix} \\ \\ \\ R4 - 2R3 \end{bmatrix}$$

3. Solution: None!

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} 2 & 3 & 5 & 4 \\ 2 & 2 & 4 & 2 \\ -4 & -5 & -9 & -6 \\ 8 & 10 & 18 & 12 \end{bmatrix}, b = \begin{bmatrix} -4 \\ -4 \\ 6 \\ -14 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 2 & 3 & 5 & 4 & -4 \\ 2 & 2 & 4 & 2 & -4 \\ -4 & -5 & -9 & -6 & 6 \\ 8 & 10 & 18 & 12 & -14 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} 2 & 3 & 5 & 4 & -4 \\ 0 & -1 & -1 & -2 & 0 \\ 0 & 1 & 1 & 2 & -2 \\ 0 & -2 & -2 & -4 & 2 \end{bmatrix}, \begin{bmatrix} \\ R2 - R1 \\ R3 + 2R1 \\ R4 - 4R1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 5 & 4 & -4 \\ 0 & -1 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 + R2 \\ R4 - 2R2 \end{bmatrix}$$

3. Solution: None!

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} -2 & 2 & 2 & -4 \\ 2 & -2 & -2 & 4 \\ 8 & -8 & -8 & 16 \\ -4 & 4 & 4 & -8 \end{bmatrix}, b = \begin{bmatrix} 6 \\ -10 \\ -8 \\ -4 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} -2 & 2 & 2 & -4 & 6 \\ 2 & -2 & -2 & 4 & -10 \\ 8 & -8 & -8 & 16 & -8 \\ -4 & 4 & 4 & -8 & -4 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} -2 & 2 & 2 & -4 & 6 \\ 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 16 \\ 0 & 0 & 0 & 0 & -16 \end{bmatrix}, \begin{bmatrix} \\ R2 + R1 \\ R3 + 4R1 \\ R4 - 2R1 \end{bmatrix}$$

3. Solution: None!

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} 2 & 2 & 1 & 1 \\ -6 & -7 & -4 & -1 \\ -2 & -6 & -6 & 8 \\ -2 & -3 & -1 & 0 \end{bmatrix}, b = \begin{bmatrix} 4 \\ -11 \\ 4 \\ -7 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 2 & 2 & 1 & 1 & 4 \\ -6 & -7 & -4 & -1 & -11 \\ -2 & -6 & -6 & 8 & 4 \\ -2 & -3 & -1 & 0 & -7 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} 2 & 2 & 1 & 1 & 4 \\ 0 & -1 & -1 & 2 & 1 \\ 0 & -4 & -5 & 9 & 8 \\ 0 & -1 & 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} \\ R2 + 3R1 \\ R1 + R3 \\ R4 + R1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 1 & 1 & 4 \\ 0 & -1 & -1 & 2 & 1 \\ 0 & 0 & -1 & 1 & 4 \\ 0 & 0 & 1 & -1 & -4 \end{bmatrix} \begin{bmatrix} \\ \\ R3 - 4R2 \\ R4 - R2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 1 & 1 & 4 \\ 0 & -1 & -1 & 2 & 1 \\ 0 & 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \\ \\ R4 + R3 \\ \end{bmatrix}$$

3. Backward eliminate

$$\begin{bmatrix} 2 & 2 & 0 & 2 & 8 \\ 0 & -1 & 0 & 1 & -3 \\ 0 & 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} R1 + R3 \\ R2 - R3 \\ \\ \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & 4 & 2 \\ 0 & -1 & 0 & 1 & -3 \\ 0 & 0 & -1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} R1 + 2R2 \\ \\ \\ \end{bmatrix}$$

4. Scale

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 1 \\ 0 & 1 & 0 & -1 & 3 \\ 0 & 0 & 1 & -1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Solution

$$x = t_4 \begin{bmatrix} -2 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \\ -4 \\ 0 \end{bmatrix}$$

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} 2 & 1 & 2 & 10 \\ -2 & -2 & 2 & -12 \\ -2 & -3 & 6 & -14 \\ 4 & 0 & 12 & 16 \end{bmatrix}, b = \begin{bmatrix} -7 \\ 10 \\ 13 \\ -8 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 2 & 1 & 2 & 10 & -7 \\ -2 & -2 & 2 & -12 & 10 \\ -2 & -3 & 6 & -14 & 13 \\ 4 & 0 & 12 & 16 & -8 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} 2 & 1 & 2 & 10 & -7 \\ 0 & -1 & 4 & -2 & 3 \\ 0 & -2 & 8 & -4 & 6 \\ 0 & -2 & 8 & -4 & 6 \end{bmatrix}, \begin{bmatrix} \\ R2 + R1 \\ R1 + R3 \\ R4 - 2R1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 2 & 10 & -7 \\ 0 & -1 & 4 & -2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} \\ \\ R3 - 2R2 \\ R4 - 2R2 \end{bmatrix}$$

