Energy-Efficient Distributed Detection Via Multi-hop Transmission in Sensor Networks

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Abstract

We investigate three multihop fusion schemes for distributed detection in geographically dispersed sensor networks, Multihop Forwarding (MF), and Log-likelihood ratio Fusion (LF), which are different in the transmitted messages, fusion rules, and communication structure. Simulation results show that transmission energy is significantly reduced by multihop fusion schemes as compared to direct transmission, with LF outperforming the others. Moreover, it is shown that LF exhibits the most favorable energy scaling law with the network size among these schemes. (EDICS: COM-NETW)

I. Introduction

Large-scale sensor networks are suitable for signal detection applications, as the detection error probability typically decays exponentially with the number of observations. Most existing works on distributed detection have focused on designing local mapping rules and fusion rules to maximize the detection performance under the assumption that a bank of independent dedicated channels (e.g., [1]) or a multi-access channel (e.g. [2]) is used to directly transmit local decisions to the fusion center. However, direct transmission is very energy-inefficient when sensors have widely varying distances towards the fusion center. Given the exponential decay in signal power with the transmission distance, a natural way for energy conservation is multi-hop fusion, where the communication structure along which messages are sent to the fusion center is a spanning tree of the network graph. A similar idea of multihop fusion has been explored in [3], where each sensor transmits the real-valued log-likelihood ratio (LLR) to the next high level sensor only when the LLR is above a threshold. However, the assumption that real

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values can be received perfectly does not hold in practice, and makes it difficult to evaluate the true detection performance and energy expenditure. In this paper, we focus on the regime where the number of sensors is large, and investigate several options for multi-hop fusion with sensors transmitting *quantized observations*. The fusion rules at the sensors and at the fusion center, as well as the communication structure of multi-hop fusion *jointly* determine the detection performance and the energy consumption.

We assume that the sensor nodes and the fusion center S constitute the vertices of a connected graph $G = (V, E)$, where $|V| = n$. The fusion center wishes to make a decision whether Hypothesis 0 (H_0) or Hypothesis 1 (H_1) is correct. Sensor j makes an observation Y_j, i.i.d. across sensors given the hypothesis. The cumulative distribution functions of Y_j under H_0 and H_1 are respectively given by F_0 and F_1, with differentiable Lebesgue densities f_0 and f_1. The weight of an edge $e = (i, j)$, denoted by $w(e)$, is defined as the energy consumed for reliably transmitting one bit between node i and j. We assume $w(e) \propto d(i, j)^\kappa$, where $d(i, j)$ is the Euclidean distance between i and j, and $\kappa \in [2, 4]$ is the pathloss exponent. Without loss of generality the received messages are assumed to be error-free. The communication structure, along which information is propagated to the fusion center, is a spanning tree (ST) on G rooted at S. The required path routes are assumed to be pre-computed, and the associated overhead is neglected.

In our study, local observations are quantized by the optimal likelihood ratio quantizer (LRQ) [4] in the sense that the exponent of detection error probability is maximized. The traditional approach of direct transmission (DT) using independent parallel channels serves as a performance baseline. In addition, we consider three multihop fusion schemes: Multihop Forwarding (MF), Multihop Histogram Fusion (HF) and Multihop LLR Fusion (LF). For MF, the quantized observations are transmitted to the fusion center through the Shortest Path Tree (SPT) without further processing at relaying nodes. The same detection performance as direct transmission is achieved, with a significantly reduced transmission energy. For HF, each sensor transmits the histogram of the quantized observations of its descendants and itself. Since the histogram is a sufficient statistic for detection with conditionally i.i.d. observations, no performance loss is incurred. Moreover, the number of bits transmitted by a node grows only logarithmically with the number of its descendants (instead of linearly for the MF scheme). HF adopts the minimum spanning tree (MST) as the transmission structure, and offers further energy saving than MF when communication

TABLE I

SUMMARY OF DISTRIBUTED DETECTION SCHEMES

Scheme	Direct Transmission	Multihop Forwarding	Histogram Fusion	LLR Fusion
Fusion Message	Quantized value	Quantized value	Histogram	LLR value
Communication structure	Depth-1 tree	SPT	MST	MST
Energy Expenditure	$k\sum_{j=1}^{n} w((j,S))$	$k\sum_{j=1}^{n} c_j^*$	$2^k \log n \sum_{e\in MST} w(e)$	$k\sum_{e\in MST} w(e)$
Detection Performance	Baseline	Same	Same	Lossy

is highly constrained. LF bears the same spirit as HF, but with the normalized LLR propagated instead, which suffers additional information loss at intermediate nodes. The latter two schemes can be viewed as the counterpart of network coding (in contrast to forwarding) in the context of distributed detection. Our results demonstrate that LF requires significantly less power to achieve the same detection performance compared with MF and HF in scenarios of practical interest. It is also shown that LF exhibits the most favorable energy scaling law with the network size n among these schemes.

