

# MA 242 Test 3

## MISO ( $u^n \rightarrow u^1$ ): Integration

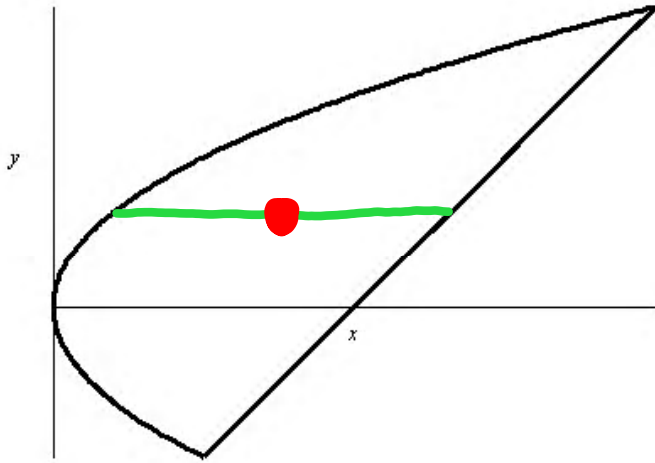
Last Name : \_\_\_\_\_ First Name: \_\_\_\_\_ Student ID: \_\_\_\_\_ Seat Code: \_\_\_\_\_

1. (1.5 points) Carry out the following iterated integration

$$\int_0^1 \int_0^z \int_0^x (2xz + 2yz) \, dy \, dx \, dz$$

$$\begin{aligned} & \int_0^x (2xz + 2yz) \, dy \\ &= \left[ 2xz y + y^2 z \right]_{y=0}^{y=x} \\ &= (2x^2 z + x^2 z) - (0 + 0) \\ &= 3x^2 z \\ & \int_0^z 3x^2 z \, dx \\ &= \left[ x^3 z \right]_{x=0}^{x=z} \\ &= z^4 - 0 \\ &= z^4 \\ & \int_0^1 z^4 \, dz \\ &= \left[ \frac{z^5}{5} \right]_0^1 \\ &= \frac{1}{5} - 0 \\ &= \frac{1}{5} \end{aligned}$$

2. (2 points)  $\int_D (x+y) dA$  where  $D = \{(x,y) : x-y^2 \geq 0, x-y-2 \leq 0\}$ .



a. Choose the “best” coordinate system out of the five that we covered in the class.

2D Rectangular

b. Choose the “best” ordering of slicing from the following options and draw the slices.

2D rectangular	2D Polar	3D Rectangular	3D Cylindrical	3D Spherical
$dy dx$	$d\theta dr$	$dx dy dz$	$dz dr d\theta$	$d\phi d\rho d\theta$
$dx dy$	$dr d\theta$	$dy dx dz$	$dr dz d\theta$	

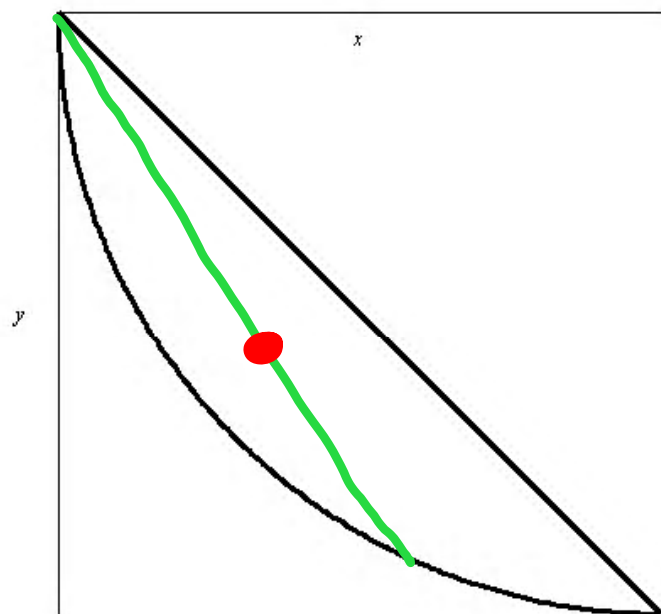
c. Show how the integration bounds are determined when necessary.

$$\begin{cases} x - y^2 = 0 & y^2 - y - 2 = 0 \\ x - y - 2 = 0 & (y-2)(y+1) = 0 \\ & y = 2, -1 \end{cases}$$

d. Write the iterated integral.

$$\int_{-1}^2 \int_{y^2}^{y+2} (x+y) dx dy$$

3. (2 points)  $\int_D (x+y) dA$  where  $D = \{(x,y) : x^2 + y^2 - 6x \leq 0, x+y \leq 0\}$ .



a. Choose the “best” coordinate system out of the five that we covered in the class.

