MA 242 Test 3

MISO ($u^n \to u^1$): Integration

Last Name : _____ First Name: _____ Student ID: _____ Seat Code: _____

(2.0 points)

1. (~~1.5~~ points) Carry out the following iterated integration

$$\int_0^1 \int_0^z \int_0^x (2xz + 2yz)\ dy\ dx\ dz$$

$$\int_0^x (2xz + 2yz)\, dy$$

$$= \left[2xzy + y^2 z\right]_{y=0}^{y=x}$$

$$= (2x^2 z + x^2 z) - (0 + 0)$$

$$= 3x^2 z$$

$$\int_0^z 3x^2 z\, dx$$

$$= \left[x^3 z\right]_{x=0}^{x=z}$$

$$= z^4 - 0$$

$$= z^4$$

$$\int_0^1 z^4\, dz$$

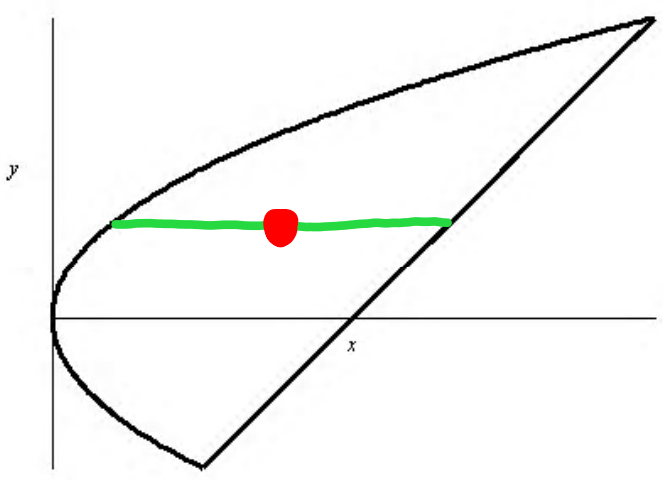
$$= \left[\frac{z^5}{5}\right]_0^1$$

$$= \frac{1}{5} - 0$$

$$= \frac{1}{5}$$

(3.0 points)

2. (~~2~~ points) $\int_D (x+y)\,dA$ where $D = \{(x,y) : x - y^2 \geq 0,\ x - y - 2 \leq 0\}$.



a. Choose the "best" coordinate system out of the five that we covered in the class.

2D Retangular

b. Choose the "best" ordering of slicing from the following options and draw the slices.

2D rectangular	2D Polar	3D Rectangular	3D Cylindrical	3D Spherical
$dy\,dx$	$d\theta\,dr$	$dx\,dy\,dz$	$dz\,dr\,d\theta$	$d\phi\,d\rho\,d\theta$
$dx\,dy$	$dr\,d\theta$	$dy\,dx\,dz$	$dr\,dz\,d\theta$	

c. Show how the integration bounds are determined when necessary.

$$\begin{cases} x - y^2 = 0 \\ x - y - 2 = 0 \end{cases} \qquad y^2 - y - 2 = 0$$

$$(y-2)(y+1) = 0$$

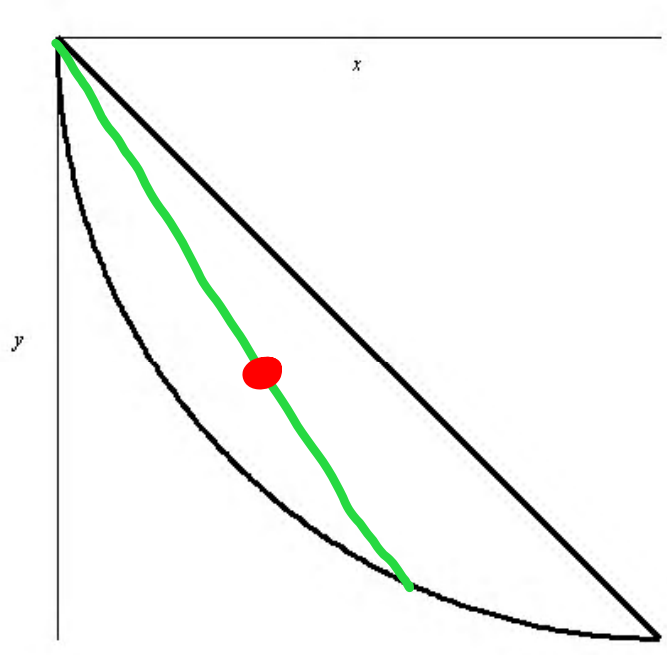
$$y = 2, -1$$

d. Write the iterated integral.

$$\int_{-1}^{2} \int_{y^2}^{y+2} (x + y)\,dx\,dy$$

(3.0 points)

3. (~~2~~ points) $\int_D (x+y)\,dA$ where $D = \{(x,y) : x^2+y^2-6x \le 0, \quad x+y \le 0\}$.



a. Choose the "best" coordinate system out of the five that we covered in the class.

2D Polar

b. Choose the "best" ordering of slicing from the following options and draw the slices.

2D rectangular	2D Polar	3D Rectangular	3D Cylindrical	3D Spherical
$dy\,dx$	$d\theta\,dr$	$dx\,dy\,dz$	$dz\,dr\,d\theta$	$d\phi\,d\rho\,d\theta$
$dx\,dy$	$dr\,d\theta$	$dy\,dx\,dz$	$dr\,dz\,d\theta$	

c. Show how the integration bounds are determined when necessary.

$r^2 - 6r\cos\theta = 0$

$r(r - 6\cos\theta) = 0$

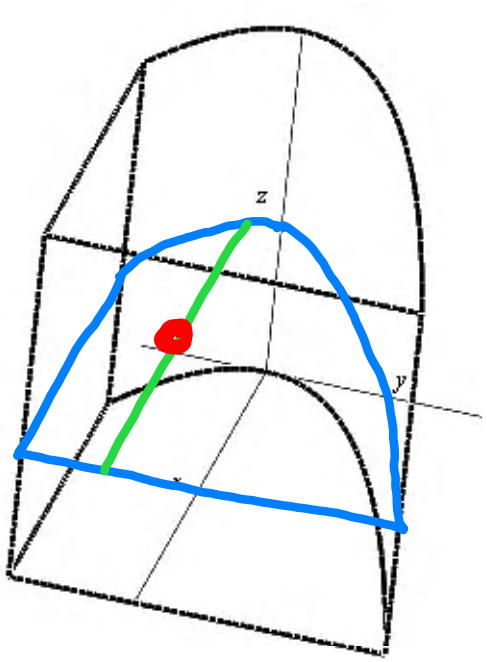
$r = 0, \quad r = 6\cos\theta$

d. Write the iterated integral.

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{4}} \int_0^{6\cos\theta} (r\cos\theta + r\sin\theta)\, r\, dr\, d\theta$$

(4.5 points)

4. (2 points) $\int_D (x+y+z)\, dV$ where $D = \{(x,y,z) : x - y^2 \geq 0,\ x \leq 4,\ y \geq -1,\ 0 \leq z \leq 3\}$



a. Choose the "best" coordinate system out of the five that we covered in the class.

3D Retangular

b. Choose the "best" ordering of slicing from the following options and draw the slices.

2D rectangular	2D Polar	3D Rectangular	3D Cylindrical	3D Spherical
$dy\ dx$	$d\theta\ dr$	$dx\ dy\ dz$	$dz\ dr\ d\theta$	$d\phi\ d\rho\ d\theta$
$dx\ dy$	$dr\ d\theta$	$dy\ dx\ dz$	$dr\ dz\ d\theta$	

c. Show how the integration bounds are determined when necessary.

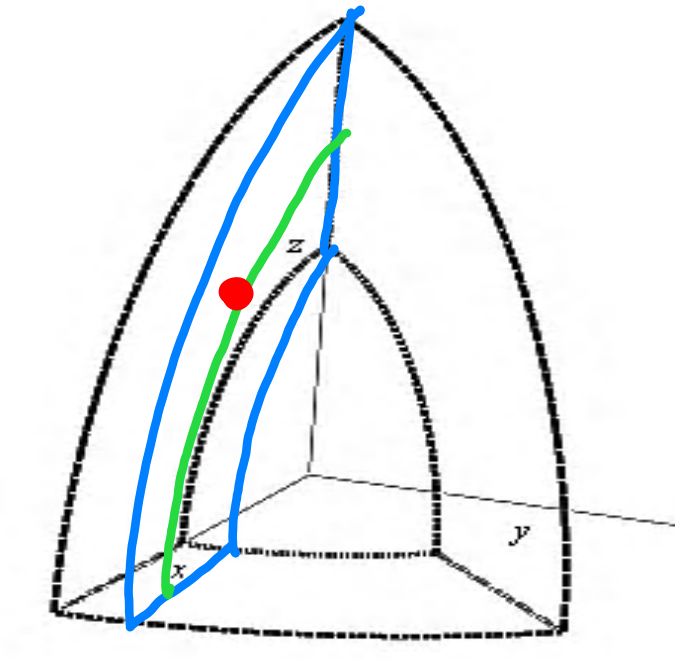
$\begin{cases} x - y^2 = 0 \\ x = 4 \end{cases}$ $\quad 4 - y^2 = 0 \quad y = 2, \cancel{-2}$

d. Write the iterated integral.

$$\int_0^3 \int_{-1}^{2} \int_{y^2}^{4} (x+y+z)\, dx\, dy\, dz$$

(4.5 points)

5. (2 points) $\int_D (x+y+z)\, dV$ where $D = \left\{(x,y,z) : \frac{1}{4} \le x^2+y^2+z^2 \le 1,\ z \ge 0,\ x \ge y\right\}$



a. Choose the "best" coordinate system out of the five that we covered in the class.

3D spherical

b. Choose the "best" ordering of slicing from the following options and draw the slices.

2D rectangular	2D Polar	3D Rectangular	3D Cylindrical	3D Spherical
dy dx	*dθ dr*	*dx dy dz*	*dz dr dθ*	*dϕ dρ dθ*
dx dy	*dr dθ*	*dy dx dz*	*dr dz dθ*	

c. Show how the integration bounds are determined when necessary.

$\frac{1}{4} \le \rho^2 \le 1$

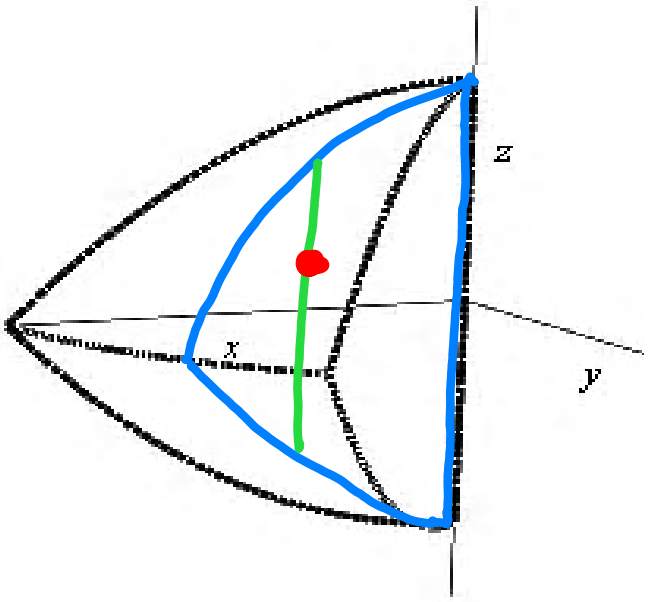
$\frac{1}{2} \le \rho \le 1$

d. Write the iterated integral.

$$\int_0^{\frac{\pi}{4}} \int_{\frac{1}{2}}^{1} \int_0^{\frac{\pi}{2}} (\rho\sin\phi\cos\theta + \rho\sin\phi\sin\theta + \rho\cos\phi)\rho^2\sin\phi \, d\phi\, d\rho\, d\theta$$

(4.5 points)

6. (~~2~~ points) $\int_D \left(x+y+z\right) dV$ where $D = \left\{(x,y,z) : x^2+y^2-\frac{1}{4} \le z \le -x^2-y^2+\frac{1}{4},\ x \ge y\right\}$



a. Choose the "best" coordinate system out of the five that we covered in the class.

3D Cylindrical

b. Choose the "best" ordering of slicing from the following options and draw the slices.

2D rectangular	2D Polar	3D Rectangular	3D Cylindrical	3D Spherical
$dy\,dx$	$d\theta\,dr$	$dx\,dy\,dz$	$dz\,dr\,d\theta$	$d\phi\,d\rho\,d\theta$
$dx\,dy$	$dr\,d\theta$	$dy\,dx\,dz$	$dr\,dz\,d\theta$	

c. Show how the integration bounds are determined when necessary.

$$\begin{cases} r^2-\frac{1}{4}=z \\ -r^2+\frac{1}{4}=z \end{cases} \qquad 2r^2-\frac{1}{2}=0 \qquad r^2=\frac{1}{4} \qquad r=\frac{1}{2}$$

d. Write the iterated integral.

$$\int_0^{\frac{\pi}{4}} \int_0^{\frac{1}{2}} \int_{r^2-\frac{1}{4}}^{-r^2+\frac{1}{4}} (r\cos\theta + r\sin\theta + z)\, r\, dz\, dr\, d\theta$$