

MA 303
Homework 2
(Non-homogeneous Linear Differential Equations)

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▼ **Undetermined Coefficient Method: 1st-order, 1 variable**

Problem:

$$y' + 3y = 5e^{2t}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda + 3 = 0$$

$$\lambda = -3$$

$$y_g = Ce^{-3t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = De^{2t}$$

$$5D = 5$$

$$D = 1$$

$$y_p = e^{2t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = Ce^{-3t} + e^{2t}$$

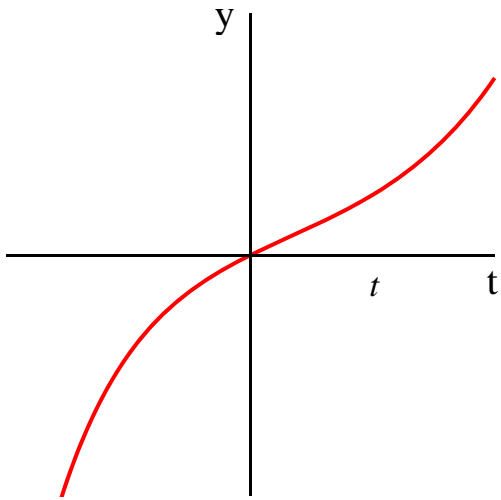
4. Find the particular solution of the non-homogeneous equation:

$$0 = C + 1$$

$$C = -1$$

$$y = -e^{-3t} + e^{2t}$$

5. Sketch the particular solution:



Problem:

$$y' + 4y = 5e^{-2t}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda + 4 = 0$$

$$\lambda = -4$$

$$y_g = Ce^{-4t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = De^{-2t}$$

$$2D = 5$$

$$D = \frac{5}{2}$$

$$y_p = \frac{5}{2}e^{-2t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = Ce^{-4t} + \frac{5}{2}e^{-2t}$$

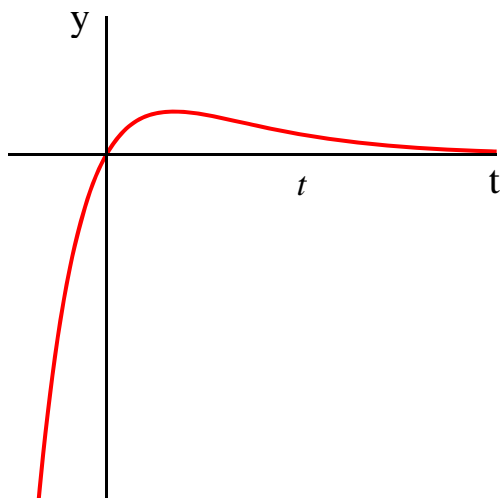
4. Find the particular solution of the non-homogeneous equation:

$$0 = C + \frac{5}{2}$$

$$C = -\frac{5}{2}$$

$$y = -\frac{5}{2}e^{-4t} + \frac{5}{2}e^{-2t}$$

5. Sketch the particular solution:



Problem:

$$y' - 4y = 5$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda - 4 = 0$$

$$\lambda = 4$$

$$y_g = C e^{4t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = D$$

$$-4D = 5$$

$$D = -\frac{5}{4}$$

$$y_p = -\frac{5}{4}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C e^{4t} - \frac{5}{4}$$

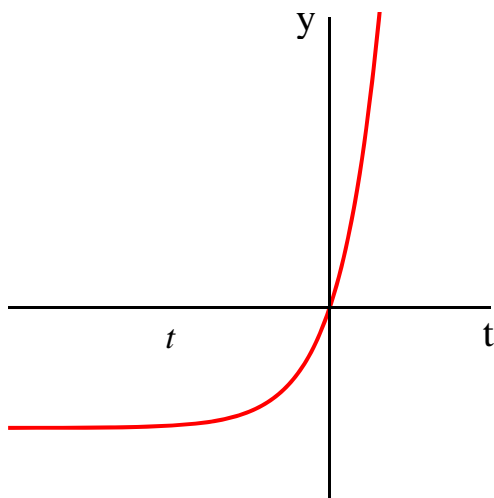
4. Find the particular solution of the non-homogeneous equation:

$$0 = C - \frac{5}{4}$$

$$C = \frac{5}{4}$$

$$y = \frac{5}{4} e^{4t} - \frac{5}{4}$$

5. Sketch the particular solution:



Problem:

$$y' + 2y = -e^{2t}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda + 2 = 0$$

$$\lambda = -2$$

$$y_g = C e^{-2t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = D e^{2t}$$

$$4D = -1$$

$$D = -\frac{1}{4}$$

$$y_p = -\frac{1}{4} e^{2t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C e^{-2t} - \frac{1}{4} e^{2t}$$

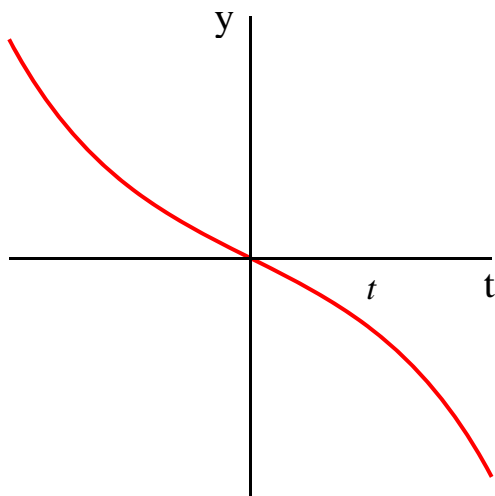
4. Find the particular solution of the non-homogeneous equation:

$$0 = C - \frac{1}{4}$$

$$C = \frac{1}{4}$$

$$y = \frac{1}{4} e^{-2t} - \frac{1}{4} e^{2t}$$

5. Sketch the particular solution:



Problem:

$$y' - 3y = 2$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda - 3 = 0$$

$$\lambda = 3$$

$$y_g = C e^{3t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = D$$

$$-3D = 2$$

$$D = -\frac{2}{3}$$

$$y_p = -\frac{2}{3}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C e^{3t} - \frac{2}{3}$$

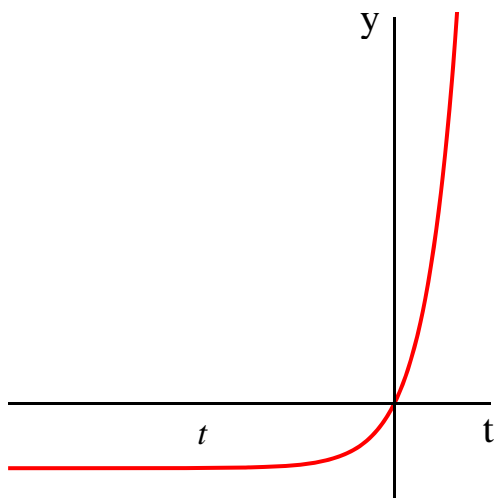
4. Find the particular solution of the non-homogeneous equation:

$$0 = C - \frac{2}{3}$$

$$C = \frac{2}{3}$$

$$y = \frac{2}{3} e^{3t} - \frac{2}{3}$$

5. Sketch the particular solution:



Problem:

$$y' + 4y = 3e^{-2t}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda + 4 = 0$$

$$\lambda = -4$$

$$y_g = Ce^{-4t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = De^{-2t}$$

$$2D = 3$$

$$D = \frac{3}{2}$$

$$y_p = \frac{3}{2}e^{-2t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = Ce^{-4t} + \frac{3}{2}e^{-2t}$$

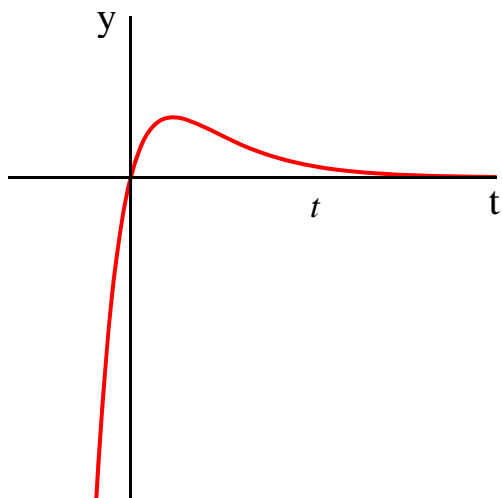
4. Find the particular solution of the non-homogeneous equation:

$$0 = C + \frac{3}{2}$$

$$C = -\frac{3}{2}$$

$$y = -\frac{3}{2}e^{-4t} + \frac{3}{2}e^{-2t}$$

5. Sketch the particular solution:



▼ Undetermined Coefficient Method: 2nd-order, 1 variable

Problem:

$$y'' + 3y' + 2y = 4e^{2t}$$

$$y(0) = 0, y'(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda^2 + 3\lambda + 2 = 0$$

$$\lambda = -1, -2$$

$$y_g = C_1 e^{-t} + C_2 e^{-2t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = D e^{2t}$$

$$12D = 4$$

$$D = \frac{1}{3}$$

$$y_p = \frac{1}{3} e^{2t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 e^{-t} + C_2 e^{-2t} + \frac{1}{3} e^{2t}$$

4. Find the particular solution of the non-homogeneous equation:

$$y' = -C_1 e^{-t} - 2C_2 e^{-2t} + \frac{2}{3} e^{2t}$$

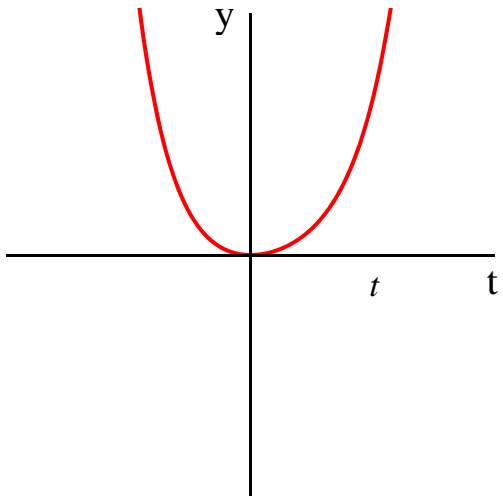
$$0 = C_1 + C_2 + \frac{1}{3}$$

$$0 = -C_1 - 2C_2 + \frac{2}{3}$$

$$C_1 = -\frac{4}{3}, C_2 = 1$$

$$y = -\frac{4}{3} e^{-t} + e^{-2t} + \frac{1}{3} e^{2t}$$

5. Sketch the particular solution:



Problem:

$$y'' - 3y' + 2y = 4e^{5t}$$

$$y(0) = 0, y'(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\lambda = 2, 1$$

$$y_g = C_1 e^{2t} + C_2 e^t$$

2. Find a particular solution of the non-homogeneous equation:

$$y = D e^{5t}$$

$$12D = 4$$

$$D = \frac{1}{3}$$

$$y_p = \frac{1}{3} e^{5t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 e^{2t} + C_2 e^t + \frac{1}{3} e^{5t}$$

4. Find the particular solution of the non-homogeneous equation:

$$y' = 2C_1 e^{2t} + C_2 e^t + \frac{5}{3} e^{5t}$$

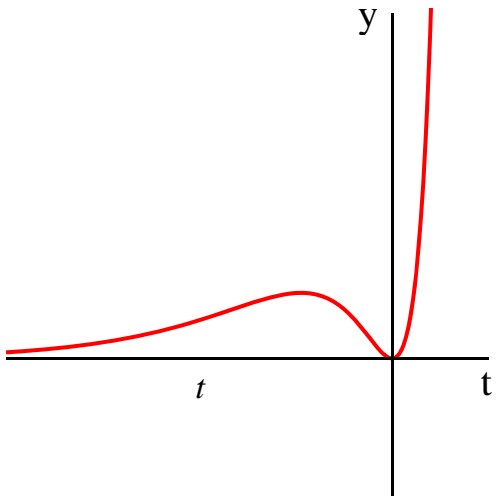
$$0 = C_1 + C_2 + \frac{1}{3}$$

$$0 = 2C_1 + C_2 + \frac{5}{3}$$

$$C_1 = -\frac{4}{3}, C_2 = 1$$

$$y = -\frac{4}{3} e^{2t} + e^t + \frac{1}{3} e^{5t}$$

5. Sketch the particular solution:



Problem:

$$y'' + 5y' + 4y = -4$$

$$y(0) = 0, y'(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda^2 + 5\lambda + 4 = 0$$

$$\lambda = -1, -4$$

$$y_g = C_1 e^{-t} + C_2 e^{-4t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = D$$

$$4D = -4$$

$$D = -1$$

$$y_p = -1$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 e^{-t} + C_2 e^{-4t} - 1$$