The following notations will be used in the paper. For node j, ch_j denotes the set of children of j, de_j denotes the set of descendants of j, pa_j denotes the parent of j, lev_j denotes the level of j (i.e., the number of hops from the fusion center), and n_j denotes the size of j (i.e., $|de_j|+1$). The rest of the paper is organized as follows. In Section II, the proposed schemes are discussed, and corresponding detection performance and energy expenditure are studied. In Section III, numerical results of various detection schemes are provided. In Section IV, we study scaling laws of energy consumption with the network size for various detection schemes. Finally, Section V concludes the paper.

II. DISTRIBUTED DETECTION SCHEMES

In this section, we first introduce direct transmission which serves as a performance baseline, followed by the three multihop fusion schemes: MF, HF and LF. A summary for the four schemes is provided Table I, where c_j^* denotes the weight of the shortest path from node j to S on G. The SPT is a superposition of all individual shortest paths for different nodes, and the MST is a spanning tree with the sum of the edge weights minimized [5].

A. Direct Transmission

We assume that each sensor quantizes its raw observation into k bits, which are transmitted directly to the fusion center. For the regime of large numbers of sensors, the natural optimization criterion for quantization is the error exponent given by the Chernoff Information. Since the Chernoff Information of the quantized values are typically difficult to evaluate, it is convenient to consider its lower bound, given by the Bhattacharyya distance. It is known that for both the Chernoff Information and the Bhattacharyya distance, the optimal local quantizers are likelihood ratio quantizers [4]. Denote the LLR function

$$L(y) \triangleq \log \frac{f_1(y)}{f_0(y)}, \tag{1}$$

and let $L_j \triangleq L(y_j)$, whose distribution functions under H_0 and H_1 are denoted by F_{L0} and F_{L1}. The LRQ is specified by the quantization vector $\mathbf{a} = [a_0, \ldots, a_M]^T$, where $M = 2^k$ and $-\infty = a_0 < a_1 \ldots < a_{M-1} < a_M = +\infty$. The quantization is defined by the mapping U:

$$u_j \triangleq U(y_j) = i, \quad \text{if } L_j \in I_i, \tag{2}$$

where $I_i = (a_{i-1}, a_i]$, $i = 1, \ldots, M$. The quantization induces the probability mass function (p.m.f.) of the quantized observations under H_0, given by

$$p_0(i) \triangleq \Pr(u_j = i | H_0) = F_{L0}(a_i) - F_{L0}(a_{i-1}) \tag{3}$$

for $1 \leq i \leq M$, and similarly define the p.m.f. p_1 under H_1. For a given M, the problem is then to find the optimal $\mathbf{a}$ that maximizes the Bhattacharyya distance:

$$B_M(\mathbf{a}) = -\log \sum_{i=1}^{M} \sqrt{p_0(i) p_1(i)}. \tag{4}$$

Newton's method may be applied to search for the minimum.

B. Multihop Forwarding (MF)

The easiest alternative to direct transmission is multihop forwarding (MF): each node simply transmits its quantized observation via the path on the communication structure to the fusion center. Since the intermediate nodes only forward the information without processing, the achievable performance is the same as direct transmission. SPT is obviously the optimal communication structure for the MF scheme.

C. Multihop Histogram Fusion (HF)

For the MF scheme, sensor j needs to send all the data of its descendants to its parent node on the SPT. Hence for a dense network, nodes with large numbers of descendants generate heavy traffic and drain power fast. Note that for detection with conditionally i.i.d. observations, the histogram of sensor observations is a sufficient statistic. Each sensor can therefore only send the histogram of its descendants (including itself)' quantized observations, without incurring any loss in the detection performance. Since binary or quadratic quantization is often sufficient for most detection scenarios when the number of sensors is large, transmitting histograms instead of the exact observations conserves energy and bandwidth.

