

Iterative Space-Time Processing for Multiuser Detection in CDMA Systems

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Abstract — Space-time processing and multiuser detection are two promising techniques for combating multipath distortion and multiple-access interference in CDMA systems. Due to the computational burden that rises very quickly with increasing numbers of users and receiver antennas, iterative implementations of various space-time multiuser detection algorithms are considered here. It is shown that fully exploiting various types of diversity through space-time processing and multiuser detection brings substantial gain over single-receiver-antenna or single-user based methods, and that iterative implementation of linear and nonlinear multiuser detection schemes approaches optimum performance with reasonable complexity.

I. INTRODUCTION

The presence of both multiple access interference (MAI) and intersymbol interference (ISI) constitutes a major impediment to reliable CDMA communications in multipath channels. These are challenges as well as opportunities for receiver designers, because through multiuser detection and space-time processing, we can exploit the inherent code, spatial, temporal and spectral diversity of multipath CDMA channels and achieve substantial gain.

Advanced signal processing often improves system performance at the cost of computational complexity. It is well known that the optimal maximum likelihood (ML) multiuser detector has prohibitive computational requirements. A variety of linear and nonlinear multiuser detectors (MUD) have been proposed to ease this computational burden while maintaining satisfactory performance [12]. However, in asynchronous multipath CDMA channels with receiver antenna arrays and large data frame length, direct implementation of these suboptimal techniques still proves to be very complex. Techniques for efficient space-time multiuser detection fall largely into two categories. One is a batch iterative method, which assumes knowledge of all signals and channels and is suitable for processing at the base station. The other is a sample-by-sample adaptive method, which requires only knowledge of the desired user and is specifically suitable for mobile-end processing, and useful for base station processing as well due to the time varying nature of wireless communications. In this paper we will consider only batch iterative space-time multiuser detectors due to space limitations. Adaptive methods will be treated in a subsequent paper.

There has been considerable research in the space-time (ST) processing area e.g., [6], [9], most of which considers

¹This work was supported by the National Science Foundation under Grant No. CCR-99-80590.

single-user-based methods. Combined multiuser detection and array processing has also been addressed recently [7], [13]. In this paper we consider iterative implementation of linear and nonlinear space-time multiuser detectors (ST MUD) in multipath CDMA channels with receiver antenna arrays.

This paper is organized as follows. In Section II a space-time multiuser signal model is presented. Iterative implementation of linear ST MUD is discussed in Section III while that of nonlinear ST MUD is dealt with in Section IV. In Section V, a new receiver front end is introduced, based on which Expectation-maximization (EM)-based iterative ST MUD is discussed. Section VI contains simulation results and Section VII concludes the paper.

II. SPACE-TIME SIGNAL MODEL

Consider a DS/CDMA communication system with K users, employing normalized spreading waveforms s_1, s_2, \dots, s_K , and transmitting sequences $b_k(i)$, $k = 1, \dots, K$, $i = 0, \dots, M - 1$, of BPSK symbols at symbol rate $1/T$ with the amplitude A_k . M is the number of data symbols per user per frame. The transmitted baseband signal due to the k -th user is given by

$$x_k(t) = A_k \sum_{i=0}^{M-1} b_k(i) s_k(t - iT), \quad k = 1, \dots, K. \quad (1)$$

Suppose the transmitted signal of each user passes through a multipath channels before it is received by a uniform linear antenna array (ULA) of P elements. Then the single-input multiple-output (SIMO) vector impulse response between the k -th user and the base station can be modeled as

$$\underline{h}_k(t) = \sum_{l=1}^L \underline{a}_{kl} g_{kl} \delta(t - \tau_{kl}), \quad (2)$$

where L is the number of paths in each user's channel, g_{kl} and τ_{kl} are respectively the complex gain and delay of the l -th path of the k -th user, and $\underline{a}_{kl} = [a_{kl,1}, \dots, a_{kl,P}]^T$ is the array response vector corresponding the l -th path of the k -th user. The received signal at the antenna array is the superposition of the channel-distorted signals from the K users and additive spatially and temporally white Gaussian noise with power spectral density σ^2

$$\underline{r}(t) = \sum_{k=1}^K x_k(t) \otimes \underline{h}_k(t) + \sigma \underline{n}(t) \quad (3)$$

where \otimes denotes convolution.

A sufficient statistic for demodulating the multiuser symbols from the space-time signal (3) is given by [13]

$$\mathbf{y} = [y_1(0) \dots y_K(0) y_1(1) \dots y_1(M-1) \dots y_K(M-1)]^T, \quad (4)$$

where $y_k(i)$ denotes a space-time matched filter output defined as follows:

$$\sum_{l=1}^L \overbrace{g_{kl}^* \underline{\mathbf{a}}_{kl}^H \int_{-\infty}^{\infty} \underline{\mathbf{r}}(t) s_k(t - iT - \tau_{kl}) dt}_{z_{kl}(i)}, \quad 1 \leq k \leq K, \quad 0 \leq i \leq M. \quad (5)$$

To produce this sufficient statistic, the received signal vector $\underline{\mathbf{r}}(t)$ is first match-filtered for each path of each user, after which beams are formed, and all the paths of each user are combined with a RAKE receiver. Since the system is asynchronous, we need to collect the statistic for all users for the whole data frame. Next we will talk about various space-time MUD receivers based on this space-time matched filter output. In Section V, however, a new front-end structure will be introduced.

An optimal ML space-time multiuser detector will maximize the following log-likelihood function [12], [13]

$$\Omega(\mathbf{b}) = 2\Re\{\mathbf{b}^T \mathbf{A} \mathbf{y}\} - \mathbf{b}^T \mathbf{A} \mathbf{H} \mathbf{A} \mathbf{b}, \quad (6)$$

where $\mathbf{b} = [b_1(0) \dots b_K(0) b_1(1) \dots b_1(M-1) \dots b_K(M-1)]^T$, \mathbf{A} is the $KM \times KM$ diagonal matrix whose $k + iK$ diagonal element is equal to A_k . The multiuser signal and channel parameters (correlation coefficients of signature waveforms, multipath delay and amplitude, array response) come into play through the $KM \times KM$ block Toeplitz system matrix $\mathbf{H} \equiv$

$$\begin{pmatrix} \underline{\mathbf{H}}^{[0]} & \underline{\mathbf{H}}^{[1]} & \dots & \underline{\mathbf{H}}^{[\Delta]} & & & \\ \underline{\mathbf{H}}^{[-1]} & \underline{\mathbf{H}}^{[0]} & \underline{\mathbf{H}}^{[1]} & \dots & \underline{\mathbf{H}}^{[\Delta]} & & \\ & \underline{\mathbf{H}}^{[-\Delta]} & \dots & \underline{\mathbf{H}}^{[0]} & \dots & \underline{\mathbf{H}}^{[\Delta]} & \\ & & \underline{\mathbf{H}}^{[-\Delta]} & \dots & \underline{\mathbf{H}}^{[-1]} & \underline{\mathbf{H}}^{[0]} & \underline{\mathbf{H}}^{[1]} \\ & & & \underline{\mathbf{H}}^{[-\Delta]} & \dots & \underline{\mathbf{H}}^{[-1]} & \underline{\mathbf{H}}^{[0]} \end{pmatrix} \quad (7)$$

where Δ denotes the multipath delay spread. The reader is referred to [13] for the details of \mathbf{H} . Dynamic programming can be applied to compute the ML estimates and the computational complexity is on the order of $\mathcal{O}(2^{(\Delta+1)K})$.

III. ITERATIVE LINEAR ST MUD

In this section, we consider the application of iterative processing to the implementation of various linear space-time multiuser detectors.

The sufficient statistic (4) can be written as (see [13])

$$\mathbf{y} = \mathbf{H} \mathbf{A} \mathbf{b} + \sigma \mathbf{v} \quad (8)$$

where $\mathbf{v} \sim \mathcal{N}(0, \mathbf{H})$. Linear multiuser detectors are of the form

$$\hat{\mathbf{b}} = \text{sgn}(\Re\{\mathbf{W} \mathbf{y}\}), \quad (9)$$

where \mathbf{W} is a suitably chosen matrix. For the linear decorrelating detector, the linear transform is given by

$$\mathbf{W}_d = \mathbf{H}^{-1} \quad (10)$$

and for the linear MMSE detector, we have

$$\mathbf{W}_m = (\mathbf{H} + \sigma^2 \mathbf{A}^{-2})^{-1}. \quad (11)$$

Direct inversion of the matrices in (10) and (11), after exploiting the block Toeplitz structure, is of complexity $\mathcal{O}(K^2 M \Delta)$ per user per symbol.

The linear multiuser detection estimates of (10) and (11) can be seen as the solution of a linear equation

$$\mathbf{C} \mathbf{x} = \mathbf{y} \quad (12)$$

with $\mathbf{C} = \mathbf{H}$ for the decorrelating detector and $\mathbf{C} = \mathbf{H} + \sigma^2 \mathbf{A}^{-2}$ for the MMSE detector.

Jacobi and Gauss-Seidel iteration are two common iterative schemes for solving linear equations such as (12) [4]. If we decompose the matrix \mathbf{C} as

$$\mathbf{C} = \mathbf{C}_L + \mathbf{D} + \mathbf{C}_U, \quad (13)$$

where \mathbf{C}_L denotes the lower triangular part, \mathbf{D} denotes the diagonal part, and \mathbf{C}_U denotes the upper triangular part, then Jacobi iteration is represented by

$$\mathbf{x}_m = -\mathbf{D}^{-1}(\mathbf{C}_L + \mathbf{C}_U)\mathbf{x}_{m-1} + \mathbf{D}^{-1}\mathbf{y} \quad (14)$$

and Gauss-Seidel iteration is represented by

$$\mathbf{x}_m = -(\mathbf{D} + \mathbf{C}_L)^{-1} \mathbf{C}_U \mathbf{x}_{m-1} + (\mathbf{D} + \mathbf{C}_L)^{-1} \mathbf{y}. \quad (15)$$

Jacobi iteration can be seen to be a form of linear parallel interference cancellation, the convergence of which is not guaranteed in general. In contrast, Gauss-Seidel iteration (linear serial interference cancellation) converges to the solution of the linear equation for any initial value.

Another approach to solving the linear equation (12) involves gradient methods, among which are steepest descent and conjugate gradient iteration. Note that solving (12) is equivalent to minimizing the cost function

$$\Phi(\mathbf{x}) = \frac{1}{2} \mathbf{x}^H \mathbf{C} \mathbf{x} - \mathbf{x}^H \mathbf{y}, \quad (16)$$

and the idea of gradient methods is the successive minimization of the above function along a set of directions $\{\mathbf{p}_m\}$ via

$$\mathbf{x}_m = \mathbf{x}_{m-1} + \alpha_m \mathbf{p}_m \quad (17)$$

with

$$\alpha_m = \mathbf{p}_m^H \mathbf{q}_{m-1} / \mathbf{p}_m^H \mathbf{C} \mathbf{p}_m \quad (18)$$

and

$$\mathbf{q}_m = -\nabla \Phi(\mathbf{x})|_{\mathbf{x}=\mathbf{x}_m} = \mathbf{y} - \mathbf{C} \mathbf{x}_m. \quad (19)$$

If we choose the search directions \mathbf{p}_m to be the negative gradient of the cost function \mathbf{q}_{m-1} directly, this is the steepest descent method, global convergence of which is guaranteed. The convergence rate may be prohibitively slow, however, due to the linear dependence of the search directions, resulting in redundant minimization. If we choose the search direction to be \mathbf{C} -conjugate as follows

$$\mathbf{p}_m = \arg \min_{\mathbf{p} \in \Lambda_{m-1}^\perp} \|\mathbf{p} - \mathbf{q}_{m-1}\|, \quad (20)$$

where $\Lambda_m = \text{span}\{\mathbf{C} \mathbf{p}_1, \dots, \mathbf{C} \mathbf{p}_{m-1}\}$, then we have the conjugate gradient method, whose convergence is guaranteed and often is very quick.

The computational complexity of Gauss-Seidel and conjugate gradient iteration are similar, which is on the order of $\mathcal{O}(K \Delta \bar{m})$, where \bar{m} is the number of iterations.

IV. ITERATIVE NONLINEAR ST-MUD

Nonlinear multiuser detectors are often based on bootstrapping techniques, which also are iterative in nature. In this section, we will consider the implementation of decision-feedback and multistage interference cancellation multiuser detection [12] in the space-time domain.

A. Cholesky Iterative Decorrelating Decision-Feedback ST MUD

Decorrelating decision feedback multiuser detection (DDF MUD) exploits the Cholesky decomposition $\mathbf{H} = \mathbf{F}^H \mathbf{F}$, where \mathbf{F} is a lower triangular matrix, to determine the feedforward and feedback matrix

$$\hat{\mathbf{b}}_m = \text{sgn}(\mathbf{F}^{-H} \mathbf{y} - (\mathbf{F} - \text{diag}(\mathbf{F})) \mathbf{A} \hat{\mathbf{b}}_{m-1}). \quad (21)$$

Suppose the user of interest is user k , the purpose of the feedforward matrix \mathbf{F}^{-H} is to whiten the noise and decorrelate against the “future users” $\{s_{k+1}, \dots, s_K\}$; while the purpose of the feedback matrix $(\mathbf{F} - \text{diag}(\mathbf{F}))$ is to cancel out the interference from “previous users” $\{s_1, \dots, s_{k-1}\}$. The recursive Cholesky factorization of the block Toeplitz matrix \mathbf{H} is given in [14] for $\Delta = 1$ as

$$\mathbf{F} = \begin{bmatrix} \underline{F}_1(0) & 0 & 0 & 0 \\ \underline{F}_2(1) & \underline{F}_2(0) & \dots & 0 \\ 0 & & & \\ \vdots & & & \\ 0 & 0 & \underline{F}_M(1) & \underline{F}_M(0) \end{bmatrix}, \quad (22)$$

where the element matrices are obtained recursively as follows:

$$\underline{V}_M = \underline{H}^{[0]},$$

and, for $i = M, M-1, \dots, 1$,

$$\underline{V}_i = \underline{F}_i^H(0) \underline{F}_i(0) \quad \underline{F}_i(1) = (\underline{F}_i^H(0))^{-1} \underline{H}^{[-1]}$$

and

$$\underline{V}_{i-1} = \underline{H}^{[0]} - \underline{H}^{[1]} \underline{V}_i^{-1} \underline{H}^{[-1]}. \quad (23)$$

The extension to $\Delta > 1$ is straightforward and is omitted here.

The performance of DDF MUD is not uniform. While the first user is demodulated by its decorrelating detector, the last detected user will achieve its single-user lower bound providing the previous decisions are correct. Note that there is another form of Cholesky decomposition, of which the feedforward matrix \mathbf{F} is upper triangular. Then the multiuser detection would be in the reverse order, as are the performances. The idea of Cholesky iterative DDF MUD is to employ these two forms of Cholesky decomposition alternatively indicated as follows:

$$\begin{cases} \bar{y}_{1,i} = F_{1,ii} A_i b_i + \sum_{j=1}^{i-1} F_{1,ij} A_j b_j + \bar{n}_i \\ u_{1,i} = \bar{y}_{1,i} - \sum_{j=1}^{i-1} F_{1,ij} A_j \hat{b}_j \approx F_{1,ii} A_i b_i + \bar{n}_i \\ i = 1, \dots, KM, \end{cases} \quad (24)$$

and

$$\begin{cases} \bar{y}_{2,i} = F_{2,ii} A_i b_i + \sum_{j=i+1}^{KM} F_{2,ij} A_j b_j + \bar{n}_i \\ u_{2,i} = \bar{y}_{2,i} - \sum_{j=i+1}^{KM} F_{2,ij} A_j \hat{b}_j \approx F_{2,ii} A_i b_i + \bar{n}_i \\ i = KM, \dots, 1, \end{cases} \quad (25)$$

where \mathbf{F}_1 is lower triangular and \mathbf{F}_2 is upper triangular; then update the log-likelihood ratio

$$L_i = 4\Re(F_{1/2,ii}^* A_i u_{1/2,i}) / \sigma^2 \quad (26)$$

if it is more reliable than last stored value, and finally make soft decisions $\hat{b}_i = \tanh(L_i/2)$ if it is an intermediate iteration, or hard decisions $\hat{b}_i = \text{sgn}(L_i)$ at the last iteration.

B. Multistage Interference Cancellation ST MUD

Multistage interference cancellation (IC) is similar to Jacobi iteration except that hard decisions are made at the end of each stage instead.

$$\hat{\mathbf{b}}_m = \text{sgn}(\mathbf{y} - (\mathbf{H} - \text{diag}(\mathbf{H})) \mathbf{A} \hat{\mathbf{b}}_{m-1}). \quad (27)$$

The underlying rationale is that such estimator-subtractor structure exploits the discrete-alphabet property of the transmitted data streams. Although the optimal decisions are a fixed point of the nonlinear transformation (27), the problem with the multistage IC is the lack of convergence and oscillatory behavior. In the following subsection we consider some improvement over space-time multistage IC MUD.

V. EM-BASED ITERATIVE ST MUD

Before discussing EM-based space-time multiuser detection, we would like to introduce a new front-end receiver structure other than that of (5). From the above discussion, the multiuser signals and multipath channel parameters come into play through the system matrix \mathbf{H} . For a CDMA system employing long spreading codes (e.g. IS-95), \mathbf{H} must be calculated for each symbol interval, which is quite cumbersome. We introduce a new structure in Fig. 1 to circumvent this problem. By this structure, a sufficient statistic for detecting one chip is given by $\tilde{\mathbf{y}} = [\tilde{y}_{11}, \dots, \tilde{y}_{1L}, \tilde{y}_{21}, \dots, \tilde{y}_{KL}]^T$ with

$$\tilde{y}_{kl}(i) = \int_{iT_c + \tau_{kl}}^{(i+1)T_c + \tau_{kl}} \mathbf{r}(t) \psi(t - iT_c - \tau_{kl}) dt \quad (28)$$

where ψ is a normalized chip waveform of duration $T_c = T/N$ (N is the spreading gain), and assumed same for all users, τ_{kl} is the multipath delay of the l th path of the k user. Note that in this structure, signals received from different paths are treated as different users. Since the front end processing is on chip level, these “users” are synchronous. Moreover, with the chip matched filter output, the noise vector components can be assumed to be independent and identically distributed (i.i.d.) (cf. (8)).

The EM algorithm [2] provides an iterative solution of maximum likelihood estimation problem such as

$$\hat{\theta}(\mathbf{Y}) = \arg \max_{\theta \in \Lambda} \log f(\mathbf{Y}; \theta) \quad (29)$$

where θ is the parameter to be estimated, and $f(\cdot)$ is the probability density of the observable \mathbf{Y} . The key element of the EM algorithm is to add in some “missing data” \mathbf{Z} to aid the parameter estimation and iteratively maximize the following new objective function

$$Q(\theta; \bar{\theta}) = E\{\log f(\mathbf{Y}, \mathbf{Z}; \theta) | \mathbf{Y} = \mathbf{y}; \bar{\theta}\}. \quad (30)$$

To be specific, given an initial estimate, the algorithm alternates between the following two steps: 1) E-step, where the complete-data sufficient statistics $Q(\theta; \theta^i)$ is computed and 2) M-step, where the estimate is refined by $\theta^{i+1} = \arg \max_{\theta \in \Lambda} Q(\theta; \theta^i)$. It has been shown that the EM estimates monotonically increase in likelihood and converges to an ML solution under mild conditions. A problem with the EM algorithm is the tradeoff between the ease of implementation and convergence rate. The space-alternating generalized

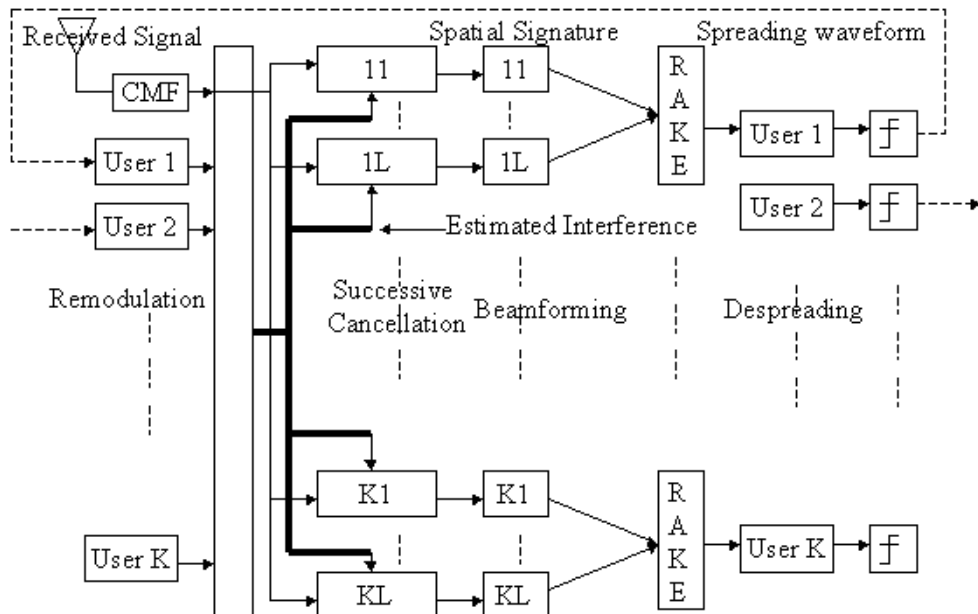


Figure 1: A New Space-Time Multiuser Receiver Structure

EM (SAGE) algorithm has been proposed [3] to improve the convergence rate for multidimensional parameter estimation. The SAGE algorithm updates the parameters sequentially (a subset for each iteration) rather than simultaneously as with EM. For each iteration, a subset of parameters S and the corresponding missing data \mathbf{Z}^S are chosen. Similar to the EM algorithm, in the E-step, a new objective function

$$Q^S(\theta_S; \bar{\theta}) = E\{\log f(\mathbf{Y}, \mathbf{Z}^S; \theta_S, \bar{\theta}_S) | \mathbf{Y} = \mathbf{y}; \bar{\theta}\} \quad (31)$$

is computed, and in the M-step, the chosen parameters are updated while the others remain unchanged

$$\begin{cases} \theta_S^{i+1} = \arg \max_{\theta_S \in \Lambda_S} Q^S(\theta_S; \theta^i) \\ \theta_{\bar{S}}^{i+1} = \theta_{\bar{S}}^i \end{cases} \quad (32)$$

The SAGE algorithm has been applied to temporal multiuser detection and the resulting receiver is similar to the multistage interference cancellation receiver, except that the bit estimates are made sequentially rather than in parallel [8].

With the new receiver structure of Fig. 1, we can apply the SAGE algorithm to space-time multiuser detection as follows. The parameters to be estimated are $\theta = \mathbf{d} \in \{\pm 1\}^{KL}$, which correspond to the chips from all paths of all users. The index sets cycle through $1, \dots, KL$ with L chips of all the paths of a user in a time. The algorithm is implemented without any missing data so there is no need for the E-step. Each iteration thus comprises the following steps:

Definition Step: $\mathbf{k} = [1, \dots, L] + (i \bmod K)L$

M-step:

$$\begin{cases} d_k^{i+1} = \text{sgn}(\tilde{y}_k - \sum_{m \neq k} \tilde{H}_{km} A_m d_m^i) & k \in \mathbf{k} \\ d_m^{i+1} = d_m^i & m \notin \mathbf{k} \end{cases} \quad (33)$$

VI. SIMULATION RESULTS

In this section the performance of the above described space-time multiuser detectors are examined through computer simulations. We assume a $K = 8$ -user CDMA system with spreading gain $N = 16$. The spreading codes are randomly generated. Each user travels through $L = 3$ paths before it reaches a uniform linear array with $P = 3$ elements and half-wavelength spacing. The complex gains and delays of the multipath and the direction of arrival are randomly generated and are kept fixed for all data symbols. We assume $A_1 = \dots = A_K$ for simplicity but the received signal powers are unequal due to the multipath effect.

First we compare the performance of various space-time multiuser receivers and some single user space-time receivers in Fig. 2. Five receivers are considered: the single-user matched filter, the single-user MMSE receiver, multiuser MMSE receiver implemented in Gauss-Seidel or conjugate gradient iteration method, the Cholesky iterative decorrelating decision-feedback multiuser receiver, and the multistage interference cancellation multiuser receiver. The reader is referred to [13] for the derivation of the single-user based receivers. The performance is evaluated after the iterative algorithms converge. Due to the bad convergence behavior of the multistage IC MUD, we test its performance after three stages. The single-user lower bound is also depicted for reference. We can see that the multiuser approach greatly outperforms the single-user based methods; nonlinear MUD offers further gain over the linear MUD; and the multistage IC seems to approach the optimal performance, when it has good convergence behavior. Note that due to the introduction of spatial (receiver antenna) and spectral diversity (RAKE combining), the SNR for the same BER is lower than that required by normal receivers without this processing.

Next we show the advantage of EM-based (SAGE) iterative method over the multistage IC method with regard to the

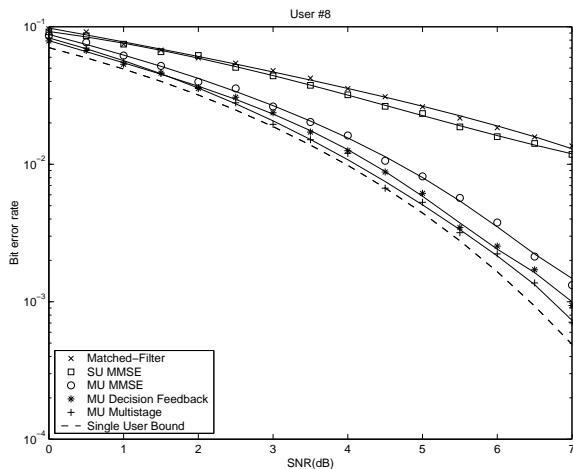


Figure 2: Comparison of Bit Error Rate (BER) versus Signal-to-Noise ratio (SNR) for Five Space-time Receivers.

convergence of the algorithms. The new receiver structure is used here. From Fig. 3 we find that while the multistage IC converges slowly and exhibits oscillatory behavior, the SAGE space-time multiuser receivers converges quickly and outperforms the multistage IC method.

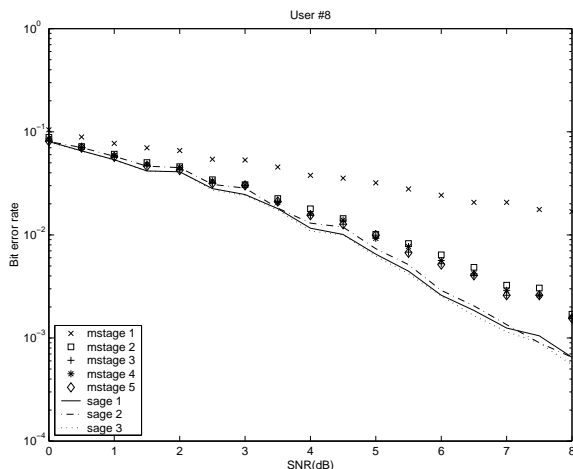


Figure 3: Comparison of convergence behavior for Multistage Interference Cancellation and SAGE Space-time Receivers.

VII. CONCLUSIONS

In this paper, we have considered various iterative space-time multiuser detection schemes in multipath CDMA channels with receiver antennas. It has been shown that fully exploiting diversity through space-time processing and multiuser detection offers substantial improvement over its counterpart, and that iterative implementation of various linear and nonlinear space-time multiuser receivers approaches the optimal performance with reasonable complexity, among which the SAGE space-time multiuser receiver outperforms the others. A new

receiver structure is also introduced for efficient implementation of iterative processing.

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