4. Find the particular solution of the non-homogeneous equation:

$$y' = -C_1 e^{-t} - 4C_2 e^{-4t}$$

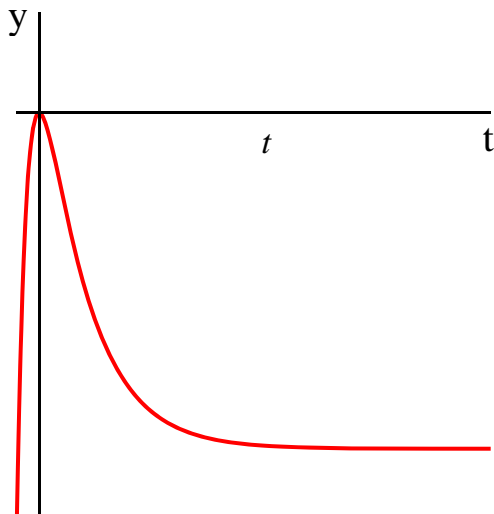
$$0 = C_1 + C_2 - 1$$

$$0 = -C_1 - 4C_2$$

$$C_1 = \frac{4}{3}, C_2 = -\frac{1}{3}$$

$$y = \frac{4}{3} e^{-t} - \frac{1}{3} e^{-4t} - 1$$

5. Sketch the particular solution:



Problem:

$$y'' + 5y' + 6y = -e^{2t}$$

$$y(0) = 0, y'(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda^2 + 5\lambda + 6 = 0$$

$$\lambda = -2, -3$$

$$y_g = C_1 e^{-2t} + C_2 e^{-3t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = D e^{2t}$$

$$20D = -1$$

$$D = -\frac{1}{20}$$

$$y_p = -\frac{1}{20} e^{2t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 e^{-2t} + C_2 e^{-3t} - \frac{1}{20} e^{2t}$$

4. Find the particular solution of the non-homogeneous equation:

$$y' = -2C_1 e^{-2t} - 3C_2 e^{-3t} - \frac{1}{10} e^{2t}$$

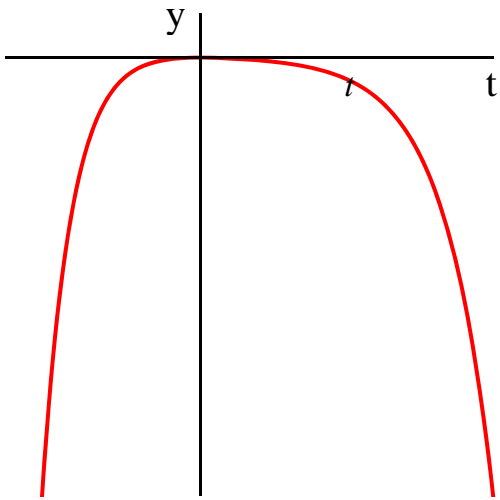
$$0 = C_1 + C_2 - \frac{1}{20}$$

$$0 = -2C_1 - 3C_2 - \frac{1}{10}$$

$$C_1 = \frac{1}{4}, C_2 = -\frac{1}{5}$$

$$y = \frac{1}{4} e^{-2t} - \frac{1}{5} e^{-3t} - \frac{1}{20} e^{2t}$$

5. Sketch the particular solution:



Problem:

$$y'' + 7y' + 10y = -10$$

$$y(0) = 0, y'(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda^2 + 7\lambda + 10 = 0$$

$$\lambda = -2, -5$$

$$y_g = C_1 e^{-2t} + C_2 e^{-5t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = D$$

$$10D = -10$$

$$D = -1$$

$$y_p = -1$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 e^{-2t} + C_2 e^{-5t} - 1$$

4. Find the particular solution of the non-homogeneous equation:

$$y' = -2C_1 e^{-2t} - 5C_2 e^{-5t}$$

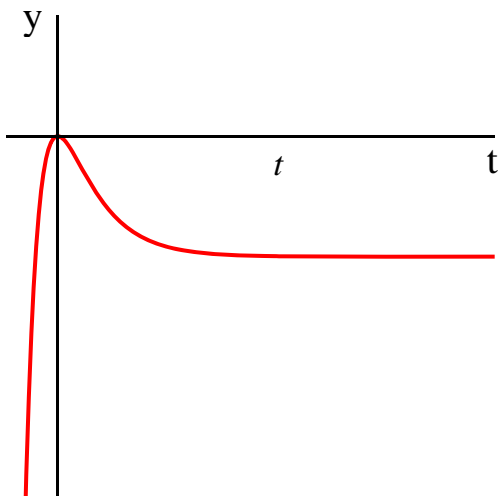
$$0 = C_1 + C_2 - 1$$

$$0 = -2C_1 - 5C_2$$

$$C_1 = \frac{5}{3}, C_2 = -\frac{2}{3}$$

$$y = \frac{5}{3} e^{-2t} - \frac{2}{3} e^{-5t} - 1$$

5. Sketch the particular solution:



Problem:

$$y'' + 3y' + 2y = -4e^{-3t}$$

$$y(0) = 0, y'(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\lambda^2 + 3\lambda + 2 = 0$$

$$\lambda = -1, -2$$

$$y_g = C_1 e^{-t} + C_2 e^{-2t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = D e^{-3t}$$

$$2D = -4$$

$$D = -2$$

$$y_p = -2e^{-3t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 e^{-t} + C_2 e^{-2t} - 2e^{-3t}$$

4. Find the particular solution of the non-homogeneous equation:

$$y' = -C_1 e^{-t} - 2C_2 e^{-2t} + 6e^{-3t}$$

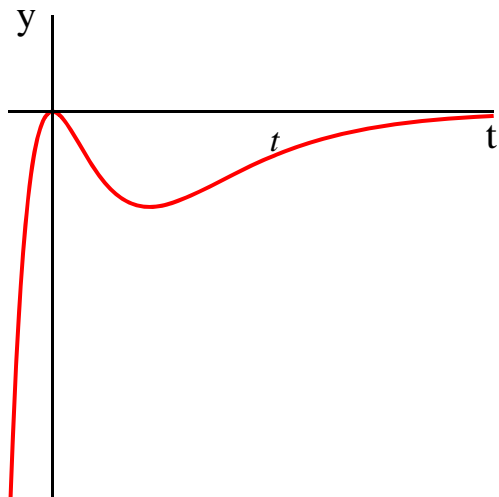
$$0 = C_1 + C_2 - 2$$

$$0 = -C_1 - 2C_2 + 6$$

$$C_1 = -2, C_2 = 4$$

$$y = -2e^{-t} + 4e^{-2t} - 2e^{-3t}$$

5. Sketch the particular solution:



▼ Undetermined Coefficient Method: 1st-order, 2 variables

Problem:

$$y' = \begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix} y + \begin{bmatrix} 2 \\ 4 \end{bmatrix} e^{2t}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\text{Det} \begin{bmatrix} 2 - \lambda & 1 \\ -3 & -2 - \lambda \end{bmatrix} = 0$$

$$\lambda^2 - 1 = 0$$

$$\lambda = 1, v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\lambda = -1, v = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + C_2 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = D e^{2t}$$

$$2D = \begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix} D + \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} D$$

$$D = \begin{bmatrix} 4 \\ -2 \end{bmatrix}, y_p = \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{2t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + C_2 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{2t}$$

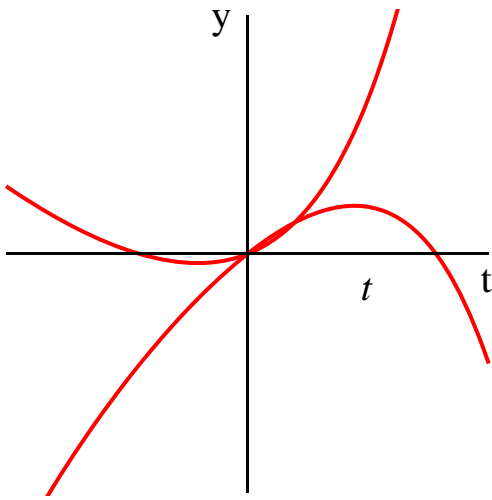
4. Find the particular solution of the non-homogeneous equation:

$$0 = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ -3 \end{bmatrix} + \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$C_1 = -5, C_2 = 1$$

$$y = \begin{bmatrix} -5 \\ 5 \end{bmatrix} e^t + \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{2t}$$

5. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 4 & 1 \\ -5 & -2 \end{bmatrix} y + \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^t$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\text{Det} \begin{bmatrix} 4 - \lambda & 1 \\ -5 & -2 - \lambda \end{bmatrix} = 0$$

$$\lambda^2 - 2\lambda - 3 = 0$$

$$\lambda = 3, v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\lambda = -1, v = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} 1 \\ -5 \end{bmatrix} e^{-t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = D e^t$$

$$D = \begin{bmatrix} 4 & 1 \\ -5 & -2 \end{bmatrix} D + \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -5 & -3 \end{bmatrix} D$$

$$D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, y_p = \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^t$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} 1 \\ -5 \end{bmatrix} e^{-t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^t$$

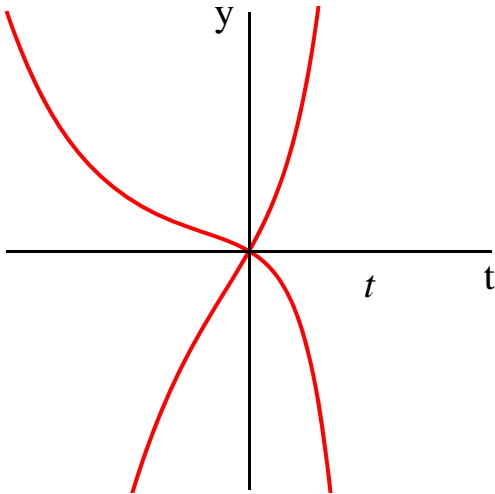
4. Find the particular solution of the non-homogeneous equation:

$$0 = C_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ -5 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C_1 = -\frac{1}{4}, C_2 = \frac{1}{4}$$

$$y = \begin{bmatrix} -\frac{1}{4} \\ \frac{1}{4} \end{bmatrix} e^{3t} + \begin{bmatrix} \frac{1}{4} \\ -\frac{5}{4} \end{bmatrix} e^{-t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^t$$

5. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} y + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\text{Det} \begin{bmatrix} 1 - \lambda & 2 \\ -1 & 4 - \lambda \end{bmatrix} = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$\lambda = 3, v = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\lambda = 2, v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 2 \\ 2 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = D$$

$$0 = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} D + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} D$$

$$D = \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}, y_p = \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 \begin{bmatrix} 2 \\ 2 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} + \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}$$

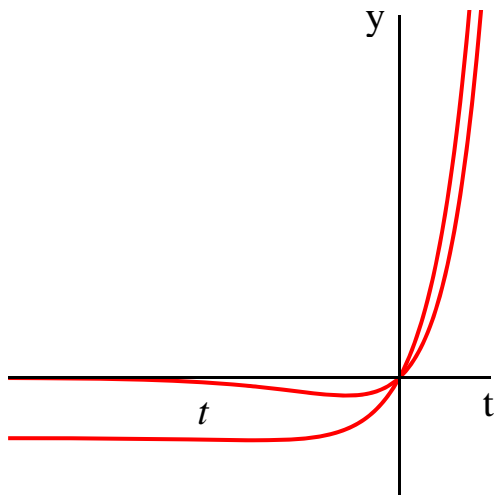
4. Find the particular solution of the non-homogeneous equation:

$$0 = C_1 \begin{bmatrix} 2 \\ 2 \end{bmatrix} + C_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}$$

$$C_1 = \frac{1}{2}, C_2 = -\frac{1}{2}$$

$$y = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + \begin{bmatrix} -1 \\ -\frac{1}{2} \end{bmatrix} e^{2t} + \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}$$

5. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -2 & -1 \\ 3 & 2 \end{bmatrix} y + \begin{bmatrix} -2 \\ -4 \end{bmatrix} e^{-2t}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\text{Det} \begin{bmatrix} -2 - \lambda & -1 \\ 3 & 2 - \lambda \end{bmatrix} = 0$$

$$\lambda^2 - 1 = 0$$

$$\lambda = 1, v = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$\lambda = -1, v = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^t + C_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = D e^{-2t}$$

$$-2D = \begin{bmatrix} -2 & -1 \\ 3 & 2 \end{bmatrix} D + \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 3 & 4 \end{bmatrix} D$$

$$D = \begin{bmatrix} 4 \\ -2 \end{bmatrix}, y_p = \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{-2t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^t + C_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{-2t}$$