Histogram fusion (HF) works as follows. Node j uses M time slots for transmission, and in the ith slot, $\lceil \log_2(n_j+1) \rceil$ bits are transmitted to indicate the number of descendants of j that has quantized observation i. Hence the transmission energy for j grows only logarithmically with n_j, instead of linearly for MF. The problem is to find the optimal spanning tree that minimizes the total transmission energy for HF:

$$ST^* = \arg\min_{ST} 2^k \sum_{j=1}^{n} w\left((j, pa(j))\right) \lceil \log_2(n_j+1) \rceil.$$

It is difficult to obtain an exact solution to this problem, as the edge weights and node sizes of the spanning tree are inter-dependent. Alternatively, we can use the upper bound of n_j+1, i.e., n, and transmit $\log n$ bits in each slot at all nodes. Under this assumption, the solution to the optimization problem is given by the MST. Thus, the transmission energy for HF is

$$E_{HF} = 2^k \log n \sum_{e \in MST} w(e). \tag{5}$$

D. Multihop LLR Fusion (LF)

The message length in the HF scheme is on the order of $O(\log n)$, which can be further constrained for better efficiency. Let us consider enforcing that each node in the communication structure transmits only k bits based on its observation and the messages from its children. Obviously such a scheme induces some form of information reduction at the intermediate nodes. For centralized detection in the Bayesian setting, the fusion center performs a threshold test on the normalized LLR $L_S \triangleq \frac{1}{n}\sum L_j \triangleq \frac{1}{n}\sum L(y_j)$, and H_1 is accepted (rejected) if $L_S > 0$ (respectively, $L_S \leq 0$) as $n \to \infty$[1]. The intention of LF is to obtain an approximation of the normalized LLR at the fusion center.

Denote the LLR for quantized values $L_q(i) \triangleq \log \frac{p_1(i)}{p_0(i)}, \quad i = 1, \cdots, M$. It can be shown using intermediate value theorem that $L_q(i) \in I_i$, $i = 1, \cdots, M$. Thus $L_q(i)$ serves as a good estimate of the LLR value that is quantized to be i, especially when M is large. Each intermediate node can thus obtain an estimate of the normalized LLR of all its descendants based on their quantized values. Eventually the fusion center obtains an estimate of L_S, to which the threshold test is applied[2].

1) Algorithm: Given the communication structure, the LLR Fusion starts from highest level nodes. The level number l is set to be $\max_j(lev_j)$ at the initial stage. During one iteration, all nodes j with $lev_j = l$ process the data from its own measurement and from their children, following the four steps below:

1) Compute the LLR of the unquantized observation of j: $L_j \Leftarrow L(y_j)$.
2) Compute the LLR of the quantized value of every child of j: $\hat{L}_t \Leftarrow L_q(u_t), \quad t \in ch_j$, where u_t is the quantized value sent by node t.
3) Estimate the normalized LLR for nodes in the subtree including j and its descendants, denoted by $\tilde{L}_j$:

$$\tilde{L}_j \Leftarrow \frac{1}{n_j} \left(L_j + \sum_{t \in ch_j} n_t \hat{L}_t \right). \tag{6}$$

4) Quantize $\tilde{L}_j$ using the LRQ, i.e., $u_j \triangleq i, \quad \text{if} \quad \tilde{L}_j \in I_i$, and send u_j to the parent node.

Then the level number l is decremented by 1 and the above process repeats until the l reaches 0. The fusion center then computes an approximation of the normalized LLR:

$$\hat{L}_S \Leftarrow \frac{1}{n} \sum_{j \in ch_S} n_j \hat{L}_j = \frac{1}{n} \sum_{j \in ch_S} n_j L_q(u_j). \tag{7}$$

A threshold test is then performed on $\hat{L}_S$ to decide the hypothesis.

[1]For finite n, the corresponding threshold is $\frac{1}{n} \log \frac{\pi_0}{\pi_1}$, where π_0 and π_1 are a priori probabilities of H_0 and H_1.

[2]Since the normalized LLRs at different nodes have different distributions depending on their positions in the communication structure, it is impossible to design a single optimal quantizer for all nodes. For simplicity and tractability, we assume that the same LRQ discussed in Section II.A is used at all nodes.

2) Analysis: **Proposition:** $\hat{L}_S \to L_S$, as $k \to \infty$.

Proof: It suffices to show that as $k \to \infty$, $\tilde{L}_j \to \frac{1}{n_j}\left(L_j + \sum_{t \in de_j} L_t\right)$ for all j. This is true for a leaf node j. Now, for any j, assume that $\tilde{L}_t \to \frac{1}{n_t}\left(L_t + \sum_{v \in de_t} L_v\right)$, $\forall t \in ch_j$. Since $\hat{L}_t \to \tilde{L}_t$ as $k \to \infty$, $\forall t \in ch_j$ (noting that $\hat{L}_j = L_q(u_j) \in I_{u_j}$ and $\tilde{L}_j \in I_{u_j}$), we have as $k \to \infty$, $\tilde{L}_j \to \frac{1}{n_j}\left[L_j + \sum_{t \in ch_j}\left(L_t + \sum_{v \in de_t} L_v\right)\right] = \frac{1}{n_j}\left(L_j + \sum_{t \in de_j} L_t\right)$. ■

The above shows that the performance of LF asymptotically approaches that of the centralized detection as the quantization level k approaches infinity. For small k, the difference between $\hat{L}_j$ and $\tilde{L}_j$ is non-negligible, and the performance of LF degrades. Unlike the MF and HF schemes, the detection performance of LF critically depends on the communication structure. Intuitively, the coarser the approximations incurred for the LLR values during fusion, the severer the distortion of the resultant normalized LLR. Hence there exists an inherent tradeoff between detection performance and energy efficiency: the best detection performance is obviously achieved when all sensor nodes are direct children of the fusion center, which corresponds to the most energy-inefficient DT scheme; on the other hand, for a given k, the best energy efficiency is achieved when the spanning tree is the MST. Extensive simulations suggest MST the best choice in the sense that it requires the least energy to attain the same error probability. Therefore, in the sequel, the communication structure for LF is implied to be MST. The total transmission energy for LF is therefore

$$E_{LF} = k \sum_{e \in MST} w(e). \tag{8}$$

Furthermore, note that messages sent by nodes with a larger number of descendants carries more weight in determining the final normalized LLR. Hence we can apply LF adaptively by allowing different quantization levels at different nodes depending on their sizes–nodes with more descendants adopt finer quantization than nodes with fewer descendants. Simulation results suggest that further performance improvement can be realized by adaptive quantization.

III. Numerical Results

As a numerical example, let us consider detection of a deterministic signal in Gaussian noise with $H_0 : y = -m + z$; $H_1 : y = m + z$, where m is the signal mean, and z is a zero-mean Gaussian noise with variance σ^2. We consider the case where the Signal-to-Noise Ratio (SNR)$\triangleq \frac{m^2}{\sigma^2} = -10$dB,

TABLE II

OPTIMAL PERFORMANCE FOR DETECTING DETERMINISTIC SIGNAL IN GAUSSIAN NOISE, $n = 100$, SNR=-10dB

k	∞	1	2	3	4	5
B distance	0.05	0.0318	0.0441	0.0483	0.0495	0.0498
P_e	7.83e-4	6e-3	1.5e-3	1.0e-3	9e-4	8e-4

i.e., a weak signal is to be detected, and the network size $n = 100$. For simplicity we only consider pathloss and the transmission is assumed error free. The optimal LRQ's are obtained by Newton's method, and the corresponding optimal Bhattacharayya distances for quantization bits $1 \sim 5$ are given in Table II. Also presented in Table II are detection error probabilities for DT/MF/HF using the optimal LRQ (obtained through simulations). The $k = \infty$ column corresponds to the analytical error exponent and error probability for the centralized detection case. We observe that the performance of distributed detection schemes converges to the centralized performance quickly with k, and the performance improvement beyond $k = 3$ is only marginal.

Now, to assess the energy consumption of various schemes for the above detection problem, we assume that 100 nodes are uniformly and randomly distributed on a square of unit area, and the pathloss exponent $\kappa = 2.5$. For LF, two quantization options are considered: 1) Fixed quantization: all nodes use the same quantization level k. 2) Adaptive quantization: for a given k, $k-1, k, k+1$ bit quantization are respectively used by nodes with $n_j < 5$, $n_j < 20$, and $n_j \geq 20$. Fig. 1 illustrates the total power consumption v.s. simulated detection error probabilities for various distributed detection schemes. It can be seen that multi-hop transmission strategies greatly reduce the transmission energy compared to direct transmission. HF is more energy efficient than MF for an error probability no smaller than 9e-4, a value very close to the centralized performance. Although LF typically requires a finer quantization to achieve the same error probability, 2 to 3 folds reduction in transmission energy can be attained compared with MF and HF. Moreover, adaptive quantization offers some further improvement than fixed quantization.

IV. ENERGY SCALING LAWS

In this section, we explore scaling laws for transmission energy with the number of nodes n for the above schemes, which provide additional insights into their comparative performance. It is instructive

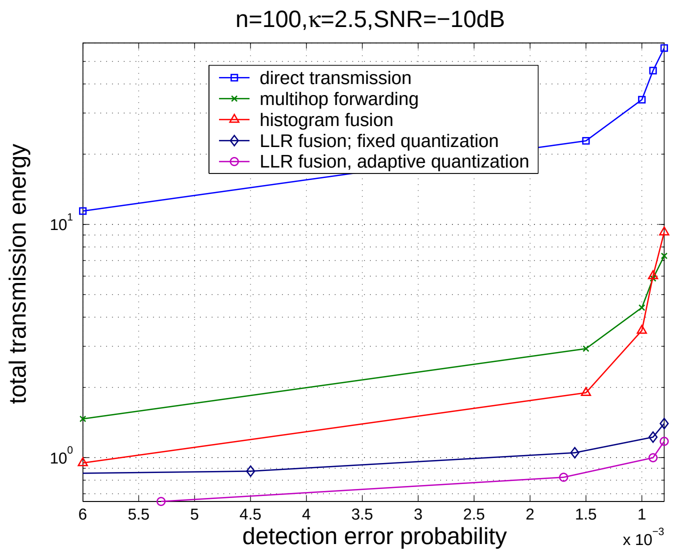

Fig. 1. Total transmission energy for direct transmission and various multihop fusion schemes, $n = 100$, $\kappa = 2.5$.

to consider a regular planar network, i.e., n sensor nodes and the fusion center (located at the center) form a grid on a square region of unit area with internode spacing $\frac{1}{\sqrt{n}-1}$. We assume without loss of generality that $\sqrt{n} = 2m + 1$. For direct transmission, it can be shown that

$$\begin{aligned} E_{DT}(n) &= 4k\left(\frac{1}{2m}\right)^{\kappa}\sum_{j=0}^{m}\sum_{i=1}^{m}\left(\sqrt{i^2+j^2}\right)^{\kappa} \\ &> 4k\left(\frac{1}{2m}\right)^{\kappa}\cdot\frac{m(m+1)}{2}\cdot\left(\frac{m}{2}\right)^{\kappa} > \left(\frac{1}{4}\right)^{\kappa}\frac{kn}{2}. \end{aligned}$$

On the other hand, we have $E_{DT}(n) < k\left(\frac{1}{2m}\right)^{\kappa}\cdot(n-1)\cdot\left(\sqrt{2}m\right)^{\kappa} < \left(\frac{1}{\sqrt{2}}\right)^{\kappa}kn$. Therefore, we get $E_{DT}(n) = \Theta(n)$. For the grid structure, the shortest path tree minimizing the Euclidean powered distances coincides with that minimizing the number of hops. Similarly as above, it can be shown that the energy consumption for the MF scheme is

$$E_{MF}(n) = 4k\left(\frac{1}{2m}\right)^{\kappa}\sum_{j=0}^{m}\sum_{i=1}^{m}(i+j) = \Theta\left(n^{\frac{3-\kappa}{2}}\right).$$

The minimum spanning tree connecting n nodes has $n-1$ edges with the weight of each edge being $\left(\frac{1}{\sqrt{n}}\right)^{\kappa}$. Thus the cost for LLR fusion is $\Theta\left(n^{1-\frac{\kappa}{2}}\right)$. For Histogram Fusion, assume $\log_2(n)$ is used at all nodes, thus the energy consumption scales like $\Theta\left(n^{1-\frac{\kappa}{2}}\log n\right)$. The above results provide interesting insights: LF exhibits favorable energy scaling behavior, and the energy consumption decreases with n as

TABLE III

SCALING LAWS OF THE TOTAL TRANSMISSION ENERGY ON A GRID WITH n NODES ON UNIT AREA

Direct Transmission	$\Theta(n)$
Multihop Forwarding	$\Theta\left(n^{\frac{3-\kappa}{2}}\right)$
Histogram Fusion	$\Theta\left(n^{1-\frac{\kappa}{2}}\log n\right)$
LLR Fusion	$\Theta\left(n^{1-\frac{\kappa}{2}}\right)$

long as $\kappa > 2$. The scaling laws of the transmission energy for various schemes under the grid deployment are summarized in Table III. For the typical value of $\kappa = 2.5$, the energy expenditure for DT, MF, HF and LF respectively scale like $\Theta(n)$, $\Theta\left(n^{\frac{1}{4}}\right)$, $\Theta\left(n^{-\frac{1}{4}}\log n\right)$ and $\Theta\left(n^{-\frac{1}{4}}\right)$.

V. CONCLUSION

Several options for energy-efficient distributed detection with multihop transmission and fusion at intermediate nodes are investigated. Among them, the LLR fusion scheme stands out in both scaling law and scenarios of practical interest.

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