2D Polar

b. Choose the “best” ordering of slicing from the following options and draw the slices.

2D rectangular	2D Polar	3D Rectangular	3D Cylindrical	3D Spherical
$dy dx$	$d\theta dr$	$dx dy dz$	$dz dr d\theta$	$d\phi d\rho d\theta$
$dx dy$	$dr d\theta$	$dy dx dz$	$dr dz d\theta$	

c. Show how the integration bounds are determined when necessary.

$$r^2 - 6r \cos \theta = 0$$

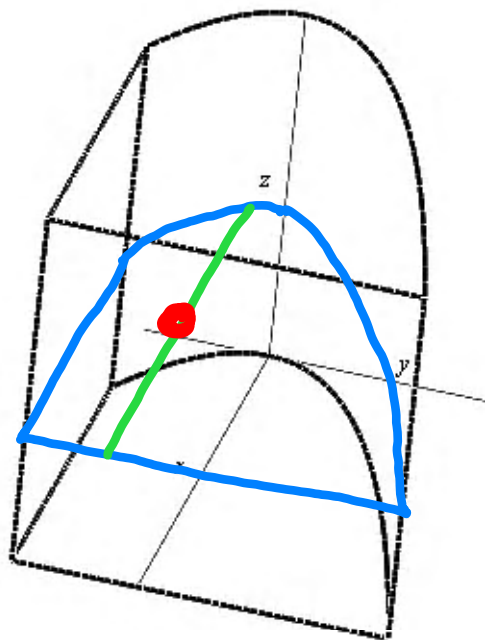
$$r(r - 6 \cos \theta) = 0$$

$$r = 0, r = 6 \cos \theta$$

d. Write the iterated integral.

$$\int_{-\frac{\pi}{2}}^{-\frac{\pi}{4}} \int_0^{6 \cos \theta} (r \cos \theta + r \sin \theta) r dr d\theta$$

4. (2 points)  $\int_D (x+y+z) dV$  where  $D = \{(x,y,z) : x-y^2 \geq 0, x \leq 4, y \geq -1, 0 \leq z \leq 3\}$



a. Choose the “best” coordinate system out of the five that we covered in the class.

3D Rectangular

b. Choose the “best” ordering of slicing from the following options and draw the slices.

2D rectangular	2D Polar	3D Rectangular	3D Cylindrical	3D Spherical
$dy dx$	$d\theta dr$	$dx dy dz$	$dz dr d\theta$	$d\phi d\rho d\theta$
$dx dy$	$dr d\theta$	$dy dx dz$	$dr dz d\theta$	

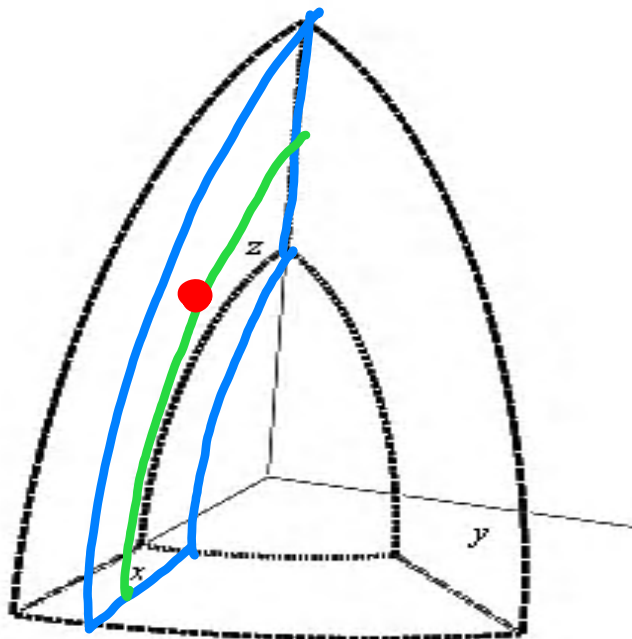
c. Show how the integration bounds are determined when necessary.