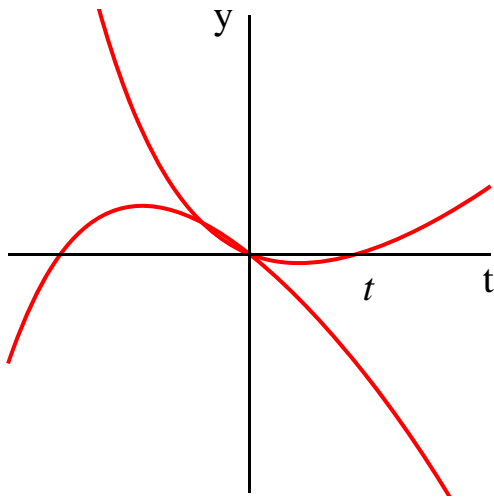
4. Find the particular solution of the non-homogeneous equation:

$$0 = C_1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$C_1 = -1, C_2 = 5$$

$$y = \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^t + \begin{bmatrix} -5 \\ 5 \end{bmatrix} e^{-t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{-2t}$$

5. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix} y + \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{2t}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\text{Det} \begin{bmatrix} 4 - \lambda & -1 \\ 5 & -2 - \lambda \end{bmatrix} = 0$$

$$\lambda^2 - 2\lambda - 3 = 0$$

$$\lambda = 3, v = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\lambda = -1, v = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} -1 \\ -1 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} -1 \\ -5 \end{bmatrix} e^{-t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = D e^{2t}$$

$$2D = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix} D + \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 5 & -4 \end{bmatrix} D$$

$$D = \begin{bmatrix} -2 \\ -3 \end{bmatrix}, y_p = \begin{bmatrix} -2 \\ -3 \end{bmatrix} e^{2t}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 \begin{bmatrix} -1 \\ -1 \end{bmatrix} e^{3t} + C_2 \begin{bmatrix} -1 \\ -5 \end{bmatrix} e^{-t} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} e^{2t}$$

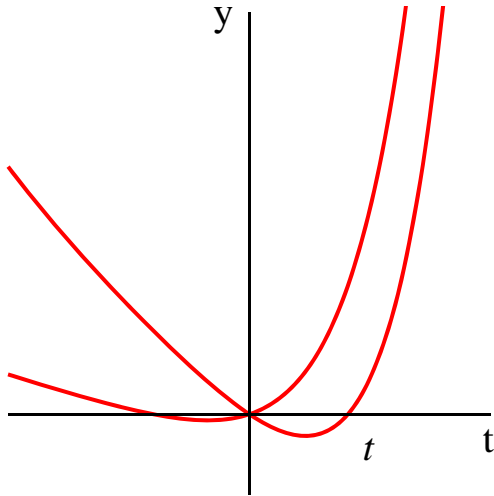
4. Find the particular solution of the non-homogeneous equation:

$$0 = C_1 \begin{bmatrix} -1 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ -5 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$C_1 = -\frac{7}{4}, C_2 = -\frac{1}{4}$$

$$y = \begin{bmatrix} \frac{7}{4} \\ \frac{7}{4} \end{bmatrix} e^{3t} + \begin{bmatrix} \frac{1}{4} \\ \frac{5}{4} \end{bmatrix} e^{-t} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} e^{2t}$$

5. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} y + \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$y(0) = 0$$

1. Find the general solution of the homogeneous equation:

$$\text{Det} \begin{bmatrix} -1 - \lambda & 2 \\ -1 & -4 - \lambda \end{bmatrix} = 0$$

$$\lambda^2 + 5\lambda + 6 = 0$$

$$\lambda = -2, v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\lambda = -3, v = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$y = C_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-2t} + C_2 \begin{bmatrix} 2 \\ -2 \end{bmatrix} e^{-3t}$$

2. Find a particular solution of the non-homogeneous equation:

$$y = D$$

$$0 = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} D + \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} D$$

$$D = \begin{bmatrix} -\frac{4}{3} \\ -\frac{1}{6} \end{bmatrix}, y_p = \begin{bmatrix} -\frac{4}{3} \\ -\frac{1}{6} \end{bmatrix}$$

3. Find the general solution of the non-homogeneous equation:

$$y = C_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-2t} + C_2 \begin{bmatrix} 2 \\ -2 \end{bmatrix} e^{-3t} + \begin{bmatrix} -\frac{4}{3} \\ -\frac{1}{6} \end{bmatrix}$$

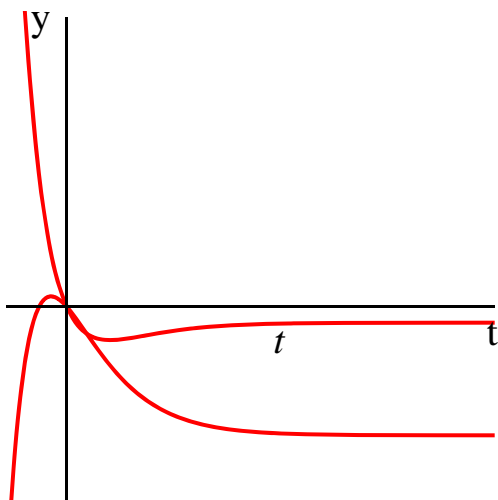
4. Find the particular solution of the non-homogeneous equation:

$$0 = C_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} + C_2 \begin{bmatrix} 2 \\ -2 \end{bmatrix} + \begin{bmatrix} -\frac{4}{3} \\ -\frac{1}{6} \end{bmatrix}$$

$$C_1 = \frac{3}{2}, C_2 = -\frac{5}{6}$$

$$y = \begin{bmatrix} 3 \\ -\frac{3}{2} \end{bmatrix} e^{-2t} + \begin{bmatrix} -\frac{5}{3} \\ \frac{5}{3} \end{bmatrix} e^{-3t} + \begin{bmatrix} -\frac{4}{3} \\ -\frac{1}{6} \end{bmatrix}$$

5. Sketch the particular solution:



▼ Laplace Transform Method: 1st-order, 1 variable

Problem:

$$y' + 3y = 5e^{2t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 3Y = \frac{5}{s-2}$$

2. Solve for Y:

$$Y = \frac{5}{(s+3)(s-2)}$$

3. Inverse Laplace Transform:

$$\frac{5}{(s+3)(s-2)} = \frac{A}{s+3} + \frac{B}{s-2}$$

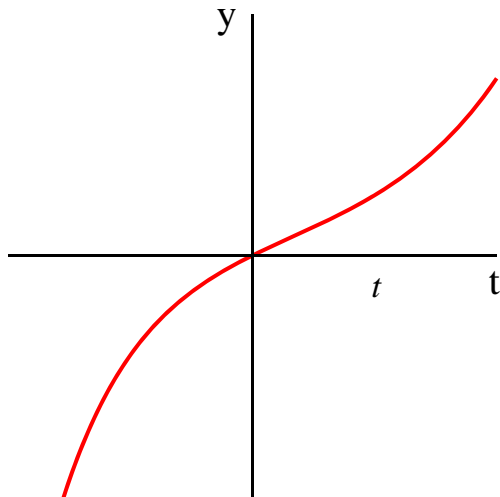
$$5 = A(s-2) + B(s+3)$$

$$s = -3, 5 = -5A, A = -1$$

$$s = 2, 5 = 5B, B = 1$$

$$y = -e^{-3t} + e^{2t}$$

4. Sketch the particular solution:



Problem:

$$y' + 4y = 5e^{-2t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 4Y = \frac{5}{s+2}$$

2. Solve for Y:

$$Y = \frac{5}{(s+4)(s+2)}$$

3. Inverse Laplace Transform:

$$\frac{5}{(s+4)(s+2)} = \frac{A}{s+4} + \frac{B}{s+2}$$

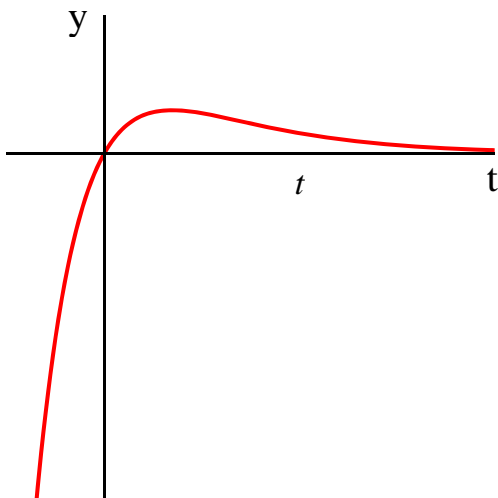
$$5 = A(s+2) + B(s+4)$$

$$s = -4, 5 = -2A, A = -\frac{5}{2}$$

$$s = -2, 5 = 2B, B = \frac{5}{2}$$

$$y = -\frac{5}{2}e^{-4t} + \frac{5}{2}e^{-2t}$$

4. Sketch the particular solution:



Problem:

$$y' - 4y = 5$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys - 4Y = \frac{5}{s}$$

2. Solve for Y:

$$Y = \frac{5}{(s-4)s}$$

3. Inverse Laplace Transform:

$$\frac{5}{(s-4)s} = \frac{A}{s-4} + \frac{B}{s}$$

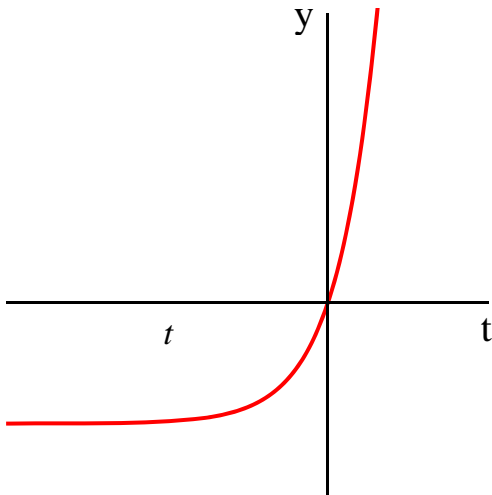
$$5 = As + B(s-4)$$

$$s=4, 5 = 4A, A = \frac{5}{4}$$

$$s=0, 5 = -4B, B = -\frac{5}{4}$$

$$y = \frac{5}{4} e^{4t} - \frac{5}{4}$$

4. Sketch the particular solution:



Problem:

$$y' + 2y = -e^{2t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 2Y = -\frac{1}{s-2}$$

2. Solve for Y:

$$Y = -\frac{1}{(s+2)(s-2)}$$

3. Inverse Laplace Transform:

$$-\frac{1}{(s+2)(s-2)} = \frac{A}{s+2} + \frac{B}{s-2}$$

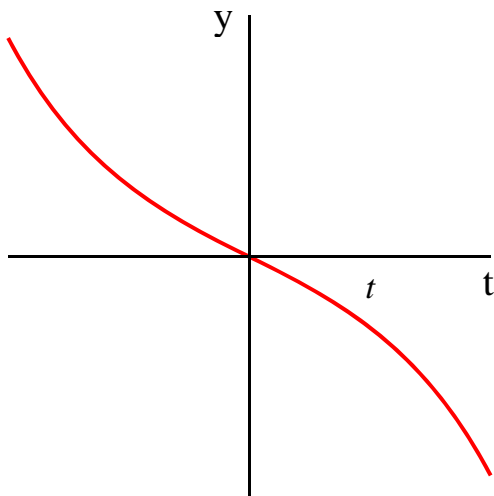
$$-1 = A(s-2) + B(s+2)$$

$$s = -2, -1 = -4A, A = \frac{1}{4}$$

$$s = 2, -1 = 4B, B = -\frac{1}{4}$$

$$y = \frac{1}{4} e^{-2t} - \frac{1}{4} e^{2t}$$

4. Sketch the particular solution:



Problem:

$$y' - 3y = 2$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys - 3Y = \frac{2}{s}$$

2. Solve for Y:

$$Y = \frac{2}{(s-3)s}$$

3. Inverse Laplace Transform:

$$\frac{2}{(s-3)s} = \frac{A}{s-3} + \frac{B}{s}$$

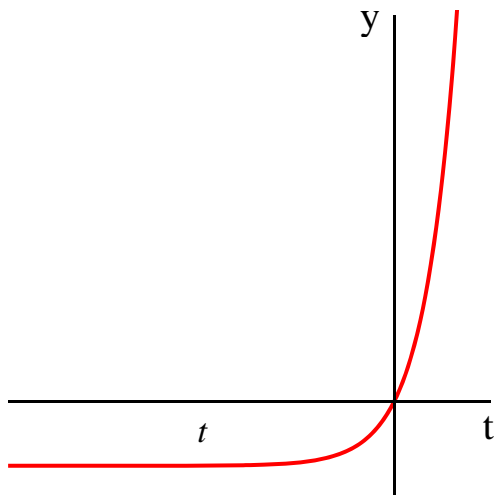
$$2 = As + B(s-3)$$

$$s=3, 2 = 3A, A = \frac{2}{3}$$

$$s=0, 2 = -3B, B = -\frac{2}{3}$$

$$y = \frac{2}{3} e^{3t} - \frac{2}{3}$$

4. Sketch the particular solution:



Problem:

$$y' + 4y = 3e^{-2t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 4Y = \frac{3}{s+2}$$

