MA 242 Test 4

Multivariate Vector Functions ($\mathbb{R}^n \to \mathbb{R}^n$)

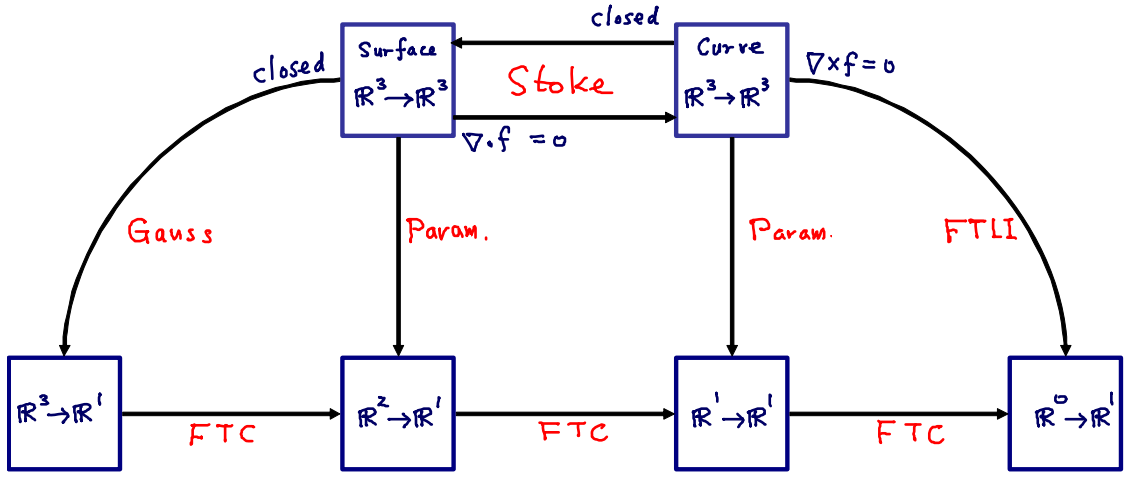
Last Name : _____ First Name: _____ Student ID: _____ Seat Code: _____

1. [1.0 points] Write the precise statement of the following theorems:

- Curve Parameterization
  $\int_C f \cdot dr = \int_{t_1}^{t_2} g\, dt$
  where
  $g = f \cdot \frac{dr}{dt}$
  $[t_1, t_2]$ is the domain for the parameter t.
- Surface Parameterization
  $\int_S f \cdot ds = \int_D g\, dA$
  where
  $g = f \cdot (r_u \times r_v)$, RHR
  D is the domain for the parameters (u, v).
- Fundamental theorem of curve (line) integral
  $\int_C f \cdot dr = [g]_{r_1}^{r_2}$
  where
  $\nabla g = f$
  $\{r_1, r_2\} = \partial C$.
- Stoke's theorem (curve integral → surface integral)
  $\int_C f \cdot dr = \int_S g \cdot ds$
  where
  $g = \nabla \times f$
  $C = \partial S$, RHR.
- Stoke's theorem (surface integral → curve integral)
  $\int_S f \cdot ds = \int_C g \cdot dr$
  where
  $\nabla \times g = f$
  $C = \partial S$, RHR.
- Gauss' theorem
  $\int_S f \cdot ds = \int_D g\, dV$
  where
  $g = \nabla \cdot f$
  $S = \partial D$, outward.

2. [1.0 points] Draw the following diagrams:

- Roadmap



- Driving Instruction for Curve Integral
  - C is open and $\nabla \times f \neq 0$: Param
  - C is open and $\nabla \times f = 0$: FTLI
  - C is closed and $\nabla \times f \neq 0$: Stoke (curve integral $\rightarrow$ surface integral)
  - C is closed and $\nabla \times f = 0$: 0
- Driving Instruction for Surface Integral
  - S is open and $\nabla \cdot f \neq 0$: Param
  - S is open and $\nabla \cdot f = 0$: Stoke (surface integral $\rightarrow$ curve integral)
  - S is closed and $\nabla \cdot f \neq 0$: Gauss
  - S is closed and $\nabla \cdot f = 0$: 0