$$\begin{cases} x - y^2 = 0 \\ x = 4 \end{cases} \quad 4 - y^2 = 0 \quad y = 2, \cancel{2}$$

d. Write the iterated integral.

$$\int_0^3 \int_{-1}^2 \int_{y^2}^4 (x+y+z) dx dy dz$$

5. (2 points)  $\int_D (x+y+z) dV$  where  $D = \{(x,y,z) : \frac{1}{4} \leq x^2 + y^2 + z^2 \leq 1, z \geq 0, x \geq y\}$



a. Choose the “best” coordinate system out of the five that we covered in the class.

3D spherical

b. Choose the “best” ordering of slicing from the following options and draw the slices.

2D rectangular	2D Polar	3D Rectangular	3D Cylindrical	3D Spherical
$dy dx$	$d\theta dr$	$dx dy dz$	$dz dr d\theta$	$d\phi d\rho d\theta$
$dx dy$	$dr d\theta$	$dy dx dz$	$dr dz d\theta$	

c. Show how the integration bounds are determined when necessary.

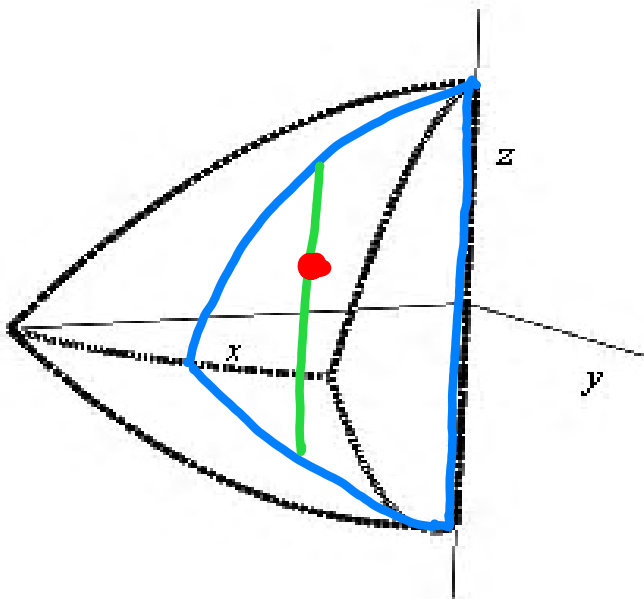
$$\frac{1}{4} \leq \rho^2 \leq 1$$

$$\frac{1}{2} \leq \rho \leq 1$$

d. Write the iterated integral.

$$\int_0^{\frac{\pi}{4}} \int_{\frac{1}{2}}^1 \int_0^{\frac{\pi}{2}} (\rho \sin\phi \cos\theta + \rho \sin\phi \sin\theta + \rho \cos\phi) \rho^2 \sin\phi d\phi d\rho d\theta$$

6. (2 points)  $\int_D (x+y+z) dV$  where  $D = \{(x,y,z) : x^2 + y^2 - \frac{1}{4} \leq z \leq -x^2 - y^2 + \frac{1}{4}, x \geq y\}$



a. Choose the “best” coordinate system out of the five that we covered in the class.

3D cylindrical

b. Choose the “best” ordering of slicing from the following options and draw the slices.

2D rectangular	2D Polar	3D Rectangular	3D Cylindrical	3D Spherical
$dy dx$	$d\theta dr$	$dx dy dz$	$dz dr d\theta$	$d\phi d\rho d\theta$
$dx dy$	$dr d\theta$	$dy dx dz$	$dr dz d\theta$	

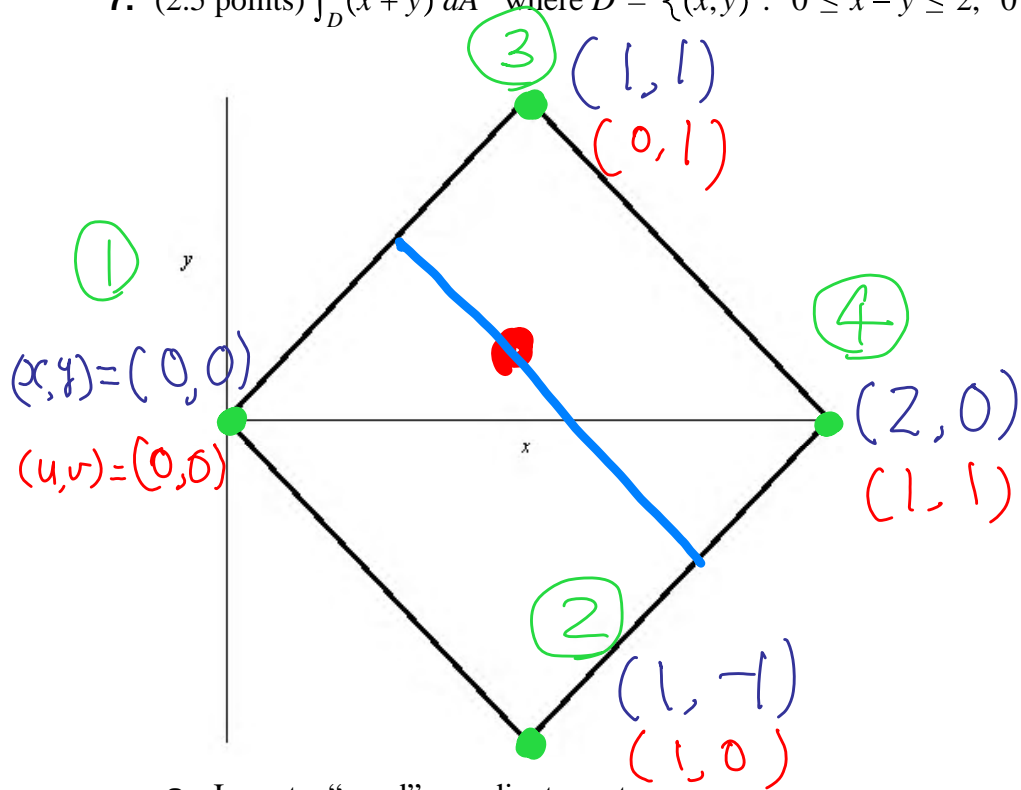
c. Show how the integration bounds are determined when necessary.

$$\begin{cases} r^2 - \frac{1}{4} = z \\ -r^2 + \frac{1}{4} = z \end{cases} \quad \begin{cases} 2r^2 - \frac{1}{2} = 0 \\ r^2 = \frac{1}{4} \\ r = \frac{1}{2} \end{cases}$$

d. Write the iterated integral.

$$\int_0^{\frac{1}{4}} \int_0^{\frac{1}{2}} \int_{r^2 - \frac{1}{4}}^{-r^2 + \frac{1}{4}} (r \cos \theta + r \sin \theta + z) r dz dr d\theta$$

7. (2.5 points)  $\int_D (x+y) dA$  where  $D = \{(x,y) : 0 \leq x-y \leq 2, 0 \leq x+y \leq 2\}$



a. Invent a "good" coordinate system.

$$\begin{cases} x = a_1 u + b_1 v + c_1 \\ y = a_2 u + b_2 v + c_2 \end{cases}$$

$$\textcircled{1} \begin{cases} 0 = c_1 \\ 0 = c_2 \end{cases} \quad \textcircled{2} \begin{cases} 1 = a_1 \\ -1 = a_2 \end{cases} \quad \textcircled{3} \begin{cases} 1 = b_1 \\ 1 = b_2 \end{cases}$$

$$J = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$\det J = 2$$

$$x+y = 2v$$

b. Choose "any" ordering of slicing and draw the slices.

$$du dv$$

c. Show how the integration bounds are determined when necessary.

$$0 \leq 2u \leq 2$$

$$0 \leq u \leq 1$$

$$0 \leq 2v \leq 2$$

$$0 \leq v \leq 1$$

d. Write the iterated integral.

$$\int_0^1 \int_0^1 (2v) \cdot 2 \cdot du dv$$