2. Solve for Y:

$$Y = \frac{3}{(s+4)(s+2)}$$

3. Inverse Laplace Transform:

$$\frac{3}{(s+4)(s+2)} = \frac{A}{s+4} + \frac{B}{s+2}$$

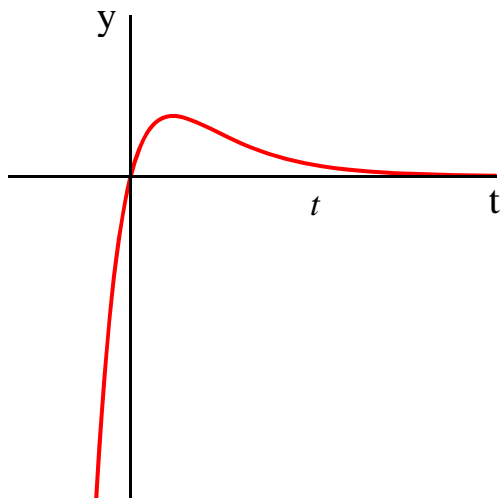
$$3 = A(s+2) + B(s+4)$$

$$s = -4, 3 = -2A, A = -\frac{3}{2}$$

$$s = -2, 3 = 2B, B = \frac{3}{2}$$

$$y = -\frac{3}{2}e^{-4t} + \frac{3}{2}e^{-2t}$$

4. Sketch the particular solution:



▼ Laplace Transform Method: 2nd-order, 1 variable

Problem:

$$y'' + 3y' + 2y = 4e^{2t}$$

$$y(0) = 0, y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 + 3Ys + 2Y = \frac{4}{s-2}$$

2. Solve for Y:

$$Y = \frac{4}{(s^2 + 3s + 2)(s-2)}$$

3. Inverse Laplace Transform:

$$\frac{4}{(s+1)(s+2)(s-2)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s-2}$$

$$4 = A(s+2)(s-2) + B(s+1)(s-2) + C(s+1)(s+2)$$

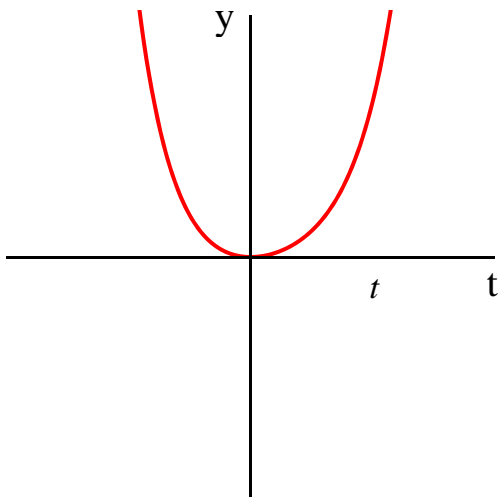
$$s = -1, 4 = -3A, A = -\frac{4}{3}$$

$$s = -2, 4 = 4B, B = 1$$

$$s = 2, 4 = 12C, C = \frac{1}{3}$$

$$y = -\frac{4}{3}e^{-t} + e^{-2t} + \frac{1}{3}e^{2t}$$

4. Sketch the particular solution:



Problem:

$$y'' - 3y' + 2y = 4e^{5t}$$

$$y(0) = 0, y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 - 3Ys + 2Y = \frac{4}{s-5}$$

2. Solve for Y:

$$Y = \frac{4}{(s^2 - 3s + 2)(s-5)}$$

3. Inverse Laplace Transform:

$$\frac{4}{(s-2)(s-1)(s-5)} = \frac{A}{s-2} + \frac{B}{s-1} + \frac{C}{s-5}$$

$$4 = A(s-1)(s-5) + B(s-2)(s-5) + C(s-2)(s-1)$$

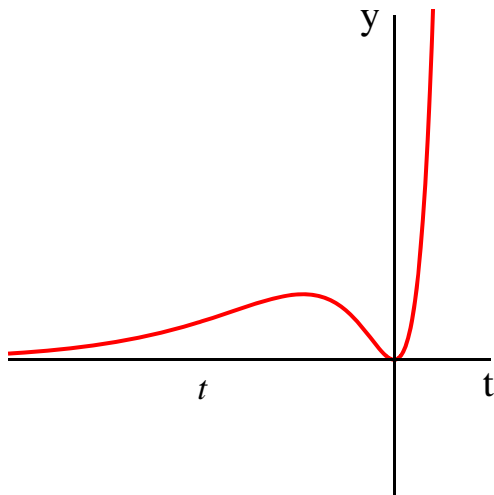
$$s=2, 4 = -3A, A = -\frac{4}{3}$$

$$s=1, 4 = 4B, B = 1$$

$$s=5, 4 = 12C, C = \frac{1}{3}$$

$$y = -\frac{4}{3}e^{2t} + e^t + \frac{1}{3}e^{5t}$$

4. Sketch the particular solution:



Problem:

$$y'' + 5y' + 4y = -4$$

$$y(0) = 0, y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 + 5Ys + 4Y = -\frac{4}{s}$$

2. Solve for Y:

$$Y = -\frac{4}{(s^2 + 5s + 4)s}$$

3. Inverse Laplace Transform:

$$-\frac{4}{(s+1)(s+4)s} = \frac{A}{s+1} + \frac{B}{s+4} + \frac{C}{s}$$

$$-4 = A(s+4)s + B(s+1)s + C(s+1)(s+4)$$

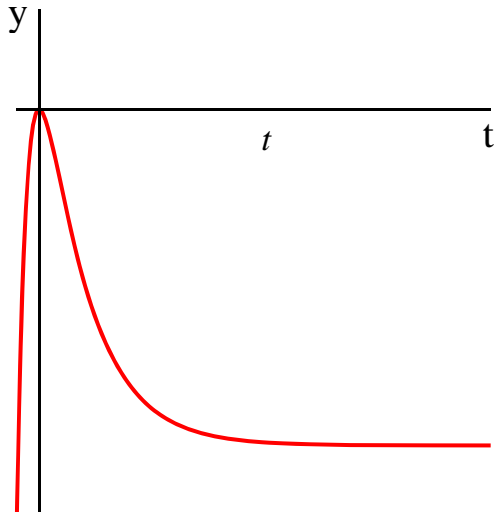
$$s = -1, -4 = -3A, A = \frac{4}{3}$$

$$s = -4, -4 = 12B, B = -\frac{1}{3}$$

$$s = 0, -4 = 4C, C = -1$$

$$y = \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t} - 1$$

4. Sketch the particular solution:



Problem:

$$y'' + 5y' + 6y = -e^{2t}$$

$$y(0) = 0, y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 + 5Ys + 6Y = -\frac{1}{s-2}$$

2. Solve for Y:

$$Y = -\frac{1}{(s^2 + 5s + 6)(s-2)}$$

3. Inverse Laplace Transform:

$$-\frac{1}{(s+2)(s+3)(s-2)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s-2}$$

$$-1 = A(s+3)(s-2) + B(s+2)(s-2) + C(s+2)(s+3)$$

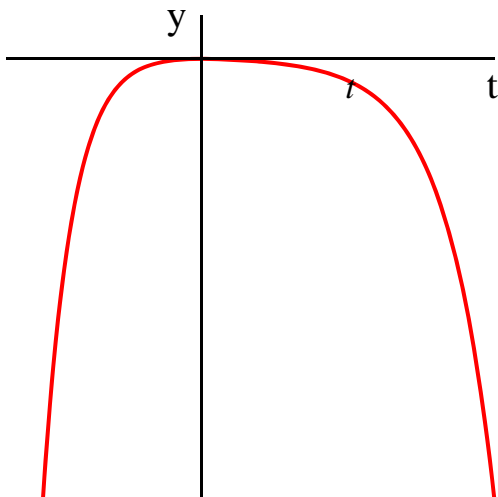
$$s = -2, -1 = -4A, A = \frac{1}{4}$$

$$s = -3, -1 = 5B, B = -\frac{1}{5}$$

$$s = 2, -1 = 20C, C = -\frac{1}{20}$$

$$y = \frac{1}{4} e^{-2t} - \frac{1}{5} e^{-3t} - \frac{1}{20} e^{2t}$$

4. Sketch the particular solution:



Problem:

$$y'' + 7y' + 10y = -10$$

$$y(0) = 0, y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 + 7Ys + 10Y = -\frac{10}{s}$$

2. Solve for Y:

$$Y = -\frac{10}{(s^2 + 7s + 10)s}$$

3. Inverse Laplace Transform:

$$-\frac{10}{(s+2)(s+5)s} = \frac{A}{s+2} + \frac{B}{s+5} + \frac{C}{s}$$

$$-10 = A(s+5)s + B(s+2)s + C(s+2)(s+5)$$

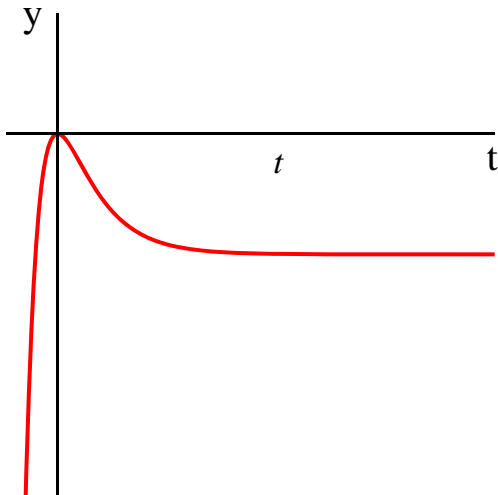
$$s = -2, -10 = -6A, A = \frac{5}{3}$$

$$s = -5, -10 = 15B, B = -\frac{2}{3}$$

$$s = 0, -10 = 10C, C = -1$$

$$y = \frac{5}{3}e^{-2t} - \frac{2}{3}e^{-5t} - 1$$

4. Sketch the particular solution:



Problem:

$$y'' + 3y' + 2y = -4e^{-3t}$$

$$y(0) = 0, y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 + 3Ys + 2Y = -\frac{4}{s+3}$$

2. Solve for Y:

$$Y = -\frac{4}{(s^2 + 3s + 2)(s + 3)}$$

3. Inverse Laplace Transform:

$$-\frac{4}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$-4 = A(s+2)(s+3) + B(s+1)(s+3) + C(s+1)(s+2)$$

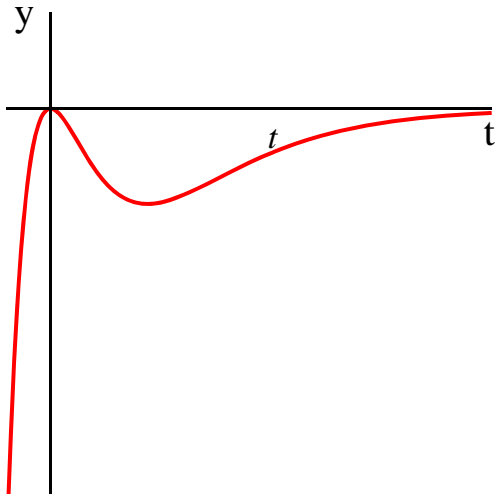
$$s = -1, -4 = 2A, A = -2$$

$$s = -2, -4 = -B, B = 4$$

$$s = -3, -4 = 2C, C = -2$$

$$y = -2e^{-t} + 4e^{-2t} - 2e^{-3t}$$

4. Sketch the particular solution:



▼ Laplace Transform Method: 1st-order, 2 variables

Problem:

$$y' = \begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix} y + \begin{bmatrix} 2 \\ 4 \end{bmatrix} e^{2t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$sY = \begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix} Y + \frac{\begin{bmatrix} 2 \\ 4 \end{bmatrix}}{s-2}$$

2. Solve for Y:

$$\begin{bmatrix} s-2 & -1 \\ 3 & s+2 \end{bmatrix} Y = \frac{\begin{bmatrix} 2 \\ 4 \end{bmatrix}}{s-2}$$

$$Y = \frac{\begin{bmatrix} 2s+8 \\ -14+4s \end{bmatrix}}{(s^2-1)(s-2)}$$

3. Inverse Laplace Transform:

$$\frac{\begin{bmatrix} 2s+8 \\ -14+4s \end{bmatrix}}{(s-1)(s+1)(s-2)} = \frac{A}{s-1} + \frac{B}{s+1} + \frac{C}{s-2}$$

$$\begin{bmatrix} 2s+8 \\ -14+4s \end{bmatrix} = A(s+1)(s-2) + B(s-1)(s-2) + C(s-1)(s+1)$$

