

# MA 242 Test 3 (Model Answer)

$$1. \int_0^y (2xz + 2yz) dx$$

$$= \left[ x^2 z + 2yzx \right]_0^y$$

$$= (y^2 z + 2y^2 z) - 0$$

$$= 3y^2 z$$

$$\int_0^z 3y^2 z dy$$

$$= \left[ y^3 z \right]_0^z$$

$$= (z^4) - 0$$

$$= z^4$$

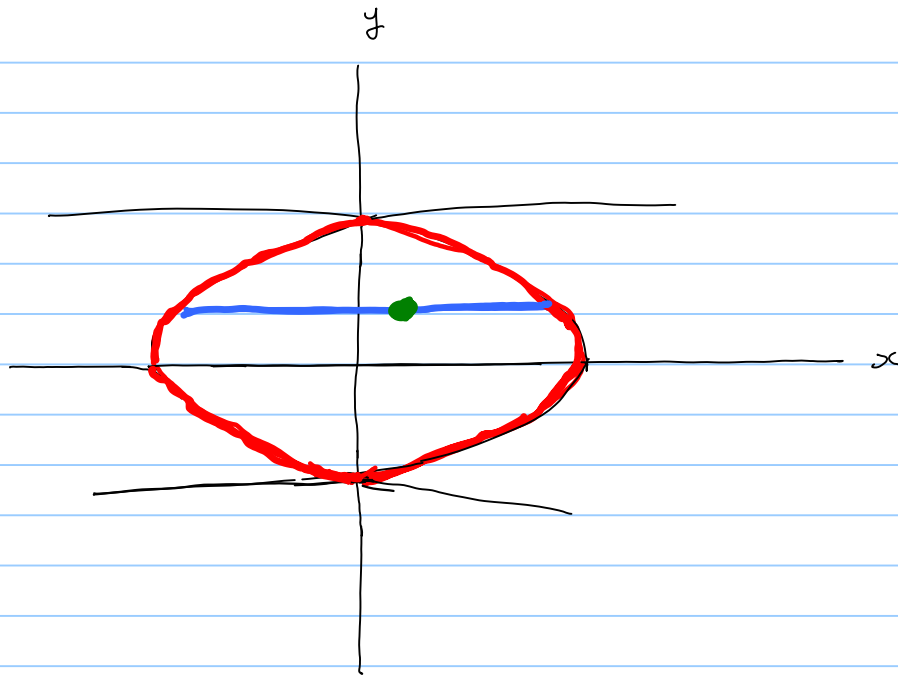
$$\int_0^1 z^4 dz$$

$$= \left[ \frac{z^5}{5} \right]_0^1$$

$$= \frac{1}{5} - 0$$

$$= \frac{1}{5}$$

2.



Rectangular

$$\int_{-1}^1 \int_{y^2-1}^{-y^2+1} (x+y) \, dx \, dy$$

$$\begin{cases} x = y^2 - 1 \\ x = -y^2 + 1 \end{cases}$$

$$y^2 - 1 = -y^2 + 1$$

$$2y^2 = 2$$

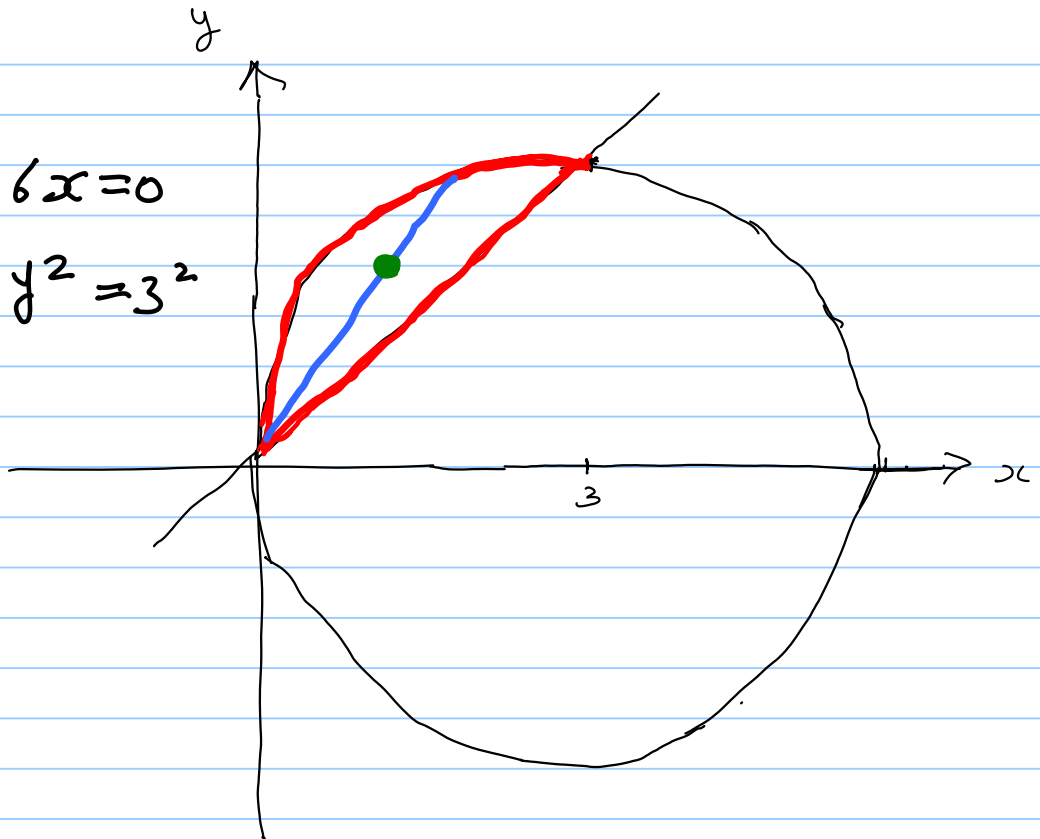
$$y^2 = 1$$

$$y = \pm 1$$

3.

$$x^2 + y^2 - 6x = 0$$

$$(x-3)^2 + y^2 = 3^2$$



Polar

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{6 \cos \theta} (r \cos \theta + r \sin \theta) r \, dr \, d\theta$$

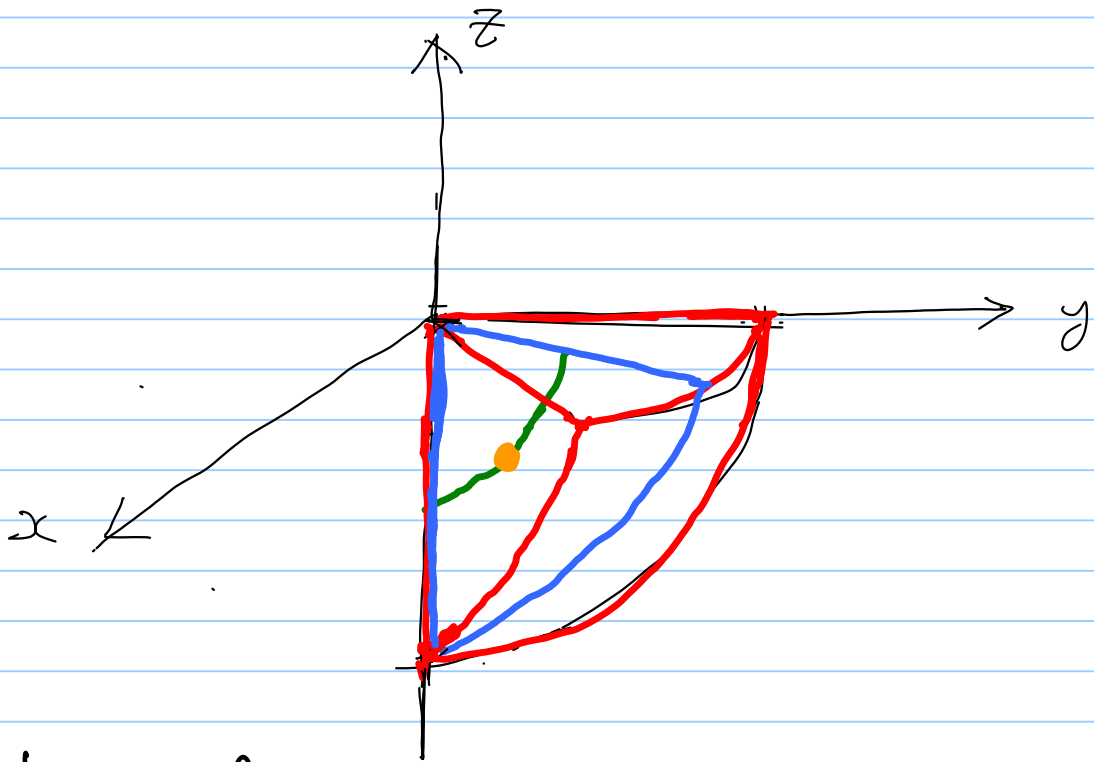
$$x^2 + y^2 - 6x = 0$$

$$r^2 - 6r \cos \theta = 0$$

$$r^2 = 6r \cos \theta$$

$$r = 6 \cos \theta$$

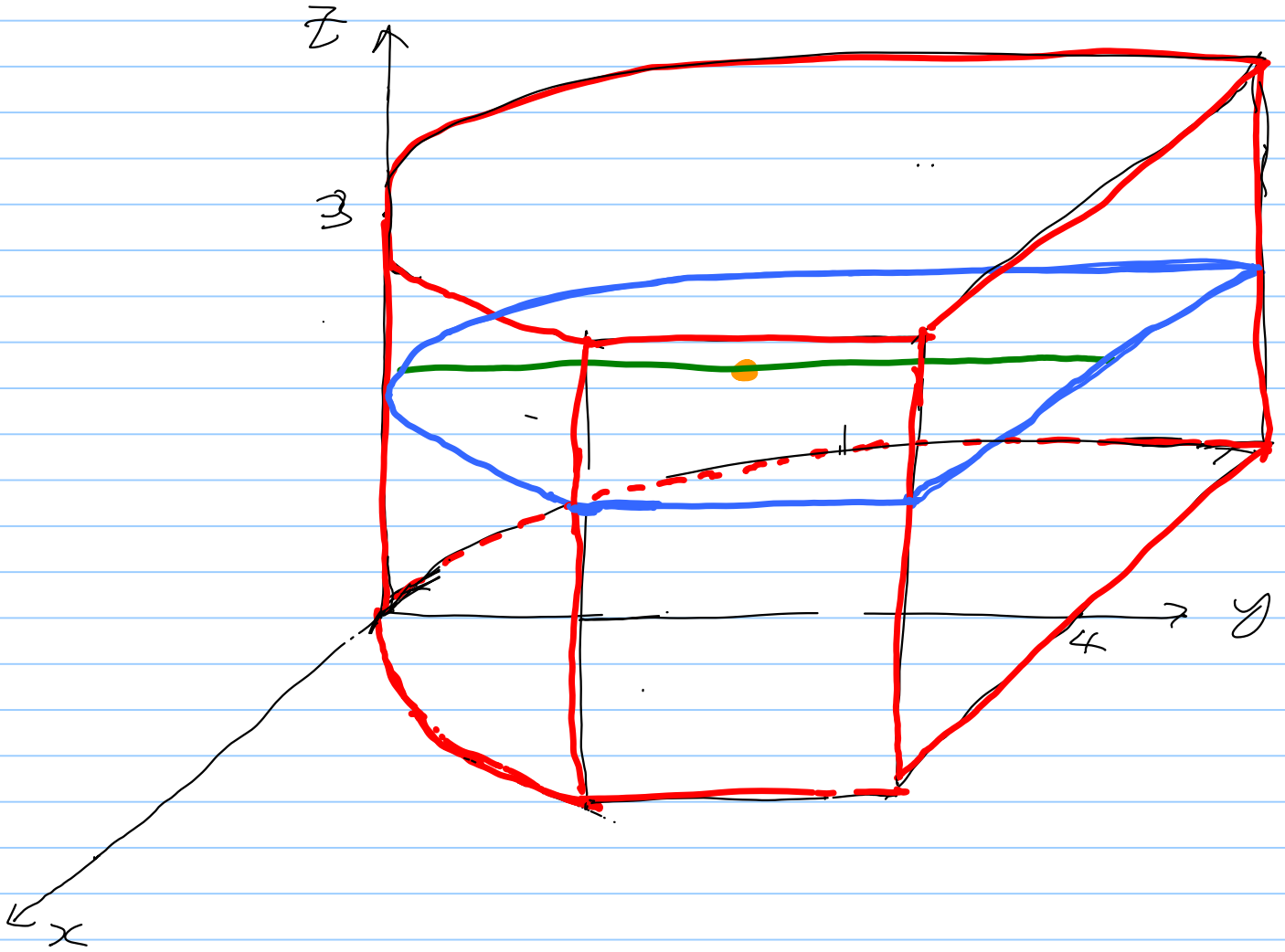
4.



Spherical

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^1 \int_{\frac{\pi}{2}}^{\pi} (\rho \sin \phi \cos \theta + \rho \sin \phi \sin \theta + \rho \cos \phi) \rho^2 \sin \phi \, d\phi \, d\rho \, d\theta$$

5.

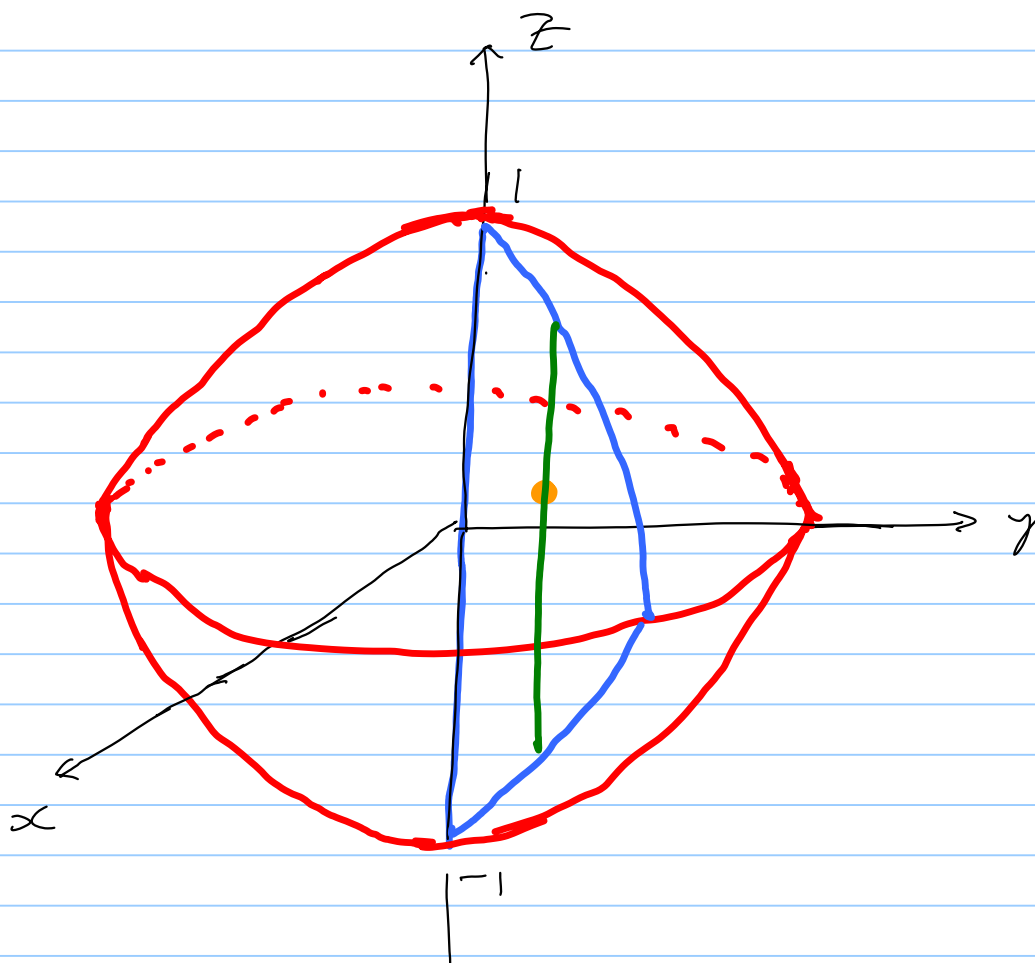


Rectangular

$$\int_0^3 \int_{-2}^1 \int_{x^2}^4 (x+y+z) dy dx dz$$

$$\begin{aligned} y &= x^2 \\ 4 &= x^2 \\ x &= \pm 2 \end{aligned}$$

6.



Cylindrical

$$\int_0^{2\pi} \int_0^1 \int_{r^2-1}^{-r^2+1} (r\cos\theta + r\sin\theta + z) r \, dz \, dr \, d\theta$$

$$\begin{cases} z = -x^2 - y^2 + 1 = -r^2 + 1 \\ z = x^2 + y^2 - 1 = r^2 - 1 \end{cases}$$

$$-r^2 + 1 = r^2 - 1$$

$$2r^2 = 2$$

$$r = 1$$