3. Backward eliminate

$$\begin{bmatrix} 2 & 0 & 6 & 8 & -4 \\ 0 & -1 & 4 & -2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} R2 + R1 \\ \\ \\ \end{bmatrix}$$

4. Scale

$$\begin{bmatrix} 1 & 0 & 3 & 4 & -2 \\ 0 & 1 & -4 & 2 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Solution

$$x = t_3 \cdot \begin{bmatrix} -3 \\ 4 \\ 1 \\ 0 \end{bmatrix} + t_4 \cdot \begin{bmatrix} -4 \\ -2 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \\ 0 \\ 0 \end{bmatrix}$$

Solve $Ax = b$ using Gauss-Jordan

$$A = \begin{bmatrix} 2 & 6 & 2 & 2 \\ 4 & 12 & 4 & 4 \\ -6 & -18 & -6 & -6 \\ 2 & 6 & 2 & 2 \end{bmatrix}, b = \begin{bmatrix} -4 \\ -8 \\ 12 \\ -4 \end{bmatrix}$$

1. Augment

$$\begin{bmatrix} 2 & 6 & 2 & 2 & -4 \\ 4 & 12 & 4 & 4 & -8 \\ -6 & -18 & -6 & -6 & 12 \\ 2 & 6 & 2 & 2 & -4 \end{bmatrix}$$

2. Forward eliminate

$$\begin{bmatrix} 2 & 6 & 2 & 2 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} R2 - 2R1 \\ R3 + 3R1 \\ R4 - R1 \end{bmatrix}$$

3. Backward eliminate

4. Scale

$$\begin{bmatrix} 1 & 3 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Solution

$$x = t_2 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t_3 \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t_4 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

▼ Determinant

Compute the determinant of A using the definition

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}$$

0. Prepare

$$\det(A) = (a_{1,1} e_1 + a_{2,1} e_2) (a_{1,2} e_1 + a_{2,2} e_2)$$

1. Apply the multilinearity

$$\det(A) = a_{1,1} a_{1,2} \det(e_1, e_1) + a_{1,1} a_{2,2} \det(e_1, e_2) + a_{2,1} a_{1,2} \det(e_2, e_1) + a_{2,1} a_{2,2} \det(e_2, e_2)$$

2. Apply the antisymmetry

$$\det(A) = a_{1,1} a_{2,2} \det(e_1, e_2) - a_{2,1} a_{1,2} \det(e_1, e_2)$$

3. Apply the normality

$$\det(A) = a_{1,1} a_{2,2} - a_{2,1} a_{1,2}$$

Compute the determinant of A using the definition

$$A = \begin{bmatrix} -1 & -8 \\ 5 & -5 \end{bmatrix}$$

0. Prepare

$$\det(A) = (-e_1 + 5e_2) \cdot (-8e_1 - 5e_2)$$

1. Apply the multilinearity

$$\det(A) = 8 \det(e_1, e_1) + 5 \det(e_1, e_2) - 40 \det(e_2, e_1) - 25 \det(e_2, e_2)$$

2. Apply the antisymmetry

$$\det(A) = 45 \det(e_1, e_2)$$

3. Apply the normality

$$\det(A) = 45$$

Compute the determinant of A using the definition

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix}$$

0. Prepare

$$\det(A) = (a_{1,1}e_1 + a_{2,1}e_2 + a_{3,1}e_3) (a_{1,2}e_1 + a_{2,2}e_2 + a_{3,2}e_3) (a_{1,3}e_1 + a_{2,3}e_2 + a_{3,3}e_3)$$

1. Apply the multilinearity

$$\begin{aligned} \det(A) = & a_{2,1}a_{2,2}a_{1,3} \det(e_2, e_2, e_1) + a_{2,1}a_{2,2}a_{2,3} \det(e_2, e_2, e_2) + a_{2,1}a_{2,2}a_{3,3} \det(e_2, e_2, \\ & e_3) + a_{2,1}a_{3,2}a_{1,3} \det(e_2, e_3, e_1) + a_{2,1}a_{3,2}a_{2,3} \det(e_2, e_3, e_2) + a_{2,1}a_{3,2}a_{3,3} \det(e_2, \\ & e_3, e_3) + a_{3,1}a_{1,2}a_{1,3} \det(e_3, e_1, e_1) + a_{3,1}a_{1,2}a_{2,3} \det(e_3, e_1, e_2) \\ & + a_{3,1}a_{1,2}a_{3,3} \det(e_3, e_1, e_3) + a_{3,1}a_{2,2}a_{1,3} \det(e_3, e_2, e_1) + a_{3,1}a_{2,2}a_{2,3} \det(e_3, e_2, e_2) \\ & + a_{3,1}a_{2,2}a_{3,3} \det(e_3, e_2, e_3) + a_{3,1}a_{3,2}a_{1,3} \det(e_3, e_3, e_1) + a_{3,1}a_{3,2}a_{2,3} \det(e_3, e_3, e_2) \\ & + a_{3,1}a_{3,2}a_{3,3} \det(e_3, e_3, e_3) + a_{1,1}a_{1,2}a_{1,3} \det(e_1, e_1, e_1) + a_{1,1}a_{1,2}a_{2,3} \det(e_1, e_1, e_2) \\ & + a_{1,1}a_{1,2}a_{3,3} \det(e_1, e_1, e_3) + a_{1,1}a_{2,2}a_{1,3} \det(e_1, e_2, e_1) + a_{1,1}a_{2,2}a_{2,3} \det(e_1, e_2, e_2) \\ & + a_{1,1}a_{2,2}a_{3,3} \det(e_1, e_2, e_3) + a_{1,1}a_{3,2}a_{1,3} \det(e_1, e_3, e_1) + a_{1,1}a_{3,2}a_{2,3} \det(e_1, e_3, e_2) \\ & + a_{1,1}a_{3,2}a_{3,3} \det(e_1, e_3, e_3) + a_{2,1}a_{1,2}a_{1,3} \det(e_2, e_1, e_1) + a_{2,1}a_{1,2}a_{2,3} \det(e_2, e_1, e_2) \\ & + a_{2,1}a_{1,2}a_{3,3} \det(e_2, e_1, e_3) \end{aligned}$$

2. Apply the antisymmetry

$$\begin{aligned} \det(A) = & a_{1,1}a_{2,2}a_{3,3} \det(e_1, e_2, e_3) - a_{1,1}a_{3,2}a_{2,3} \det(e_1, e_2, e_3) - a_{2,1}a_{1,2}a_{3,3} \det(e_1, e_2, \\ & e_3) + a_{2,1}a_{3,2}a_{1,3} \det(e_1, e_2, e_3) + a_{3,1}a_{1,2}a_{2,3} \det(e_1, e_2, e_3) - a_{3,1}a_{2,2}a_{1,3} \det(e_1, \\ & e_2, e_3) \end{aligned}$$

3. Apply the normality

$$\begin{aligned} \det(A) = & a_{1,1}a_{2,2}a_{3,3} - a_{1,1}a_{3,2}a_{2,3} - a_{2,1}a_{1,2}a_{3,3} + a_{2,1}a_{3,2}a_{1,3} + a_{3,1}a_{1,2}a_{2,3} \\ & - a_{3,1}a_{2,2}a_{1,3} \end{aligned}$$

Compute the determinant of A using the definition

$$A = \begin{bmatrix} 5 & -6 & 3 \\ 7 & 5 & 8 \\ -6 & -8 & -2 \end{bmatrix}$$

0. Prepare

$$\det(A) = (5 e_1 + 7 e_2 - 6 e_3) (-6 e_1 + 5 e_2 - 8 e_3) (3 e_1 + 8 e_2 - 2 e_3)$$

1. Apply the multilinearity

$$\begin{aligned} \det(A) = & -70 \det(e_2, e_2, e_3) - 168 \det(e_2, e_3, e_1) - 448 \det(e_2, e_3, e_2) + 112 \det(e_2, e_3, e_3) \\ & + 108 \det(e_3, e_1, e_1) + 288 \det(e_3, e_1, e_2) - 72 \det(e_3, e_1, e_3) - 90 \det(e_3, e_2, e_1) \\ & - 240 \det(e_3, e_2, e_2) + 60 \det(e_3, e_2, e_3) + 144 \det(e_3, e_3, e_1) + 384 \det(e_3, e_3, e_2) \\ & - 96 \det(e_3, e_3, e_3) - 90 \det(e_1, e_1, e_1) - 240 \det(e_1, e_1, e_2) + 60 \det(e_1, e_1, e_3) \\ & + 75 \det(e_1, e_2, e_1) + 200 \det(e_1, e_2, e_2) - 50 \det(e_1, e_2, e_3) - 120 \det(e_1, e_3, e_1) \\ & - 320 \det(e_1, e_3, e_2) + 80 \det(e_1, e_3, e_3) - 126 \det(e_2, e_1, e_1) - 336 \det(e_2, e_1, e_2) \\ & + 84 \det(e_2, e_1, e_3) + 105 \det(e_2, e_2, e_1) + 280 \det(e_2, e_2, e_2) \end{aligned}$$

2. Apply the antisymmetry

$$\det(A) = 396 \det(e_1, e_2, e_3)$$

3. Apply the normality

$$\det(A) = 396$$

▼ Solve using Cramer's Rule

Solve $Ax = b$ using Cramer

$$A = \begin{bmatrix} -3 & -2 \\ 3 & 1 \end{bmatrix}, b = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$$

1. Denominator

$$\det \left(\begin{bmatrix} -3 & -2 \\ 3 & 1 \end{bmatrix} \right) = 3$$

2. Numerators

$$\det \left(\begin{bmatrix} -5 & -2 \\ 4 & 1 \end{bmatrix} \right) = 3$$

$$\det \left(\begin{bmatrix} -3 & -5 \\ 3 & 4 \end{bmatrix} \right) = 3$$

3. Solution

$$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Solve $Ax = b$ using Cramer

$$A = \begin{bmatrix} 2 & -4 \\ 6 & -8 \end{bmatrix}, b = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

1. Denominator

$$\det \left(\begin{bmatrix} 2 & -4 \\ 6 & -8 \end{bmatrix} \right) = 8$$

2. Numerators

$$\det \left(\begin{bmatrix} 0 & -4 \\ -4 & -8 \end{bmatrix} \right) = -16$$

$$\det \left(\begin{bmatrix} 2 & 0 \\ 6 & -4 \end{bmatrix} \right) = -8$$

3. Solution

$$x = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

Solve $Ax = b$ using Cramer

$$A = \begin{bmatrix} -1 & 4 \\ 1 & -5 \end{bmatrix}, b = \begin{bmatrix} 7 \\ -9 \end{bmatrix}$$

1. Denominator

$$\det \left(\begin{bmatrix} -1 & 4 \\ 1 & -5 \end{bmatrix} \right) = 1$$

2. Numerators

$$\det \left(\begin{bmatrix} 7 & 4 \\ -9 & -5 \end{bmatrix} \right) = 1$$

$$\det \left(\begin{bmatrix} -1 & 7 \\ 1 & -9 \end{bmatrix} \right) = 2$$

3. Solution

$$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Solve $Ax = b$ using Cramer

$$A = \begin{bmatrix} -2 & 4 \\ 2 & 0 \end{bmatrix}, b = \begin{bmatrix} -8 \\ -4 \end{bmatrix}$$

1. Denominator

$$\det \left(\begin{bmatrix} -2 & 4 \\ 2 & 0 \end{bmatrix} \right) = -8$$

2. Numerators

$$\det \left(\begin{bmatrix} -8 & 4 \\ -4 & 0 \end{bmatrix} \right) = 16$$

$$\det \left(\begin{bmatrix} -2 & -8 \\ 2 & -4 \end{bmatrix} \right) = 24$$

3. Solution

$$x = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

Solve $Ax = b$ using Cramer

$$A = \begin{bmatrix} -1 & 4 & -4 \\ -1 & 0 & -2 \\ -3 & 0 & -4 \end{bmatrix}, b = \begin{bmatrix} -4 \\ 0 \\ -4 \end{bmatrix}$$

1. Denominator

$$\det \begin{pmatrix} \begin{bmatrix} -1 & 4 & -4 \\ -1 & 0 & -2 \\ -3 & 0 & -4 \end{bmatrix} \end{pmatrix} = 8$$

2. Numerators

$$\det \begin{pmatrix} \begin{bmatrix} -4 & 4 & -4 \\ 0 & 0 & -2 \\ -4 & 0 & -4 \end{bmatrix} \end{pmatrix} = 32$$

$$\det \begin{pmatrix} \begin{bmatrix} -1 & -4 & -4 \\ -1 & 0 & -2 \\ -3 & -4 & -4 \end{bmatrix} \end{pmatrix} = -16$$

$$\det \begin{pmatrix} \begin{bmatrix} -1 & 4 & -4 \\ -1 & 0 & 0 \\ -3 & 0 & -4 \end{bmatrix} \end{pmatrix} = -16$$

3. Solution

$$x = \begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix}$$

Solve $Ax = b$ using Cramer

$$A = \begin{bmatrix} 3 & -6 & 6 \\ -3 & 9 & -8 \\ 3 & -9 & 6 \end{bmatrix}, b = \begin{bmatrix} -6 \\ 0 \\ 6 \end{bmatrix}$$

1. Denominator

$$\det \begin{pmatrix} \begin{bmatrix} 3 & -6 & 6 \\ -3 & 9 & -8 \\ 3 & -9 & 6 \end{bmatrix} \end{pmatrix} = -18$$

2. Numerators

$$\det \begin{pmatrix} \begin{bmatrix} -6 & -6 & 6 \\ 0 & 9 & -8 \\ 6 & -9 & 6 \end{bmatrix} \end{pmatrix} = 72$$

$$\det \begin{pmatrix} \begin{bmatrix} 3 & -6 & 6 \\ -3 & 0 & -8 \\ 3 & 6 & 6 \end{bmatrix} \end{pmatrix} = 72$$

$$\det \begin{pmatrix} \begin{bmatrix} 3 & -6 & -6 \\ -3 & 9 & 0 \\ 3 & -9 & 6 \end{bmatrix} \end{pmatrix} = 54$$

3. Solution

$$x = \begin{bmatrix} -4 \\ -4 \\ -3 \end{bmatrix}$$

Solve $Ax = b$ using Cramer

$$A = \begin{bmatrix} 4 & -6 & 8 \\ -4 & 3 & -2 \\ 4 & 0 & -2 \end{bmatrix}, b = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

1. Denominator

$$\det \begin{pmatrix} \begin{bmatrix} 4 & -6 & 8 \\ -4 & 3 & -2 \\ 4 & 0 & -2 \end{bmatrix} \end{pmatrix} = -24$$

2. Numerators

$$\det \begin{pmatrix} \begin{bmatrix} -2 & -6 & 8 \\ -1 & 3 & -2 \\ 0 & 0 & -2 \end{bmatrix} \end{pmatrix} = 24$$

$$\det \begin{pmatrix} \begin{bmatrix} 4 & -2 & 8 \\ -4 & -1 & -2 \\ 4 & 0 & -2 \end{bmatrix} \end{pmatrix} = 72$$

$$\det \begin{pmatrix} \begin{bmatrix} 4 & -6 & -2 \\ -4 & 3 & -1 \\ 4 & 0 & 0 \end{bmatrix} \end{pmatrix} = 48$$

3. Solution

$$x = \begin{bmatrix} -1 \\ -3 \\ -2 \end{bmatrix}$$

Solve $Ax = b$ using Cramer

$$A = \begin{bmatrix} -3 & 1 & 2 \\ 9 & -4 & -8 \\ -9 & 5 & 8 \end{bmatrix}, b = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$$

1. Denominator

$$\det \begin{pmatrix} \begin{bmatrix} -3 & 1 & 2 \\ 9 & -4 & -8 \\ -9 & 5 & 8 \end{bmatrix} \end{pmatrix} = -6$$

2. Numerators

$$\det \begin{pmatrix} \begin{bmatrix} -3 & 1 & 2 \\ 3 & -4 & -8 \\ 1 & 5 & 8 \end{bmatrix} \end{pmatrix} = -18$$

$$\det \begin{pmatrix} \begin{bmatrix} -3 & -3 & 2 \\ 9 & 3 & -8 \\ -9 & 1 & 8 \end{bmatrix} \end{pmatrix} = -24$$

$$\det \begin{pmatrix} \begin{bmatrix} -3 & 1 & -3 \\ 9 & -4 & 3 \\ -9 & 5 & 1 \end{bmatrix} \end{pmatrix} = -6$$

3. Solution

$$x = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

▼ Invert matrix using determinants

Invert A using determinant

$$A = \begin{bmatrix} -3 & -2 \\ 3 & 1 \end{bmatrix}$$

1. Determinant of A

$$\det \left(\begin{bmatrix} -3 & -2 \\ 3 & 1 \end{bmatrix} \right) = 3$$

2. Adjugate matrix of A

$$\begin{bmatrix} \det \left(\begin{bmatrix} 1 \end{bmatrix} \right) & -\det \left(\begin{bmatrix} -2 \end{bmatrix} \right) \\ -\det \left(\begin{bmatrix} 3 \end{bmatrix} \right) & \det \left(\begin{bmatrix} -3 \end{bmatrix} \right) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -3 & -3 \end{bmatrix}$$

3. Divide

$$\text{inverse}(A) = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ -1 & -1 \end{bmatrix}$$

Invert A using determinant

$$A = \begin{bmatrix} 2 & -4 \\ 6 & -8 \end{bmatrix}$$

1. Determinant of A

$$\det\left(\begin{bmatrix} 2 & -4 \\ 6 & -8 \end{bmatrix}\right) = 8$$

2. Adjugate matrix of A

$$\begin{bmatrix} \det\left(\begin{bmatrix} -8 \end{bmatrix}\right) & -\det\left(\begin{bmatrix} -4 \end{bmatrix}\right) \\ -\det\left(\begin{bmatrix} 6 \end{bmatrix}\right) & \det\left(\begin{bmatrix} 2 \end{bmatrix}\right) \end{bmatrix} = \begin{bmatrix} -8 & 4 \\ -6 & 2 \end{bmatrix}$$

3. Divide

$$\text{inverse}(A) = \begin{bmatrix} -1 & \frac{1}{2} \\ -\frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

Invert A using determinant

$$A = \begin{bmatrix} -1 & 4 \\ 1 & -5 \end{bmatrix}$$

1. Determinant of A

$$\det \left(\begin{bmatrix} -1 & 4 \\ 1 & -5 \end{bmatrix} \right) = 1$$

2. Adjugate matrix of A

$$\begin{bmatrix} \det \left(\begin{bmatrix} -5 \end{bmatrix} \right) & -\det \left(\begin{bmatrix} 4 \end{bmatrix} \right) \\ -\det \left(\begin{bmatrix} 1 \end{bmatrix} \right) & \det \left(\begin{bmatrix} -1 \end{bmatrix} \right) \end{bmatrix} = \begin{bmatrix} -5 & -4 \\ -1 & -1 \end{bmatrix}$$

3. Divide

$$\text{inverse}(A) = \begin{bmatrix} -5 & -4 \\ -1 & -1 \end{bmatrix}$$

Invert A using determinant

$$A = \begin{bmatrix} -2 & 4 \\ 2 & 0 \end{bmatrix}$$

1. Determinant of A

$$\det\left(\begin{bmatrix} -2 & 4 \\ 2 & 0 \end{bmatrix}\right) = -8$$

2. Adjugate matrix of A

$$\begin{bmatrix} \det\left(\begin{bmatrix} 0 \end{bmatrix}\right) & -\det\left(\begin{bmatrix} 4 \end{bmatrix}\right) \\ -\det\left(\begin{bmatrix} 2 \end{bmatrix}\right) & \det\left(\begin{bmatrix} -2 \end{bmatrix}\right) \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ -2 & -2 \end{bmatrix}$$

3. Divide

$$\text{inverse}(A) = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

Invert A using determinant

$$A = \begin{bmatrix} -1 & 4 & -4 \\ -1 & 0 & -2 \\ -3 & 0 & -4 \end{bmatrix}$$

1. Determinant of A

$$\det \left(\begin{bmatrix} -1 & 4 & -4 \\ -1 & 0 & -2 \\ -3 & 0 & -4 \end{bmatrix} \right) = 8$$

2. Adjugate matrix of A

$$\begin{bmatrix} \det \left(\begin{bmatrix} 0 & -2 \\ 0 & -4 \end{bmatrix} \right) & -\det \left(\begin{bmatrix} 4 & -4 \\ 0 & -4 \end{bmatrix} \right) & \det \left(\begin{bmatrix} 4 & -4 \\ 0 & -2 \end{bmatrix} \right) \\ -\det \left(\begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix} \right) & \det \left(\begin{bmatrix} -1 & -4 \\ -3 & -4 \end{bmatrix} \right) & -\det \left(\begin{bmatrix} -1 & -4 \\ -1 & -2 \end{bmatrix} \right) \\ \det \left(\begin{bmatrix} -1 & 0 \\ -3 & 0 \end{bmatrix} \right) & -\det \left(\begin{bmatrix} -1 & 4 \\ -3 & 0 \end{bmatrix} \right) & \det \left(\begin{bmatrix} -1 & 4 \\ -1 & 0 \end{bmatrix} \right) \end{bmatrix} = \begin{bmatrix} 0 & 16 & -8 \\ 2 & -8 & 2 \\ 0 & -12 & 4 \end{bmatrix}$$

3. Divide

$$\text{inverse}(A) = \begin{bmatrix} 0 & 2 & -1 \\ \frac{1}{4} & -1 & \frac{1}{4} \\ 0 & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

Invert A using determinant

$$A = \begin{bmatrix} 3 & -6 & 6 \\ -3 & 9 & -8 \\ 3 & -9 & 6 \end{bmatrix}$$

1. Determinant of A

$$\det \left(\begin{bmatrix} 3 & -6 & 6 \\ -3 & 9 & -8 \\ 3 & -9 & 6 \end{bmatrix} \right) = -18$$

2. Adjugate matrix of A

$$\begin{bmatrix} \det \left(\begin{bmatrix} 9 & -8 \\ -9 & 6 \end{bmatrix} \right) & -\det \left(\begin{bmatrix} -6 & 6 \\ -9 & 6 \end{bmatrix} \right) & \det \left(\begin{bmatrix} -6 & 6 \\ 9 & -8 \end{bmatrix} \right) \\ -\det \left(\begin{bmatrix} -3 & -8 \\ 3 & 6 \end{bmatrix} \right) & \det \left(\begin{bmatrix} 3 & 6 \\ 3 & 6 \end{bmatrix} \right) & -\det \left(\begin{bmatrix} 3 & 6 \\ -3 & -8 \end{bmatrix} \right) \\ \det \left(\begin{bmatrix} -3 & 9 \\ 3 & -9 \end{bmatrix} \right) & -\det \left(\begin{bmatrix} 3 & -6 \\ 3 & -9 \end{bmatrix} \right) & \det \left(\begin{bmatrix} 3 & -6 \\ -3 & 9 \end{bmatrix} \right) \end{bmatrix} = \begin{bmatrix} -18 & -18 & -6 \\ -6 & 0 & 6 \\ 0 & 9 & 9 \end{bmatrix}$$

3. Divide

$$\text{inverse}(A) = \begin{bmatrix} 1 & 1 & \frac{1}{3} \\ \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

Invert A using determinant

$$A = \begin{bmatrix} 4 & -6 & 8 \\ -4 & 3 & -2 \\ 4 & 0 & -2 \end{bmatrix}$$

1. Determinant of A

$$\det \left(\begin{bmatrix} 4 & -6 & 8 \\ -4 & 3 & -2 \\ 4 & 0 & -2 \end{bmatrix} \right) = -24$$

2. Adjugate matrix of A

$$\begin{bmatrix} \det \begin{pmatrix} 3 & -2 \\ 0 & -2 \end{pmatrix} & -\det \begin{pmatrix} -6 & 8 \\ 0 & -2 \end{pmatrix} & \det \begin{pmatrix} -6 & 8 \\ 3 & -2 \end{pmatrix} \\ -\det \begin{pmatrix} -4 & -2 \\ 4 & -2 \end{pmatrix} & \det \begin{pmatrix} 4 & 8 \\ 4 & -2 \end{pmatrix} & -\det \begin{pmatrix} 4 & 8 \\ -4 & -2 \end{pmatrix} \\ \det \begin{pmatrix} -4 & 3 \\ 4 & 0 \end{pmatrix} & -\det \begin{pmatrix} 4 & -6 \\ 4 & 0 \end{pmatrix} & \det \begin{pmatrix} 4 & -6 \\ -4 & 3 \end{pmatrix} \end{bmatrix} = \begin{bmatrix} -6 & -12 & -12 \\ -16 & -40 & -24 \\ -12 & -24 & -12 \end{bmatrix}$$

3. Divide

$$\text{inverse}(A) = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{5}{3} & 1 \\ \frac{1}{2} & 1 & \frac{1}{2} \end{bmatrix}$$

Invert A using determinant

$$A = \begin{bmatrix} -3 & 1 & 2 \\ 9 & -4 & -8 \\ -9 & 5 & 8 \end{bmatrix}$$

1. Determinant of A

$$\det \left(\begin{bmatrix} -3 & 1 & 2 \\ 9 & -4 & -8 \\ -9 & 5 & 8 \end{bmatrix} \right) = -6$$

2. Adjugate matrix of A

$$\begin{bmatrix} \det \left(\begin{bmatrix} -4 & -8 \\ 5 & 8 \end{bmatrix} \right) & -\det \left(\begin{bmatrix} 1 & 2 \\ 5 & 8 \end{bmatrix} \right) & \det \left(\begin{bmatrix} 1 & 2 \\ -4 & -8 \end{bmatrix} \right) \\ -\det \left(\begin{bmatrix} 9 & -8 \\ -9 & 8 \end{bmatrix} \right) & \det \left(\begin{bmatrix} -3 & 2 \\ -9 & 8 \end{bmatrix} \right) & -\det \left(\begin{bmatrix} -3 & 2 \\ 9 & -8 \end{bmatrix} \right) \\ \det \left(\begin{bmatrix} 9 & -4 \\ -9 & 5 \end{bmatrix} \right) & -\det \left(\begin{bmatrix} -3 & 1 \\ -9 & 5 \end{bmatrix} \right) & \det \left(\begin{bmatrix} -3 & 1 \\ 9 & -4 \end{bmatrix} \right) \end{bmatrix} = \begin{bmatrix} 8 & 2 & 0 \\ 0 & -6 & -6 \\ 9 & 6 & 3 \end{bmatrix}$$

3. Divide

$$\text{inverse}(A) = \begin{bmatrix} -\frac{4}{3} & -\frac{1}{3} & 0 \\ 0 & 1 & 1 \\ -\frac{3}{2} & -1 & -\frac{1}{2} \end{bmatrix}$$

▼ Volume of linear object using determinant (generalized Pythagorean Theorem)

Find the 1-volume (length) of the linear object defined by the vectors

$$\begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

2. Compute $B = A^t A$

$$B = \begin{bmatrix} 25 \end{bmatrix}$$

3. Compute $D = \det(B)$

$$D = 25$$

4. Compute the square-root of D

$$5$$

Find the 1-volume (length) of the linear object defined by the vectors

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

2. Compute $B = A^t A$

$$B = \begin{bmatrix} a_1^2 + a_2^2 \end{bmatrix}$$

3. Compute $D = \det(B)$

$$D = a_1^2 + a_2^2$$

4. Compute the square-root of D

$$\sqrt{a_1^2 + a_2^2}$$

Find the 1-volume (length) of the linear object defined by the vectors

$$\begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix}$$

2. Compute $B = A^t A$

$$B = \begin{bmatrix} 21 \end{bmatrix}$$

3. Compute $D = \det(B)$

$$D = 21$$

4. Compute the square-root of D

$$\sqrt{21}$$

Find the 1-volume (length) of the linear object defined by the vectors

$$\begin{bmatrix} -5 \\ 5 \\ -4 \\ 5 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} -5 \\ 5 \\ -4 \\ 5 \end{bmatrix}$$

2. Compute $B = A^t A$

$$B = \begin{bmatrix} 91 \end{bmatrix}$$

3. Compute $D = \det(B)$

$$D = 91$$

4. Compute the square-root of D

$$\sqrt{91}$$

Find the 2-volume (area) of the linear object defined by the vectors

$$\begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ -1 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} -2 & -4 \\ 3 & 1 \\ 2 & -1 \end{bmatrix}$$

2. Compute $B = A^t A$

$$B = \begin{bmatrix} 17 & 9 \\ 9 & 18 \end{bmatrix}$$

3. Compute $D = \det(B)$

$$D = 225$$

4. Compute the square-root of D

$$15$$

Find the 2-volume (area) of the linear object defined by the vectors

$$\begin{bmatrix} 5 \\ 5 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} 5 & 2 \\ 5 & 2 \\ -2 & 2 \end{bmatrix}$$

2. Compute $B = A^t A$

$$B = \begin{bmatrix} 54 & 16 \\ 16 & 12 \end{bmatrix}$$

3. Compute $D = \det(B)$

$$D = 392$$

4. Compute the square-root of D

$$14\sqrt{2}$$

Find the 2-volume (area) of the linear object defined by the vectors

$$\begin{bmatrix} 4 \\ -5 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \\ -4 \\ -1 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} 4 & 4 \\ -5 & -4 \\ -3 & -4 \\ -2 & -1 \end{bmatrix}$$

2. Compute $B = A^t A$

$$B = \begin{bmatrix} 54 & 50 \\ 50 & 49 \end{bmatrix}$$

3. Compute $D = \det(B)$

$$D = 146$$

4. Compute the square-root of D

$$\sqrt{146}$$

Find the 3-volume (volume) of the linear object defined by the vectors

$$\begin{bmatrix} -5 \\ -2 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ -5 \\ -3 \\ 5 \end{bmatrix}, \begin{bmatrix} -5 \\ -2 \\ -1 \\ -3 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} -5 & 5 & -5 \\ -2 & -5 & -2 \\ -1 & -3 & -1 \\ 5 & 5 & -3 \end{bmatrix}$$

2. Compute $B = A^t A$

$$B = \begin{bmatrix} 55 & 13 & 15 \\ 13 & 84 & -27 \\ 15 & -27 & 39 \end{bmatrix}$$

3. Compute $D = \det(B)$

$$D = 104064$$

4. Compute the square-root of D

$$8\sqrt{1626}$$

Find the 2-volume (area) of the linear object defined by the vectors

$$\begin{bmatrix} 1 \\ -4 \\ 5 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \\ -2 \\ 5 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} 1 & 3 \\ -4 & 3 \\ 5 & 3 \\ 2 & -2 \\ -3 & 5 \end{bmatrix}$$

2. Compute $B = A^t A$

$$B = \begin{bmatrix} 55 & -13 \\ -13 & 56 \end{bmatrix}$$

3. Compute $D = \det(B)$

$$D = 2911$$

4. Compute the square-root of D

$$\sqrt{2911}$$

Find the 3-volume (volume) of the linear object defined by the vectors

$$\begin{bmatrix} -3 \\ -4 \\ 3 \\ -4 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \\ -1 \\ -5 \\ 1 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} -3 & 3 & 4 \\ -4 & 1 & -4 \\ 3 & 2 & -1 \\ -4 & -3 & -5 \\ 5 & 3 & 1 \end{bmatrix}$$

2. Compute $B = A^t A$

$$B = \begin{bmatrix} 75 & 20 & 26 \\ 20 & 32 & 24 \\ 26 & 24 & 59 \end{bmatrix}$$

3. Compute $D = \det(B)$

$$D = 78128$$

4. Compute the square-root of D

$$4 \sqrt{4883}$$

Find the 4-volume of the linear object defined by the vectors

$$\begin{bmatrix} -2 \\ -1 \\ 5 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 4 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ -1 \\ -4 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 4 \\ 1 \\ -5 \end{bmatrix}$$

1. Form a matrix A

$$A = \begin{bmatrix} -2 & 2 & -4 & 1 \\ -1 & -4 & -1 & -1 \\ 5 & 4 & -4 & 4 \\ -2 & 1 & -2 & 1 \\ -3 & 3 & 5 & -5 \end{bmatrix}$$

2. Compute $B = A^t A$

$$B = \begin{bmatrix} 43 & 9 & -22 & 32 \\ 9 & 46 & -7 & 8 \\ -22 & -7 & 62 & -46 \\ 32 & 8 & -46 & 44 \end{bmatrix}$$

3. Compute $D = \det(B)$

$$D = 277200$$

4. Compute the square-root of D

$$60\sqrt{77}$$