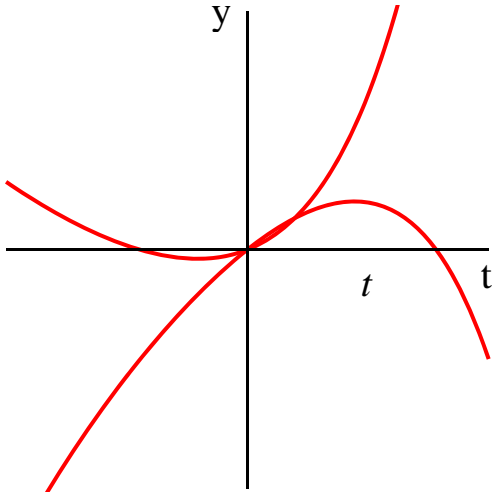
$$s=1, \begin{bmatrix} 10 \\ -10 \end{bmatrix} = -2A, A = \begin{bmatrix} -5 \\ 5 \end{bmatrix}$$

$$s=-1, \begin{bmatrix} 6 \\ -18 \end{bmatrix} = 6B, B = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$s=2, \begin{bmatrix} 12 \\ -6 \end{bmatrix} = 3C, C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$y = \begin{bmatrix} -5 \\ 5 \end{bmatrix} e^t + \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{2t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 4 & 1 \\ -5 & -2 \end{bmatrix} y + \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{2t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$sY = \begin{bmatrix} 4 & 1 \\ -5 & -2 \end{bmatrix} Y + \frac{\begin{bmatrix} -1 \\ 3 \end{bmatrix}}{s-2}$$

2. Solve for Y:

$$\begin{bmatrix} s-4 & -1 \\ 5 & s+2 \end{bmatrix} Y = \frac{\begin{bmatrix} -1 \\ 3 \end{bmatrix}}{s-2}$$

$$Y = \frac{\begin{bmatrix} -s+1 \\ -7+3s \end{bmatrix}}{(s^2-2s-3)(s-2)}$$

3. Inverse Laplace Transform:

$$\frac{\begin{bmatrix} -s+1 \\ -7+3s \end{bmatrix}}{(s+1)(s-3)(s-2)} = \frac{A}{s-3} + \frac{B}{s+1} + \frac{C}{s-2}$$

$$\begin{bmatrix} -s+1 \\ -7+3s \end{bmatrix} = A(s+1)(s-2) + B(s-3)(s-2) + C(s-3)(s+1)$$

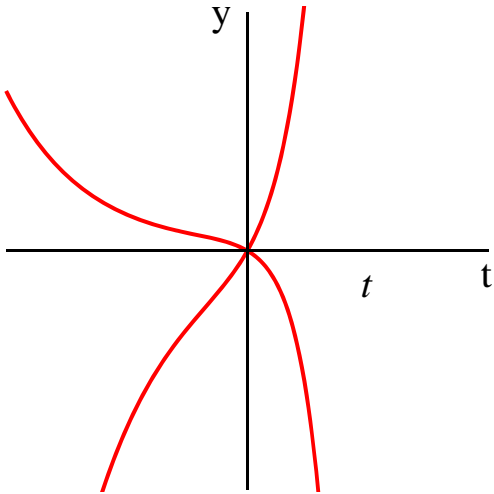
$$s=3, \begin{bmatrix} -2 \\ 2 \end{bmatrix} = 4A, A = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$s=-1, \begin{bmatrix} 2 \\ -10 \end{bmatrix} = 12B, B = \begin{bmatrix} \frac{1}{6} \\ -\frac{5}{6} \end{bmatrix}$$

$$s=2, \begin{bmatrix} -1 \\ -1 \end{bmatrix} = -3C, C = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$y = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix} e^{3t} + \begin{bmatrix} \frac{1}{6} \\ -\frac{5}{6} \end{bmatrix} e^{-t} + \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} e^{2t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} y + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$y(0) = 0$$

1. Laplace Transform:

$$sY = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} Y + \frac{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}{s}$$

2. Solve for Y:

$$\begin{bmatrix} s-1 & -2 \\ 1 & s-4 \end{bmatrix} Y = \frac{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}{s}$$

$$Y = \frac{\begin{bmatrix} s \\ -3+2s \end{bmatrix}}{(s^2-5s+6)s}$$

3. Inverse Laplace Transform:

$$\frac{\begin{bmatrix} s \\ -3+2s \end{bmatrix}}{(s-2)(s-3)s} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s}$$

$$\begin{bmatrix} s \\ -3+2s \end{bmatrix} = A(s-2)s + B(s-3)s + C(s-3)(s-2)$$

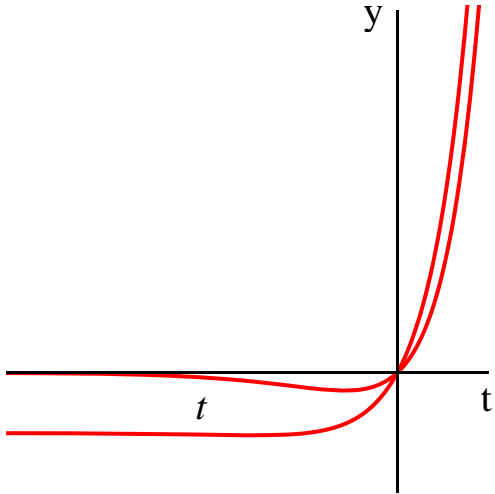
$$s=3, \begin{bmatrix} 3 \\ 3 \end{bmatrix} = 3A, A = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$s=2, \begin{bmatrix} 2 \\ 1 \end{bmatrix} = -2B, B = \begin{bmatrix} -1 \\ -\frac{1}{2} \end{bmatrix}$$

$$s=0, \begin{bmatrix} 0 \\ -3 \end{bmatrix} = 6C, C = \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}$$

$$y = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + \begin{bmatrix} -1 \\ -\frac{1}{2} \end{bmatrix} e^{2t} + \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -2 & -1 \\ 3 & 2 \end{bmatrix} y + \begin{bmatrix} -2 \\ -4 \end{bmatrix} e^{-2t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$sY = \begin{bmatrix} -2 & -1 \\ 3 & 2 \end{bmatrix} Y + \frac{\begin{bmatrix} -2 \\ -4 \end{bmatrix}}{s+2}$$

2. Solve for Y:

$$\begin{bmatrix} s+2 & 1 \\ -3 & s-2 \end{bmatrix} Y = \frac{\begin{bmatrix} -2 \\ -4 \end{bmatrix}}{s+2}$$

$$Y = \frac{\begin{bmatrix} -2s+8 \\ -14-4s \end{bmatrix}}{(s^2-1)(s+2)}$$

3. Inverse Laplace Transform:

$$\frac{\begin{bmatrix} -2s+8 \\ -14-4s \end{bmatrix}}{(s-1)(s+1)(s+2)} = \frac{A}{s-1} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$\begin{bmatrix} -2s+8 \\ -14-4s \end{bmatrix} = A(s+1)(s+2) + B(s-1)(s+2) + C(s-1)(s+1)$$

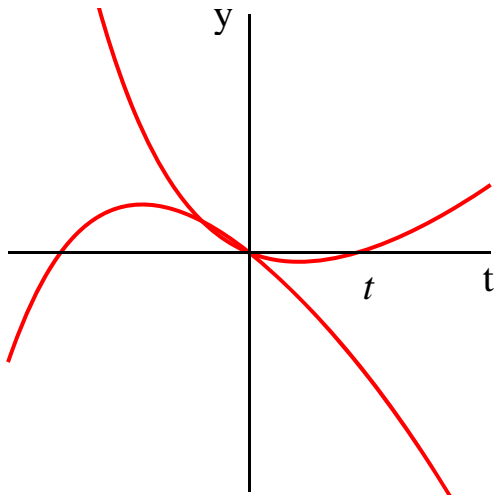
$$s=1, \begin{bmatrix} 6 \\ -18 \end{bmatrix} = 6A, A = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$s=-1, \begin{bmatrix} 10 \\ -10 \end{bmatrix} = -2B, B = \begin{bmatrix} -5 \\ 5 \end{bmatrix}$$

$$s=-2, \begin{bmatrix} 12 \\ -6 \end{bmatrix} = 3C, C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^t + \begin{bmatrix} -5 \\ 5 \end{bmatrix} e^{-t} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} e^{-2t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix} y + \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{2t}$$

$$y(0) = 0$$

1. Laplace Transform:

$$sY = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix} Y + \frac{\begin{bmatrix} 1 \\ -2 \end{bmatrix}}{s-2}$$

2. Solve for Y:

$$\begin{bmatrix} s-4 & 1 \\ -5 & s+2 \end{bmatrix} Y = \frac{\begin{bmatrix} 1 \\ -2 \end{bmatrix}}{s-2}$$

$$Y = \frac{\begin{bmatrix} s+4 \\ 13-2s \end{bmatrix}}{(s^2-2s-3)(s-2)}$$

3. Inverse Laplace Transform:

$$\frac{\begin{bmatrix} s+4 \\ 13-2s \end{bmatrix}}{(s+1)(s-3)(s-2)} = \frac{A}{s-3} + \frac{B}{s+1} + \frac{C}{s-2}$$

$$\begin{bmatrix} s+4 \\ 13-2s \end{bmatrix} = A(s+1)(s-2) + B(s-3)(s-2) + C(s-3)(s+1)$$

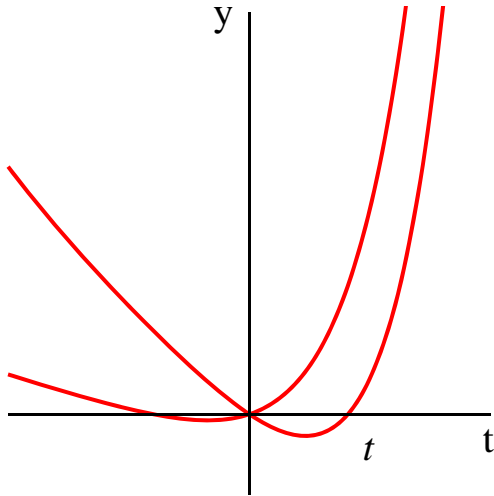
$$s=3, \begin{bmatrix} 7 \\ 7 \end{bmatrix} = 4A, A = \begin{bmatrix} \frac{7}{4} \\ \frac{7}{4} \end{bmatrix}$$

$$s=-1, \begin{bmatrix} 3 \\ 15 \end{bmatrix} = 12B, B = \begin{bmatrix} \frac{1}{4} \\ \frac{5}{4} \end{bmatrix}$$

$$s=2, \begin{bmatrix} 6 \\ 9 \end{bmatrix} = -3C, C = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$y = \begin{bmatrix} \frac{7}{4} \\ \frac{7}{4} \end{bmatrix} e^{3t} + \begin{bmatrix} \frac{1}{4} \\ \frac{5}{4} \end{bmatrix} e^{-t} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} e^{2t}$$

4. Sketch the particular solution:



Problem:

$$y' = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} y + \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$y(0) = 0$$

1. Laplace Transform:

$$sY = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} Y + \frac{\begin{bmatrix} -1 \\ -2 \end{bmatrix}}{s}$$

2. Solve for Y:

$$\begin{bmatrix} s+1 & -2 \\ 1 & s+4 \end{bmatrix} Y = \frac{\begin{bmatrix} -1 \\ -2 \end{bmatrix}}{s}$$

$$Y = \frac{\begin{bmatrix} -s-8 \\ -1-2s \end{bmatrix}}{(s^2+5s+6)s}$$

3. Inverse Laplace Transform:

$$\frac{\begin{bmatrix} -s-8 \\ -1-2s \end{bmatrix}}{(s+3)(s+2)s} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s}$$

$$\begin{bmatrix} -s-8 \\ -1-2s \end{bmatrix} = A(s+3)s + B(s+2)s + C(s+2)(s+3)$$

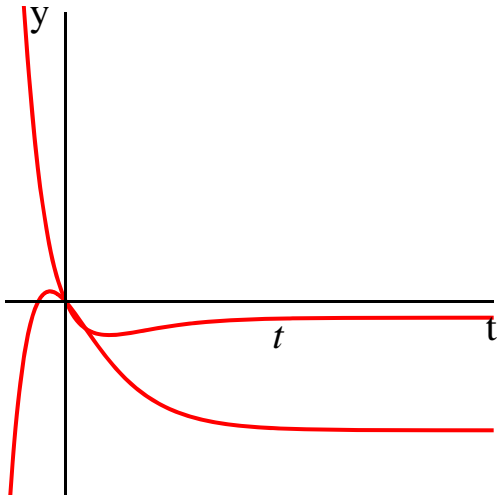
$$s = -2, \begin{bmatrix} -6 \\ 3 \end{bmatrix} = -2A, A = \begin{bmatrix} 3 \\ -\frac{3}{2} \end{bmatrix}$$

$$s = -3, \begin{bmatrix} -5 \\ 5 \end{bmatrix} = 3B, B = \begin{bmatrix} -\frac{5}{3} \\ \frac{5}{3} \end{bmatrix}$$

$$s = 0, \begin{bmatrix} -8 \\ -1 \end{bmatrix} = 6C, C = \begin{bmatrix} -\frac{4}{3} \\ -\frac{1}{6} \end{bmatrix}$$

$$y = \begin{bmatrix} 3 \\ -\frac{3}{2} \end{bmatrix} e^{-2t} + \begin{bmatrix} -\frac{5}{3} \\ \frac{5}{3} \end{bmatrix} e^{-3t} + \begin{bmatrix} -\frac{4}{3} \\ -\frac{1}{6} \end{bmatrix}$$

