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# Linear Algebra Homework 1

(no answers)

Hoon Hong

## ▼ Solve using Gauss-Jordan elimination

Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} -3 & -2 \\ 3 & 1 \end{bmatrix}, b = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$$

Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} 2 & -4 \\ 6 & -8 \end{bmatrix}, b = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} -1 & 4 \\ 1 & -5 \end{bmatrix}, b = \begin{bmatrix} 7 \\ -9 \end{bmatrix}$$

Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} -2 & 4 \\ 2 & 0 \end{bmatrix}, b = \begin{bmatrix} -8 \\ -4 \end{bmatrix}$$

Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} -1 & 4 & -4 \\ -1 & 0 & -2 \\ -3 & 0 & -4 \end{bmatrix}, b = \begin{bmatrix} -4 \\ 0 \\ -4 \end{bmatrix}$$

Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} 3 & -6 & 6 \\ -3 & 9 & -8 \\ 3 & -9 & 6 \end{bmatrix}, b = \begin{bmatrix} -6 \\ 0 \\ 6 \end{bmatrix}$$



Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} 4 & -6 & 8 \\ -4 & 3 & -2 \\ 4 & 0 & -2 \end{bmatrix}, b = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} -3 & 1 & 2 \\ 9 & -4 & -8 \\ -9 & 5 & 8 \end{bmatrix}, b = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$$

**▼ Solve using Gauss-Jordan elimination (None/Many Solutions)**Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} -2 & 6 & -4 \\ 2 & -4 & 2 \\ 2 & -4 & 2 \end{bmatrix}, b = \begin{bmatrix} -8 \\ -6 \\ -2 \end{bmatrix}$$

Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} -3 & -9 & 12 \\ 3 & 9 & -12 \\ 3 & 9 & -12 \end{bmatrix}, b = \begin{bmatrix} -6 \\ 15 \\ 9 \end{bmatrix}$$

Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} 4 & -2 & 2 \\ 4 & 0 & 4 \\ 16 & -16 & 0 \end{bmatrix}, b = \begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix}$$

Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} -2 & -6 & -4 \\ 6 & 18 & 12 \\ 2 & 6 & 4 \end{bmatrix}, b = \begin{bmatrix} -4 \\ 12 \\ 4 \end{bmatrix}$$

Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} 3 & 2 & -2 & 8 \\ 12 & 7 & -12 & 19 \\ -12 & -6 & 18 & 0 \\ -9 & -4 & 18 & 14 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 13 \\ -6 \\ 2 \end{bmatrix}$$

Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} 2 & 3 & 5 & 4 \\ 2 & 2 & 4 & 2 \\ -4 & -5 & -9 & -6 \\ 8 & 10 & 18 & 12 \end{bmatrix}, b = \begin{bmatrix} -4 \\ -4 \\ 6 \\ -14 \end{bmatrix}$$



Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} -2 & 2 & 2 & -4 \\ 2 & -2 & -2 & 4 \\ 8 & -8 & -8 & 16 \\ -4 & 4 & 4 & -8 \end{bmatrix}, b = \begin{bmatrix} 6 \\ -10 \\ -8 \\ -4 \end{bmatrix}$$

Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} 2 & 2 & 1 & 1 \\ -6 & -7 & -4 & -1 \\ -2 & -6 & -6 & 8 \\ -2 & -3 & -1 & 0 \end{bmatrix}, b = \begin{bmatrix} 4 \\ -11 \\ 4 \\ -7 \end{bmatrix}$$

[extra space to work]

Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} 2 & 1 & 2 & 10 \\ -2 & -2 & 2 & -12 \\ -2 & -3 & 6 & -14 \\ 4 & 0 & 12 & 16 \end{bmatrix}, b = \begin{bmatrix} -7 \\ 10 \\ 13 \\ -8 \end{bmatrix}$$

[extra space to work]

Solve  $Ax = b$  using Gauss-Jordan

$$A = \begin{bmatrix} 2 & 6 & 2 & 2 \\ 4 & 12 & 4 & 4 \\ -6 & -18 & -6 & -6 \\ 2 & 6 & 2 & 2 \end{bmatrix}, b = \begin{bmatrix} -4 \\ -8 \\ 12 \\ -4 \end{bmatrix}$$

## ▼ Determinant

Compute the determinant of A using the definition

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}$$

Compute the determinant of A using the definition

$$A = \begin{bmatrix} -1 & -8 \\ 5 & -5 \end{bmatrix}$$



Compute the determinant of A using the definition

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix}$$

Compute the determinant of A using the definition

$$A = \begin{bmatrix} 5 & -6 & 3 \\ 7 & 5 & 8 \\ -6 & -8 & -2 \end{bmatrix}$$

## ▼ Solve using Cramer's Rule

Solve  $Ax = b$  using Cramer

$$A = \begin{bmatrix} -3 & -2 \\ 3 & 1 \end{bmatrix}, b = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$$

Solve  $Ax = b$  using Cramer

$$A = \begin{bmatrix} 2 & -4 \\ 6 & -8 \end{bmatrix}, b = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

Solve  $Ax = b$  using Cramer

$$A = \begin{bmatrix} -1 & 4 \\ 1 & -5 \end{bmatrix}, b = \begin{bmatrix} 7 \\ -9 \end{bmatrix}$$

Solve  $Ax = b$  using Cramer

$$A = \begin{bmatrix} -2 & 4 \\ 2 & 0 \end{bmatrix}, b = \begin{bmatrix} -8 \\ -4 \end{bmatrix}$$

Solve  $Ax = b$  using Cramer

$$A = \begin{bmatrix} -1 & 4 & -4 \\ -1 & 0 & -2 \\ -3 & 0 & -4 \end{bmatrix}, b = \begin{bmatrix} -4 \\ 0 \\ -4 \end{bmatrix}$$

Solve  $Ax = b$  using Cramer

$$A = \begin{bmatrix} 3 & -6 & 6 \\ -3 & 9 & -8 \\ 3 & -9 & 6 \end{bmatrix}, b = \begin{bmatrix} -6 \\ 0 \\ 6 \end{bmatrix}$$



Solve  $Ax = b$  using Cramer

$$A = \begin{bmatrix} 4 & -6 & 8 \\ -4 & 3 & -2 \\ 4 & 0 & -2 \end{bmatrix}, b = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

Solve  $Ax = b$  using Cramer

$$A = \begin{bmatrix} -3 & 1 & 2 \\ 9 & -4 & -8 \\ -9 & 5 & 8 \end{bmatrix}, b = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$$

## ▼ Invert matrix using determinants

Invert A using determinant

$$A = \begin{bmatrix} -3 & -2 \\ 3 & 1 \end{bmatrix}$$

Invert  $A$  using determinant

$$A = \begin{bmatrix} 2 & -4 \\ 6 & -8 \end{bmatrix}$$

Invert A using determinant

$$A = \begin{bmatrix} -1 & 4 \\ 1 & -5 \end{bmatrix}$$

Invert  $A$  using determinant

$$A = \begin{bmatrix} -2 & 4 \\ 2 & 0 \end{bmatrix}$$

Invert  $A$  using determinant

$$A = \begin{bmatrix} -1 & 4 & -4 \\ -1 & 0 & -2 \\ -3 & 0 & -4 \end{bmatrix}$$

Invert  $A$  using determinant

$$A = \begin{bmatrix} 3 & -6 & 6 \\ -3 & 9 & -8 \\ 3 & -9 & 6 \end{bmatrix}$$



Invert  $A$  using determinant

$$A = \begin{bmatrix} 4 & -6 & 8 \\ -4 & 3 & -2 \\ 4 & 0 & -2 \end{bmatrix}$$

Invert  $A$  using determinant

$$A = \begin{bmatrix} -3 & 1 & 2 \\ 9 & -4 & -8 \\ -9 & 5 & 8 \end{bmatrix}$$

## ▼ Volume of linear object using determinant (generalized Pythagorean Theorem)

Find the 1-volume (length) of the linear object defined by the vectors

$$\begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

Find the 1-volume (length) of the linear object defined by the vectors

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Find the 1-volume (length) of the linear object defined by the vectors

$$\begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix}$$

Find the 1-volume (length) of the linear object defined by the vectors

$$\begin{bmatrix} -5 \\ 5 \\ -4 \\ 5 \end{bmatrix}$$

Find the 2-volume (area) of the linear object defined by the vectors

$$\begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ -1 \end{bmatrix}$$

Find the 2-volume (area) of the linear object defined by the vectors

$$\begin{bmatrix} 5 \\ 5 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$



Find the 2-volume (area) of the linear object defined by the vectors

$$\begin{bmatrix} 4 \\ -5 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \\ -4 \\ -1 \end{bmatrix}$$

Find the 3-volume (volume) of the linear object defined by the vectors

$$\begin{bmatrix} -5 \\ -2 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ -5 \\ -3 \\ 5 \end{bmatrix}, \begin{bmatrix} -5 \\ -2 \\ -1 \\ -3 \end{bmatrix}$$

Find the 2-volume (area) of the linear object defined by the vectors

$$\begin{bmatrix} 1 \\ -4 \\ 5 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \\ -2 \\ 5 \end{bmatrix}$$

Find the 3-volume (volume) of the linear object defined by the vectors

$$\begin{bmatrix} -3 \\ -4 \\ 3 \\ -4 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \\ -1 \\ -5 \\ 1 \end{bmatrix}$$

Find the 4-volume of the linear object defined by the vectors

$$\begin{bmatrix} -2 \\ -1 \\ 5 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 4 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ -1 \\ -4 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 4 \\ 1 \\ -5 \end{bmatrix}$$