4. Sketch the particular solution:



▼ Laplace Transform Method: 1st-order, 1 variable, Trigonometric function

Problem:

$$y' + 2y = 3e^{-t} \cos(10t)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 2Y = \frac{3s + 3}{(s + 1)^2 + 100}$$

2. Solve for Y:

$$Y = \frac{3s + 3}{(s + 2)((s + 1)^2 + 100)}$$

3. Inverse Laplace Transform:

$$\frac{3s + 3}{(s + 2)((s + 1)^2 + 100)} = \frac{A}{s + 2} + \frac{B(s + 1) + 10C}{(s + 1)^2 + 100}$$

$$3s + 3 = A((s + 1)^2 + 100) + (B(s + 1) + 10C)(s + 2)$$

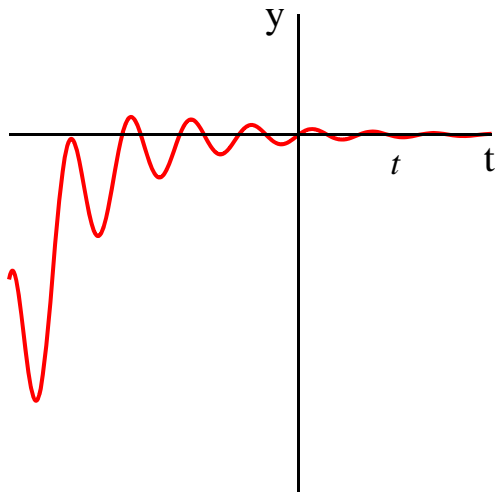
$$s = -2, -3 = 101A, A = -\frac{3}{101}$$

$$s = -1, 0 = 100A + 10C, C = \frac{30}{101}$$

$$s = 0, 3 = 101A + 2B + 20C, B = \frac{3}{101}$$

$$y = -\frac{3}{101}e^{-2t} + e^{-t} \left(\frac{3}{101} \cos(10t) + \frac{30}{101} \sin(10t) \right)$$

4. Sketch the particular solution:



Problem:

$$y' - 2y = 4e^t \sin(6t)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys - 2Y = \frac{24}{(s-1)^2 + 36}$$

2. Solve for Y:

$$Y = \frac{24}{(s-2)((s-1)^2 + 36)}$$

3. Inverse Laplace Transform:

$$\frac{24}{(s-2)((s-1)^2 + 36)} = \frac{A}{s-2} + \frac{B(s-1) + 6C}{(s-1)^2 + 36}$$

$$24 = A((s-1)^2 + 36) + (B(s-1) + 6C)(s-2)$$

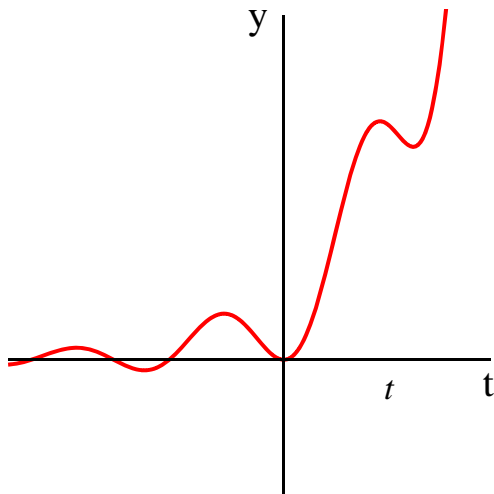
$$s=2, 24 = 37A, A = \frac{24}{37}$$

$$s=1, 24 = 36A - 6C, C = -\frac{4}{37}$$

$$s=0, 24 = 37A + 2B - 12C, B = -\frac{24}{37}$$

$$y = \frac{24}{37} e^{2t} + e^t \left(-\frac{24}{37} \cos(6t) - \frac{4}{37} \sin(6t) \right)$$

4. Sketch the particular solution:



Problem:

$$y' + y = e^{2t} (2 \cos(10t) - 3 \sin(10t))$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + Y = \frac{2s - 34}{(s - 2)^2 + 100}$$

2. Solve for Y:

$$Y = \frac{2s - 34}{(s + 1)((s - 2)^2 + 100)}$$

3. Inverse Laplace Transform:

$$\frac{2s - 34}{(s + 1)((s - 2)^2 + 100)} = \frac{A}{s + 1} + \frac{B(s - 2) + 10C}{(s - 2)^2 + 100}$$

$$2s - 34 = A((s - 2)^2 + 100) + (B(s - 2) + 10C)(s + 1)$$

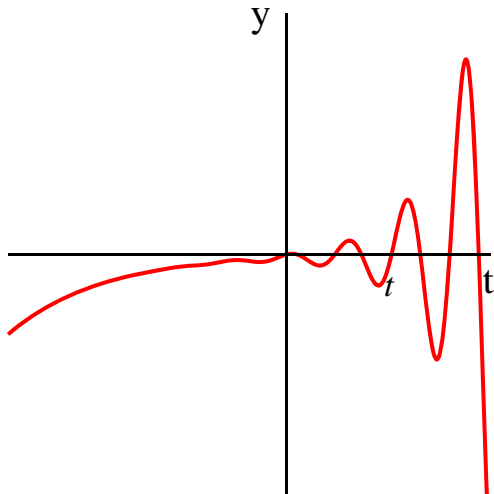
$$s = -1, -36 = 109A, A = -\frac{36}{109}$$

$$s = 2, -30 = 100A + 30C, C = \frac{11}{109}$$

$$s = 0, -34 = 104A - 2B + 10C, B = \frac{36}{109}$$

$$y = -\frac{36}{109} e^{-t} + e^{2t} \left(\frac{36}{109} \cos(10t) + \frac{11}{109} \sin(10t) \right)$$

4. Sketch the particular solution:



Problem:

$$y' + y = -2 \cos(2t)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + Y = -\frac{2s}{s^2 + 4}$$

2. Solve for Y:

$$Y = -\frac{2s}{(s+1)(s^2+4)}$$

3. Inverse Laplace Transform:

$$-\frac{2s}{(s+1)(s^2+4)} = \frac{A}{s+1} + \frac{Bs+2C}{s^2+4}$$

$$-2s = A(s^2+4) + (Bs+2C)(s+1)$$

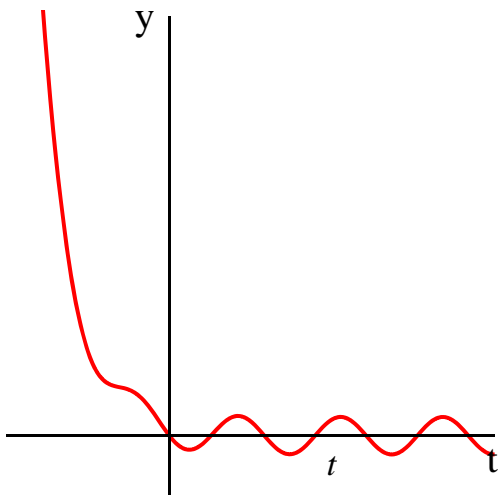
$$s = -1, 2 = 5A, A = \frac{2}{5}$$

$$s = 0, 0 = 4A + 2C, C = -\frac{4}{5}$$

$$s = 1, -2 = 5A + 2B + 4C, B = -\frac{2}{5}$$

$$y = \frac{2}{5} e^{-t} - \frac{2}{5} \cos(2t) - \frac{4}{5} \sin(2t)$$

4. Sketch the particular solution:



Problem:

$$y' - 2y = 4 \sin(10t) e^t$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys - 2Y = \frac{40}{(s-1)^2 + 100}$$

2. Solve for Y:

$$Y = \frac{40}{(s-2)((s-1)^2 + 100)}$$

3. Inverse Laplace Transform:

$$\frac{40}{(s-2)((s-1)^2 + 100)} = \frac{A}{s-2} + \frac{B(s-1) + 10C}{(s-1)^2 + 100}$$

$$40 = A((s-1)^2 + 100) + (B(s-1) + 10C)(s-2)$$

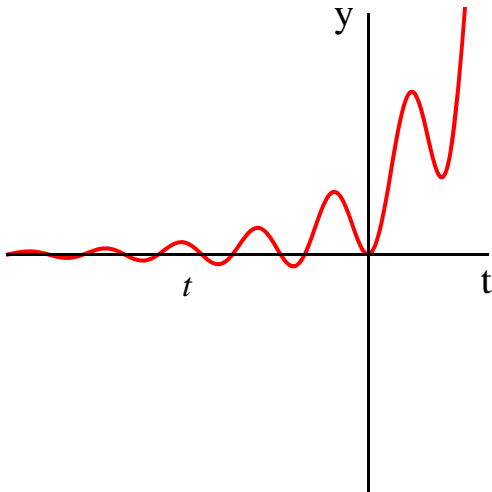
$$s=2, 40 = 101A, A = \frac{40}{101}$$

$$s=1, 40 = 100A - 10C, C = -\frac{4}{101}$$

$$s=0, 40 = 101A + 2B - 20C, B = -\frac{40}{101}$$

$$y = \frac{40}{101} e^{2t} + e^t \left(-\frac{40}{101} \cos(10t) - \frac{4}{101} \sin(10t) \right)$$

4. Sketch the particular solution:



Problem:

$$y' + y = e^{-2t} (\cos(10t) - \sin(10t))$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + Y = \frac{s - 8}{(s + 2)^2 + 100}$$

2. Solve for Y:

$$Y = \frac{s - 8}{(s + 1) ((s + 2)^2 + 100)}$$

3. Inverse Laplace Transform:

$$\frac{s - 8}{(s + 1) ((s + 2)^2 + 100)} = \frac{A}{s + 1} + \frac{B(s + 2) + 10C}{(s + 2)^2 + 100}$$

$$s - 8 = A((s + 2)^2 + 100) + (B(s + 2) + 10C)(s + 1)$$

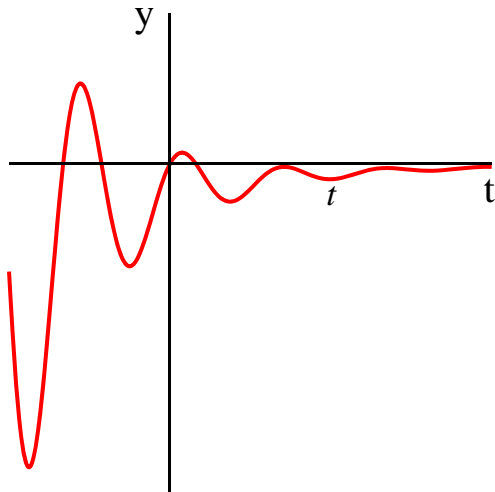
$$s = -1, -9 = 101A, A = -\frac{9}{101}$$

$$s = -2, -10 = 100A - 10C, C = \frac{11}{101}$$

$$s = 0, -8 = 104A + 2B + 10C, B = \frac{9}{101}$$

$$y = -\frac{9}{101} e^{-t} + e^{-2t} \left(\frac{9}{101} \cos(10t) + \frac{11}{101} \sin(10t) \right)$$

4. Sketch the particular solution:



▼ Laplace Transform Method: 2nd-order, 1 variable, Exponential function, Non-real eigenvalues

Problem:

$$y'' + 2y' + 26y = 2e^{-5t}$$

$$y(0) = 0, y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 + 2Ys + 26Y = \frac{2}{s+5}$$

2. Solve for Y:

$$Y = \frac{2}{(s^2 + 2s + 26)(s+5)}$$

3. Inverse Laplace Transform:

$$\alpha = -1, \beta = 5$$

$$\frac{2}{((s+1)^2 + 25)(s+5)} = \frac{A(s+1) + 5B}{(s+1)^2 + 25} + \frac{C}{s+5}$$

$$2 = (A(s+1) + 5B)(s+5) + C((s+1)^2 + 25)$$

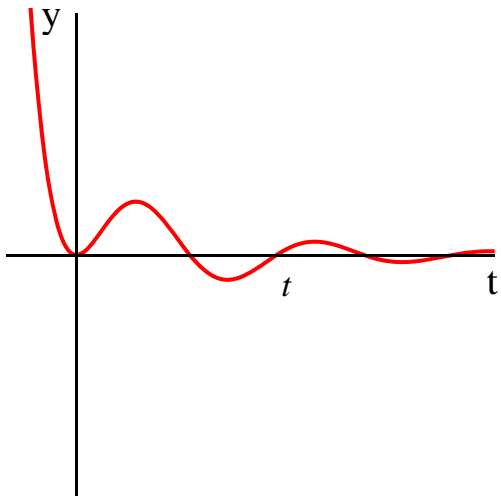
$$s = -5, 2 = 41C, C = \frac{2}{41}$$

$$s = -1, 2 = 20B + 25C, B = \frac{8}{205}$$

$$s = 0, 2 = 5A + 25B + 26C, A = -\frac{2}{41}$$

$$y = e^{-t} \left(-\frac{2}{41} \cos(5t) + \frac{8}{205} \sin(5t) \right) + \frac{2}{41} e^{-5t}$$

4. Sketch the particular solution:



Problem:

$$y'' - 4y' + 53y = -e^{-t}$$

$$y(0) = 0, y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 - 4Ys + 53Y = -\frac{1}{s+1}$$

2. Solve for Y:

$$Y = -\frac{1}{(s^2 - 4s + 53)(s+1)}$$

3. Inverse Laplace Transform:

$$\alpha = 2, \beta = 7$$

$$-\frac{1}{((s-2)^2 + 49)(s+1)} = \frac{A(s-2) + 7B}{(s-2)^2 + 49} + \frac{C}{s+1}$$

$$-1 = (A(s-2) + 7B)(s+1) + C((s-2)^2 + 49)$$

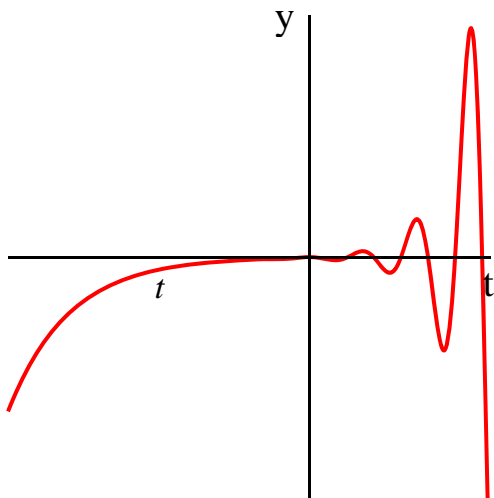
$$s = -1, -1 = 58C, C = -\frac{1}{58}$$

$$s = 2, -1 = 21B + 49C, B = -\frac{3}{406}$$

$$s = 0, -1 = -2A + 7B + 53C, A = \frac{1}{58}$$

$$y = e^{2t} \left(\frac{1}{58} \cos(7t) - \frac{3}{406} \sin(7t) \right) - \frac{1}{58} e^{-t}$$

4. Sketch the particular solution:



Problem:

$$y'' + 2y' + 26y = 2$$

$$y(0) = 0, y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 + 2Ys + 26Y = \frac{2}{s}$$

2. Solve for Y:

$$Y = \frac{2}{(s^2 + 2s + 26)s}$$

3. Inverse Laplace Transform:

$$\alpha = -1, \beta = 5$$

$$\frac{2}{((s+1)^2 + 25)s} = \frac{A(s+1) + 5B}{(s+1)^2 + 25} + \frac{C}{s}$$

$$2 = (A(s+1) + 5B)s + C((s+1)^2 + 25)$$

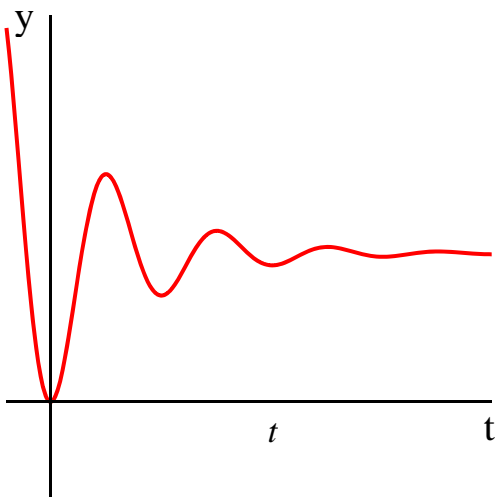
$$s=0, 2 = 26C, C = \frac{1}{13}$$

$$s=-1, 2 = -5B + 25C, B = -\frac{1}{65}$$

$$s=1, 2 = 2A + 5B + 29C, A = -\frac{1}{13}$$

$$y = e^{-t} \left(-\frac{1}{13} \cos(5t) - \frac{1}{65} \sin(5t) \right) + \frac{1}{13}$$

4. Sketch the particular solution:



Problem:

$$y'' - 2y' + 26y = 2e^{2t}$$

$$y(0) = 0, y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 - 2Ys + 26Y = \frac{2}{s-2}$$

2. Solve for Y:

$$Y = \frac{2}{(s^2 - 2s + 26)(s-2)}$$

3. Inverse Laplace Transform:

$$\alpha = 1, \beta = 5$$

$$\frac{2}{((s-1)^2 + 25)(s-2)} = \frac{A(s-1) + 5B}{(s-1)^2 + 25} + \frac{C}{s-2}$$

$$2 = (A(s-1) + 5B)(s-2) + C((s-1)^2 + 25)$$

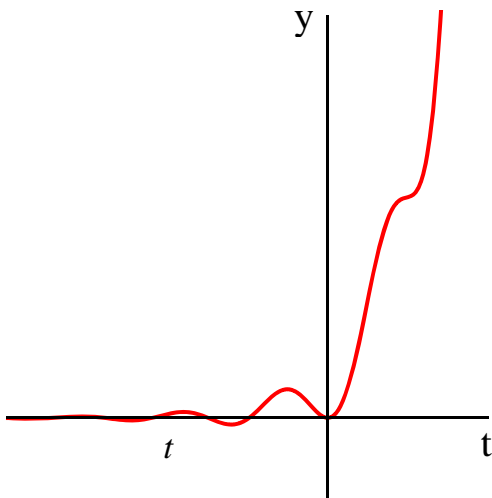
$$s=2, 2 = 26C, C = \frac{1}{13}$$

$$s=1, 2 = -5B + 25C, B = -\frac{1}{65}$$

$$s=0, 2 = 2A - 10B + 26C, A = -\frac{1}{13}$$

$$y = e^t \left(-\frac{1}{13} \cos(5t) - \frac{1}{65} \sin(5t) \right) + \frac{1}{13} e^{2t}$$

4. Sketch the particular solution:



Problem:

$$y'' - 2y' + 26y = 2$$

$$y(0) = 0, y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 - 2Ys + 26Y = \frac{2}{s}$$

2. Solve for Y:

$$Y = \frac{2}{(s^2 - 2s + 26)s}$$

3. Inverse Laplace Transform:

$$\alpha = 1, \beta = 5$$

$$\frac{2}{((s-1)^2 + 25)s} = \frac{A(s-1) + 5B}{(s-1)^2 + 25} + \frac{C}{s}$$

$$2 = (A(s-1) + 5B)s + C((s-1)^2 + 25)$$

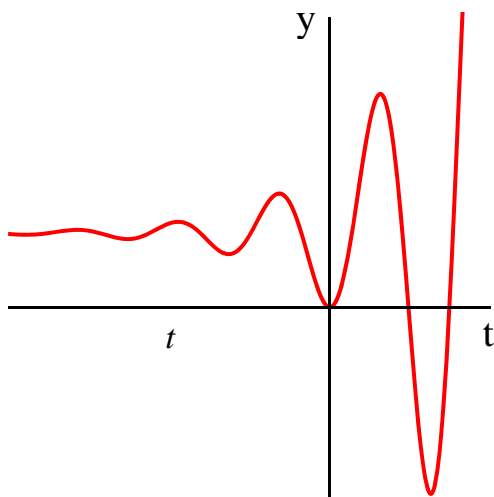
$$s=0, 2 = 26C, C = \frac{1}{13}$$

$$s=1, 2 = 5B + 25C, B = \frac{1}{65}$$

$$s=2, 2 = 2A + 10B + 26C, A = -\frac{1}{13}$$

$$y = e^t \left(-\frac{1}{13} \cos(5t) + \frac{1}{65} \sin(5t) \right) + \frac{1}{13}$$

4. Sketch the particular solution:



Problem:

$$y'' + 2y' + 37y = -e^{-2t}$$

$$y(0) = 0, y'(0) = 0$$

1. Laplace Transform:

$$Ys^2 + 2Ys + 37Y = -\frac{1}{s+2}$$

2. Solve for Y:

$$Y = -\frac{1}{(s^2 + 2s + 37)(s+2)}$$

3. Inverse Laplace Transform:

$$\alpha = -1, \beta = 6$$

$$-\frac{1}{((s+1)^2 + 36)(s+2)} = \frac{A(s+1) + 6B}{(s+1)^2 + 36} + \frac{C}{s+2}$$

$$-1 = (A(s+1) + 6B)(s+2) + C((s+1)^2 + 36)$$

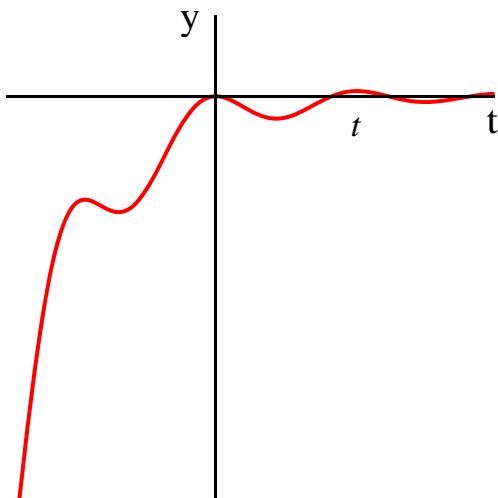
$$s = -2, -1 = 37C, C = -\frac{1}{37}$$

$$s = -1, -1 = 6B + 36C, B = -\frac{1}{222}$$

$$s = 0, -1 = 2A + 12B + 37C, A = \frac{1}{37}$$

$$y = e^{-t} \left(\frac{1}{37} \cos(6t) - \frac{1}{222} \sin(6t) \right) - \frac{1}{37} e^{-2t}$$

4. Sketch the particular solution:



▼ Laplace Transform Method: 1st-order, 1 variable, Step function

Problem:

$$y' + 2y = 3u(t-5)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 2Y = \frac{3e^{-5s}}{s}$$

2. Solve for Y:

$$Y = \frac{3e^{-5s}}{(s+2)s}$$

3. Inverse Laplace Transform:

$$\frac{3}{(s+2)s} = \frac{A}{s+2} + \frac{B}{s}$$

$$3 = As + B(s+2)$$

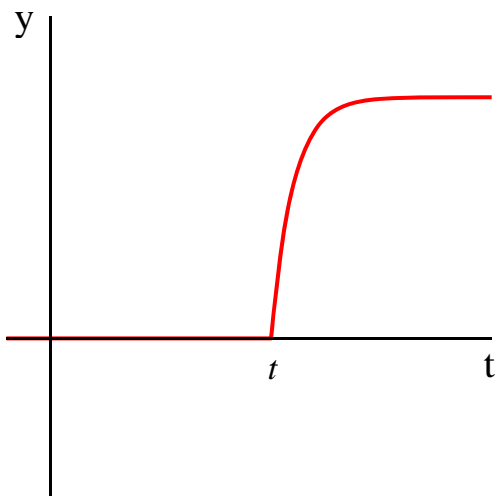
$$s = -2, 3 = -2A, A = -\frac{3}{2}$$

$$s = 0, 3 = 2B, B = \frac{3}{2}$$

$$f = -\frac{3}{2}e^{-2t} + \frac{3}{2}$$

$$y = \left(-\frac{3}{2}e^{-2t+10} + \frac{3}{2}\right)u(t-5)$$

4. Sketch the particular solution:



Problem:

$$y' + 4y = -3e^{-2t+2}u(t-1)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 4Y = -\frac{3e^{-s}}{s+2}$$

2. Solve for Y:

$$Y = -\frac{3e^{-s}}{(s+4)(s+2)}$$

3. Inverse Laplace Transform:

$$-\frac{3}{(s+4)(s+2)} = \frac{A}{s+4} + \frac{B}{s+2}$$

$$-3 = A(s+2) + B(s+4)$$

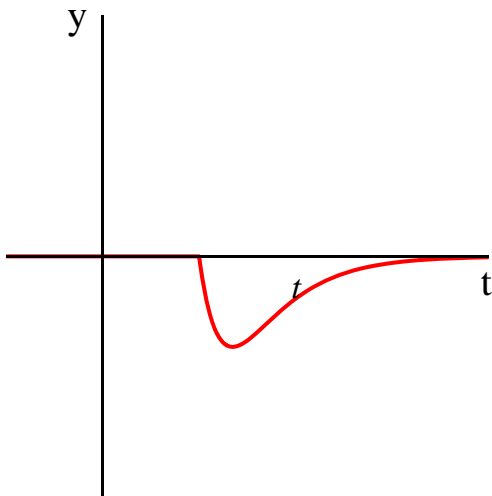
$$s = -4, -3 = -2A, A = \frac{3}{2}$$

$$s = -2, -3 = 2B, B = -\frac{3}{2}$$

$$f = \frac{3}{2}e^{-4t} - \frac{3}{2}e^{-2t}$$

$$y = \left(\frac{3}{2}e^{-4t+4} - \frac{3}{2}e^{-2t+2} \right) u(t-1)$$

4. Sketch the particular solution:



Problem:

$$y' - y = -e^{-2t+2} u(t-1)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys - Y = -\frac{e^{-s}}{s+2}$$

2. Solve for Y:

$$Y = -\frac{e^{-s}}{(s-1)(s+2)}$$

3. Inverse Laplace Transform:

$$-\frac{1}{(s-1)(s+2)} = \frac{A}{s-1} + \frac{B}{s+2}$$

$$-1 = A(s+2) + B(s-1)$$

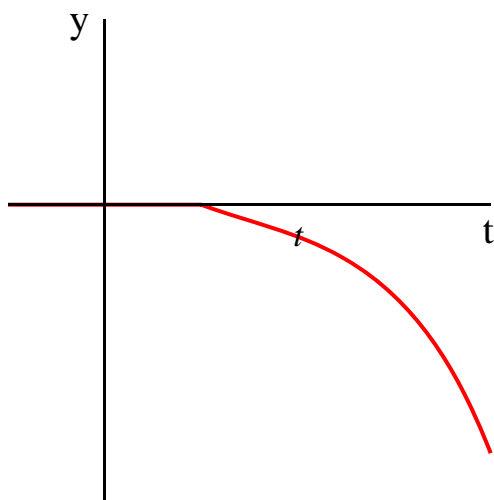
$$s=1, -1 = 3A, A = -\frac{1}{3}$$

$$s=-2, -1 = -3B, B = \frac{1}{3}$$

$$f = -\frac{1}{3} e^t + \frac{1}{3} e^{-2t}$$

$$y = \left(-\frac{1}{3} e^{t-1} + \frac{1}{3} e^{-2t+2} \right) u(t-1)$$

4. Sketch the particular solution:



Problem:

$$y' + y = 3 u(t - 2)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + Y = \frac{3 e^{-2s}}{s}$$

2. Solve for Y:

$$Y = \frac{3 e^{-2s}}{(s+1)s}$$

3. Inverse Laplace Transform:

$$\frac{3}{(s+1)s} = \frac{A}{s+1} + \frac{B}{s}$$

$$3 = As + B(s+1)$$

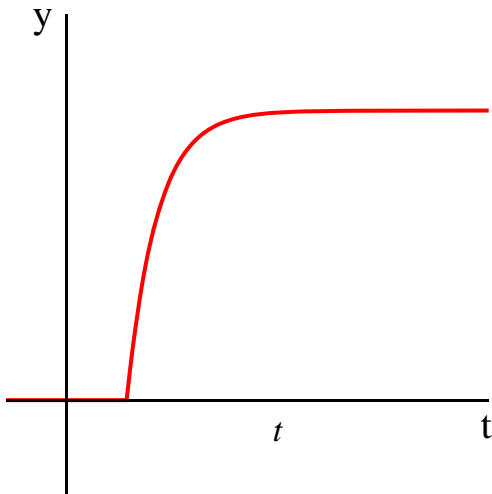
$$s = -1, 3 = -A, A = -3$$

$$s = 0, 3 = B, B = 3$$

$$f = -3e^{-t} + 3$$

$$y = (-3e^{-t+2} + 3) u(t-2)$$

4. Sketch the particular solution:



Problem:

$$y' - 2y = -3e^{-2t+2}u(t-1)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys - 2Y = -\frac{3e^{-s}}{s+2}$$

2. Solve for Y:

$$Y = -\frac{3e^{-s}}{(s-2)(s+2)}$$

3. Inverse Laplace Transform:

$$-\frac{3}{(s-2)(s+2)} = \frac{A}{s-2} + \frac{B}{s+2}$$

$$-3 = A(s+2) + B(s-2)$$

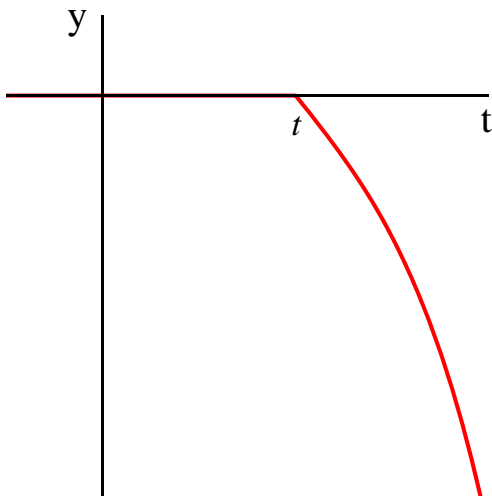
$$s=2, -3=4A, A = -\frac{3}{4}$$

$$s=-2, -3=-4B, B = \frac{3}{4}$$

$$f = -\frac{3}{4}e^{2t} + \frac{3}{4}e^{-2t}$$

$$y = \left(-\frac{3}{4}e^{2t-2} + \frac{3}{4}e^{-2t+2}\right)u(t-1)$$

4. Sketch the particular solution:



Problem:

$$y' + y = 5 e^{-2t+2} u(t-1)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + Y = \frac{5 e^{-s}}{s+2}$$

2. Solve for Y:

$$Y = \frac{5 e^{-s}}{(s+1)(s+2)}$$

3. Inverse Laplace Transform:

$$\frac{5}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$5 = A(s+2) + B(s+1)$$

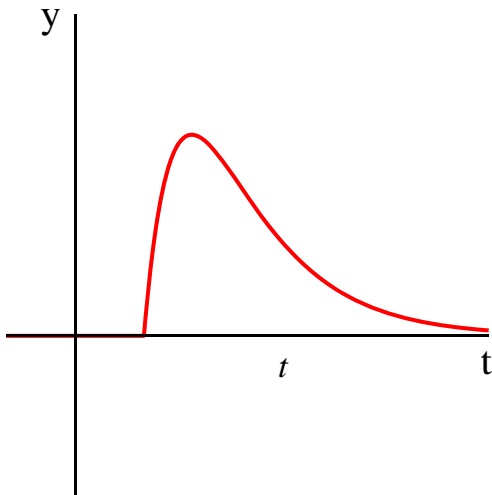
$$s = -1, 5 = A, A = 5$$

$$s = -2, 5 = -B, B = -5$$

$$f = 5 e^{-t} - 5 e^{-2t}$$

$$y = (5 e^{-t+1} - 5 e^{-2t+2}) u(t-1)$$

4. Sketch the particular solution:



▼ Laplace Transform Method: 1st-order, 1 variable, Impulse function

Problem:

$$y' + 2y = 2\delta(t-1)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 2Y = 2e^{-s}$$

2. Solve for Y:

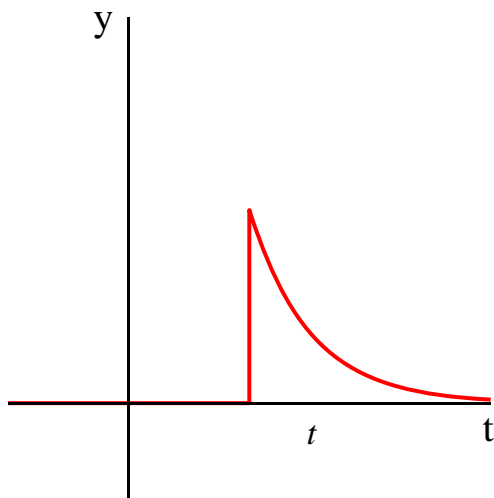
$$Y = \frac{2e^{-s}}{s+2}$$

3. Inverse Laplace Transform:

$$f = 2e^{-2t}$$

$$y = 2e^{-2t+2}u(t-1)$$

4. Sketch the particular solution:



Problem:

$$y' - 2y = -3\delta(t - 3)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys - 2Y = -3e^{-3s}$$

2. Solve for Y:

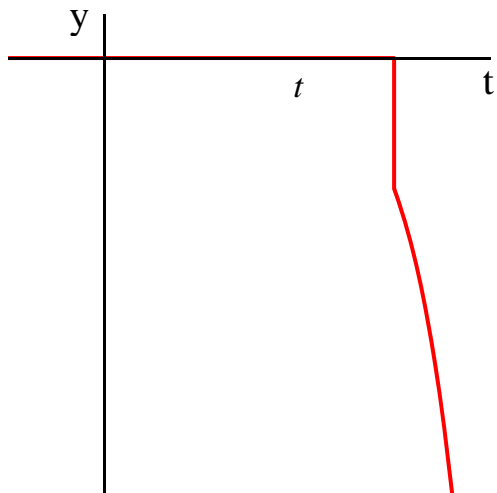
$$Y = -\frac{3e^{-3s}}{s-2}$$

3. Inverse Laplace Transform:

$$f = -3e^{2t}$$

$$y = -3e^{2t-6}u(t-3)$$

4. Sketch the particular solution:



Problem:

$$y' + 2y = -2\delta(t - 3)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 2Y = -2e^{-3s}$$

2. Solve for Y:

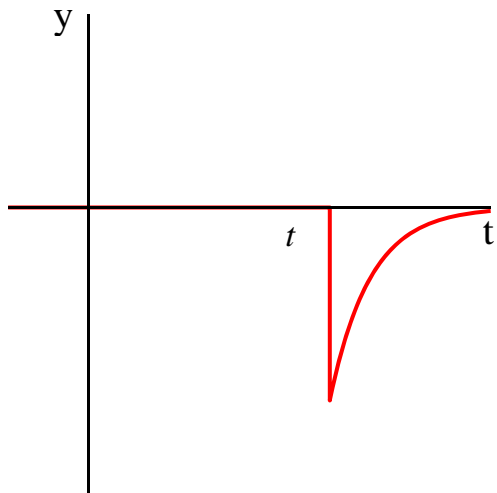
$$Y = -\frac{2e^{-3s}}{s+2}$$

3. Inverse Laplace Transform:

$$f = -2e^{-2t}$$

$$y = -2e^{-2t+6}u(t-3)$$

4. Sketch the particular solution:



Problem:

$$y' - 2y = 3\delta(t-1)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys - 2Y = 3e^{-s}$$

2. Solve for Y:

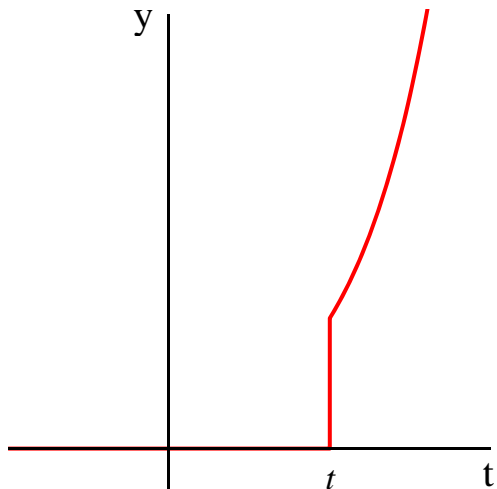
$$Y = \frac{3e^{-s}}{s-2}$$

3. Inverse Laplace Transform:

$$f = 3e^{2t}$$

$$y = 3e^{2t-2}u(t-1)$$

4. Sketch the particular solution:



Problem:

$$y' + y = 2 \delta(t - 2)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + Y = 2e^{-2s}$$

2. Solve for Y:

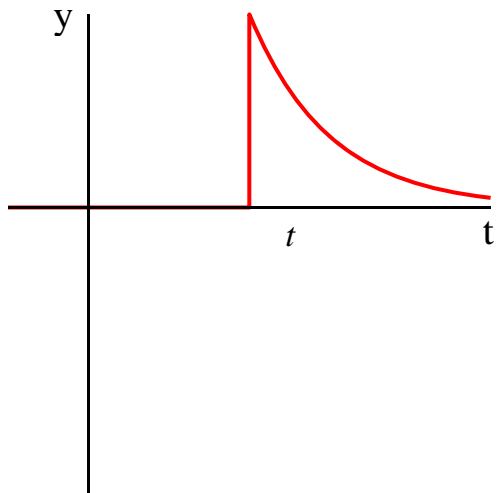
$$Y = \frac{2e^{-2s}}{s+1}$$

3. Inverse Laplace Transform:

$$f = 2e^{-t}$$

$$y = 2e^{-t+2}u(t-2)$$

4. Sketch the particular solution:



Problem:

$$y' + 2y = -2\delta(t-1)$$

$$y(0) = 0$$

1. Laplace Transform:

$$Ys + 2Y = -2e^{-s}$$

2. Solve for Y:

$$Y = -\frac{2e^{-s}}{s+2}$$

3. Inverse Laplace Transform:

$$f = -2e^{-2t}$$

$$y = -2e^{-2t+2}u(t-1)$$

4. Sketch the particular solution:

