THROUGHPUT AND ENERGY EFFICIENCY OF SENSOR NETWORKS WITH MULTIUSER RECEIVERS AND SPATIAL DIVERSITY

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ABSTRACT

Linear multiuser detectors and receive antenna array enhance the received signal-to-noise ratio, thereby improving both the throughput and the energy efficiency of sensor networks. We assume Rayleigh flat-fading and no uplink channel state information, and derive analytically the throughput of large sensor networks with linear multiuser detector and spatial diversity, for both deterministic scheduling and slotted ALOHA multiple access. We introduce the notion of the effective energy, and show that it can be minimized by judiciously choosing the number of simultaneous transmissions and the transmission power.

1. INTRODUCTION

Throughput and energy efficiency are two important performance measures in the design of the physical and the MAC layer of sensor networks. Since sensors are extremely power-limited, it is desired that each successful transmission consumes the least energy possible. On the other hand, the network usually poses some minimum throughput requirement, which may arise from a mild delay constraint,or from the stability concerns to avoid the buffer overflow. The use of multiuser CDMA in the sensor network is therefore justified by the fact that spread-spectrum is inherently energy-efficient and that multiple packets can be successfully demodulated during one time slot. Moreover, it is well-known that antenna array at the receiver enhances the performance through the effect of resource pooling [1].

In this work, we focus on the uplink transmission, and assume that a large number of sensors transmit to a common receiver, which has high data rate connection to remote control centers, and has replenishable power supply; an example of such a network structure is the Sensor Networks with Mobile Agent (SENMA) [2]. We assume that the receiver is equipped with multiple antennas, and is capable of multiuser detection. We also assume that the sensors have no knowledge of the uplink channel state information, and transmit with the same power P_T. We consider two types of multiple access schemes, one is deterministic scheduling and the other is slotted ALOHA. Denote $C_k^{(L)}$ as the average number of successfully decoded packets given k transmissions and L receive antennas. $C_k^{(L)}$ is therefore the throughput of a network which deterministically schedules k nodes to transmit in each slot. If slotted ALOHA multiple access is employed, the maximum stable throughput without transmission control is shown to be $C_\infty^{(L)} = \lim_{k\to\infty} C_k^{(L)}$ [3], while the maximum stable throughput with optimal decentralized transmission control is a function of $C_k^{(L)}$, $k = 0, 1, 2, \cdots$ [4]. This holds for any random arrival distribution if delayed first transmission (DFT) is used, i.e., the new arrivals are transmitted with the same probability as the backlogged packets at the beginning of each time slot. Moreover, The energy efficiency of the network is also directly related $C_k^{(L)}$.

The main contribution of this work is the analytical expression of $C_k^{(L)}$ in the Rayleigh flat-fading scenario, which we derive in section 2 using recent results on large CDMA networks [1,5], as well as the metric of the effective energy, which we introduce and optimize with respect to the number of simultaneous transmissions k and the transmission power P_T in section 3.

Here are some further assumptions in our study. We assume that a packet can be successfully decoded if its SIR at the detector output is above a certain threshold β,which is determined by the modulation type and the error-correction coding scheme employed. Prior to transmission, the sensor node randomly chooses a spreading sequence of length N. Without loss of generality we assume similar distances from the sensor nodes to the common receicer, and the path loss is normalized to 1. The channel gains between each transmitter and the receiver are independent and Rayleigh distributed. If there are L receive antennas, the received power of each user is the sum of the power received on all L antennas, given by $P = P_T\gamma$, where the channel state γ is chi-square distributed:

$$f_\gamma(\gamma) = \frac{\gamma^{L-1}e^{-\gamma}}{(L-1)!}. \tag{1}$$

2. THROUGHPUT ANALYSIS

2.1. Deterministic Scheduling

It is proved in [1] that for linear multiuser detectors and microdiversity (the channel gain distribution for all receive antennas are identical), if $N, k \to \infty$ with $k/N = \alpha$ and L fixed, then SIR_1/P_1 converges in probability to a positive non-random value a. For finite-sized systems, the SIR_1/P_1 fluctuates around a, and the variance of such fluctuation diminishes as k and N increases. For reasonably large N, using the asymptotic limit in the analysis of finite-sized network can be justified [1, 5].

With $P_1 = P_T\gamma_1$ and denoting $x = \frac{1}{aP_T}$, we have $\mathrm{SIR}_1 = \frac{\gamma_1}{x}$. The successful reception of user 1 requires that $\mathrm{SIR}_1 = \frac{\gamma_1}{x_k^{(L)}} > \beta$, so the probability of success is

$$P[\gamma_1 > \beta x] = e^{-\beta x}\sum_{l=0}^{L-1}\frac{1}{l!}(\beta x)^l, \tag{2}$$

hence the average number of successes given k transmissions is

$$C_k^{(L)} = ke^{-\beta x}\sum_{l=0}^{L-1}\frac{1}{l!}(\beta x)^l. \tag{3}$$

Therefore the expressions of $C_k^{(L)}$ of the three types of linear detectors assume the same form, and the only difference lies in the expression of x, which we derive below.

For the matched filter receiver, a is given by [1]

$$a = \frac{1}{\sigma^2 + \frac{\alpha}{L}\mathbf{E}_P[P]}, \tag{4}$$

where $\mathbf{E}$ denotes expectation. Since $P = P_T\gamma$, and $\mathbf{E}_\gamma[\gamma] = L$, we have

$$x = \frac{\sigma^2}{P_T} + \alpha.$$

Therefore

$$C_{k,\mathrm{mf}}^{(L)} = ke^{-\beta(\frac{\sigma^2}{P_T}+\frac{k}{N})}\sum_{l=0}^{L-1}\frac{1}{l!}[\beta(\frac{\sigma^2}{P_T}+\frac{k}{N})]^l. \tag{5}$$

It can be seen that, for any fixed N and L, $C_{\infty,\mathrm{mf}}^{(L)} = 0$.

For the decorrelator detector, a is given by [5]

$$a = \begin{cases} \frac{1-\alpha}{\sigma^2} & \alpha < 1 \\ 0, & \alpha \geq 1, \end{cases} \tag{6}$$

so

$$x = \begin{cases} \frac{\sigma^2}{P_T(1-\alpha)}, & \alpha < 1 \\ +\infty, & \alpha \geq 1. \end{cases}$$

Therefore

$$C_{k,\mathrm{dec}}^{(L)} = \begin{cases} ke^{-\frac{\beta\sigma^2/P_T}{1-k/N}}\sum_{l=0}^{L-1}\frac{1}{l!}[\frac{\beta\sigma^2/P_T}{1-k/N}]^l, & k < N \\ 0, & k \geq N. \end{cases} \tag{7}$$

For the linear MMSE detector, a is the unique fixed point of the equation [5]

$$a = \frac{1}{\sigma^2 + \frac{\alpha}{L}\mathbf{E}_P[\frac{P}{1+Pa}]}. \tag{8}$$

Noting that $x = \frac{1}{aP_T}$ and $P = P_T\gamma$, we can write (8) as

$$x = \frac{\sigma^2}{P_T} + \frac{\alpha}{L}x\mathbf{E}_\gamma[\frac{\gamma}{x+\gamma}], \tag{9}$$

with

$$\begin{aligned} \mathbf{E}_\gamma[\frac{\gamma}{x+\gamma}] &= \int_0^\infty \frac{\gamma}{x+\gamma}\frac{\gamma^{L-1}e^{\gamma}}{(L-1)!}d\gamma \\ &= \frac{e^x}{(L-1)!}\int_1^\infty \frac{(t-1)^L x^L e^{-xt}}{t}dt \\ &= Le^x\int_1^\infty \frac{e^{-xt}}{t^{L+1}}dt = Le^xE_{L+1}(x), \end{aligned} \tag{10}$$

where $E_n(x)$ is the exponential integral function defined as $E_n(x) = \int_1^\infty e^{-xt}/t^n dt$, $x > 0$, and the result follows from repeating integration by parts.

Denoting the right-hand-side of (9) by $T_L(x)$, we have

$$T_L(x) = \frac{\sigma^2}{P_T} + \alpha xe^xE_{L+1}(x). \tag{11}$$

Proposition 3.1 The equation $x = T_L(x)$ has a unique fixed point x^* on the interval $(0, +\infty)$, and x^* is an attractive fixed point, i.e., $|T_L'(x)|_{x=x^*} < 1$.

Proof The following properties of the $E_n(x)$ function are useful, and we state them without proof:

$$(a) E_n'(x) = -E_{n-1}(x)$$

$$(b) xE_n(x) = e^{-x} - nE_{n+1}(x)$$

$$(c) \frac{1}{x+n} < e^xE_n(x) < \frac{1}{x+n-1}$$

$$(d) e^x[E_n(x) - E_{n+1}(x)] > \frac{1}{(x+n-1)(x+n)}$$

$$(e) \lim_{x\to\infty} xe^xE_n(x) = 1$$

Using (a) (b) and (c), we have

$$T_L'(x) = \alpha[(x+L+1)e^xE_{L+1}(x) - 1] > 0. \tag{12}$$

Using (a) (c) and (d), we have

$$\begin{aligned} T_L''(x) &= \alpha\{e^xE_{L+1}(x) - (x+L+1)e^x[E_L(x) - E_{L+1}(x)]\} \\ &< \alpha[\frac{1}{x+L} - \frac{x+L+1}{(x+L-1)(x+L)}] < 0. \end{aligned} \tag{13}$$

Therefore $T_L(x)$ is a strictly increasing, concave function on the interval $(0, +\infty)$. Note that

$$\lim_{x\to 0} T_L(x) = \frac{\sigma^2}{P_T} > 0, \tag{14}$$

and (e) implies that

$$\lim_{x\to\infty} T_L(x) = \frac{\sigma^2}{P_T} + \alpha. \tag{15}$$

From (12)-(15) it is clear that $y(x) = T_L(x)$ and $y_1(x) = x$ must have a unique intersection, and the slope at the intersection must be less than 1.

The fact that the fixed point is unique and attractive enables us to solve x^* with fixed-point iteration [6]: Start with an arbitrary positive x_0, and successively compute $x_1 = T_L(x_0)$, $x_2 = T_L(x_1), \cdots$, and the sequence (x_n) converges to x^*. When N is fixed, x^* is a function of k and L. We henceforth denote the fixed point of $T_L(x) = x$ corresponding to k transmissions and L receive antennas by $x_k^{(L)*}$. Since $x_k^{(L)*}$ is usually close to $x_{k-1}^{(L)*}$, when solving $x_k^{(L)*}$ it is computationally efficient to choose $x_{k-1}^{(L)*}$ as the initial value of iteration. Finally we have

$$C_{k,\mathrm{mmse}}^{(L)} = k e^{-\beta x_k^{(L)*}} \sum_{l=0}^{L-1} \frac{1}{l!}(\beta x_k^{(L)*})^l. \tag{16}$$

Using (c), we have $x_k^{(L)*} > \frac{\sigma^2}{P_T} + \frac{k}{N}\frac{x_k^{(L)*}}{x_k^{(L)*}+L+1}$, from which we can obtain $x_k^{(L)*} > \frac{\sigma^2}{P_T} + \frac{k}{N} - L - 1$. Recall that $x_k^{(L)*} < \frac{\sigma^2}{P_T} + \frac{k}{N}$. Thus we have

$$C_{k,\mathrm{mmse}}^{(L)} < k e^{-\beta(\frac{\sigma^2}{P_T}+\frac{k}{N}-L-1)} \sum_{l=1}^{L-1} \frac{1}{l!}[\beta(\frac{\sigma^2}{P_T}+\frac{k}{N})]^l.$$

The right-hand-side of the above inequality goes to zero as k approaches infinity, so $C_{\infty,\mathrm{mmse}}^{(L)} = 0$.

Fig. 1. compares the $C_k^{(L)}$'s of the three types of detectors, where $N = 16$, $\beta = 3\mathrm{dB}$ and $P_T/\sigma^2 = 3\mathrm{dB}$. The figure clearly demonstrates the advantage of the linear MMSE detector over the traditional matched filter, as well as the tremendous throughput increase that is possible by adding receive antennas. Note that for the decorrelator, the multiple antennas only enhance the received power, but does not improve the multiuser interference suppression ability, and $C_k^{(L)}$ is always zero when k is greater than N. However, for both the linear MMSE and the match filter, the multiple antenna provides not only power gain, but also multiuser diversity gain, and the system behaves like a single antenna system with spreading gain LN.

2.2. Slotted ALOHA

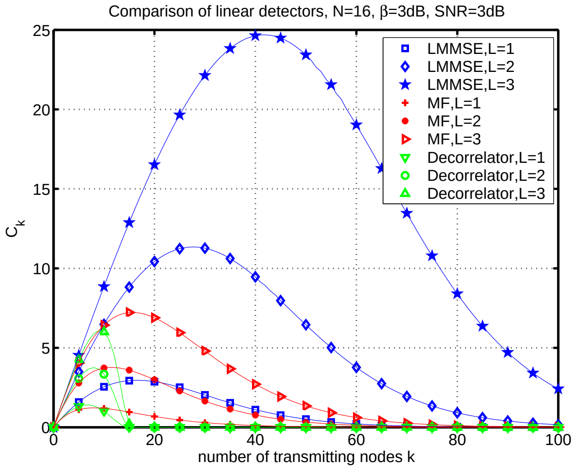

Fig. 1. Throughput comparison of linear multiuser detectors, $N = 16$, $\beta = 3\mathrm{dB}$, $P_T/\sigma^2 = 3\mathrm{dB}$

It is proved in [3] that, the maximum stable throughput of slotted ALOHA multiple access without transmission control for an infinite-user system is $C_\infty^{(L)}$, which can be taken as a good approximation of the maximum stable throughput of networks with a large number of transmitters. In the above we have shown that, $C_\infty^{(L)} = 0$ for all the three types of linear detectors. Therefore decentralized transmission control that uses the knowledge of the channel backlog is necessary to stabilize slotted ALOHA [4]. Denote the transmission probability by p, and the channel backlog by n, the throughput is given by

$$\lambda_n = \sum_{k=1}^{n} \binom{n}{k} p^k (1-p)^{n-k} C_k^{(L)}. \tag{17}$$

The maximum asymptotic stable throughput achievable by a decentralized control algorithm is [4]

$$\lambda_\infty = \lim_{n\to\infty} \lambda_n = \sup_{x>0} e^{-x} \sum_{k=1}^{\infty} C_k^{(L)} \frac{x^k}{k!}. \tag{18}$$

Moreover, the constant $x = A$ at which the supremum is attained is the optimum number of transmissions per slot, which is typically close to the value of k at which $C_k^{(L)}$ reaches maximum. The optimal transmission probability is $p = \min(A/n, 1)$.

3. ENERGY EFFICIENCY ANALYSIS

The energy efficiency of the SENMA model has been previously studied in [7], where efficiency, defined as the average number of successes over the total number of transmissions, is studied. However, a high efficiency does not necessarily mean low energy expenditure, as we can always make the efficiency close to 1 if the transmission power is high enough. Therefore we propose the metric of effective energy, defined as the average transmission energy for each successful transmission, i.e.,

$$E_e = \frac{kP_T T}{C_k^{(L)}}, \tag{19}$$

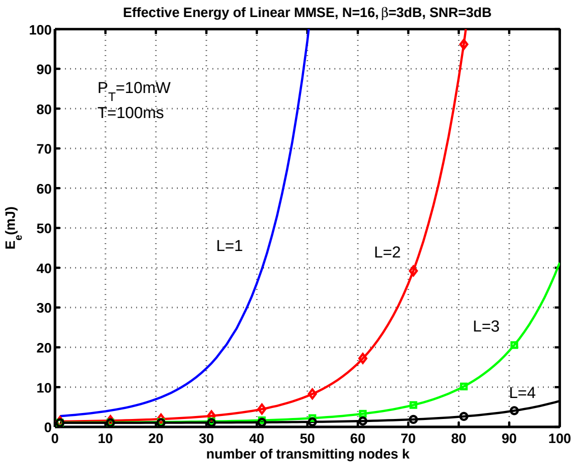

Fig. 2. Effective energy of the linear MMSE detector, $N = 16$, $\beta = 3\text{dB}$, $P_T/\sigma^2 = 3\text{dB}$

where T is the length of each time slot. The effective energy directly determines how many packets a sensor can successfully transmit during its lifetime.

3.1. The Optimal k

Since $C_k^{(L)}/k$ is simply the probability of success, which obviously decreases with the number of transmissions k, the effective energy E_e increases with k when P_T is fixed. Therefore when there is no throughput requirement, it is most energy-efficient to allow only one node to transmit in each slot. Sometimes the network poses a minimum throughput requirement Λ due to a delay constraint or stability concerns. Denote the maximum of $C_k^{(L)}$ across k by $C_{k\,\max}^{(L)}$. For a given P_T, if the desired throughput $\Lambda \leq C_{k\,\max}^{(L)}$, then the admissible values of k that satisfy the throughput requirement form an interval $[k_{\min}, k_{\max}]$, and the effective energy is minimized by $k = k_{\min}$; if $\Lambda > C_{k\,\max}^{(L)}$, then the throughput requirement can not be met with P_T.

Fig.2 shows the effective energy for the linear MMSE detector, where the values $P_T = 10\text{mW}$ and $T = 100\text{ms}$ are used. We observe that the effective energy is significantly reduced with more antennas. The increase of E_e with k is very slow when $L = 4$. This has the significance for sensor networks as a sizable throughput can be achieved while being energy efficient.

3.2. The Optimal $\mathbf{P_T}$

If we assume k is fixed, and the transmission power P_T is adjustable, it can be shown that the effective energy is a convex function of P_T, so there exists a value of $P_{T,\min}$ that minimizes E_e. For the matched filter and $L = 1$, we have $E_e = P_T T e^{\beta(\frac{\sigma^2}{P_T} + \frac{k}{N})}$. Differentiating with respect to P_T and setting it to zero, we obtain that E_e is minimized by $P_{T,\min} = \beta\sigma^2$, which means that the average received SNR is the same as the required SIR β. For a general L, and for the decorrelator and the linear MMSE detector, $P_{T,\min}$ can be obtained numerically. Note that when there is a minimum throughput requirement, P_T has to be above a threshold P_{th} such that $C_{k\,\max}^{(L)} \geq \Lambda$. Thus the optimal P_T is given by $P_{T,\text{opt}} = \max(P_{T,\min}, P_{\text{th}})$.

4. CONCLUSION

We showed analytically that multiuser detection and receive antenna array significantly improve the throughput and energy efficiency of sensor networks with deterministic scheduling, which suggests that such improvement is also possible for the slotted ALOHA system if optimal decentralized transmission control is applied. We proposed the metric of effective energy, and showed that for a fixed transmission power, the effective energy is minimized by the smallest number of tranmissions that achieves the throughput requirement, while for a fixed number of transmissions, it can be minimized by a certain transmission power.

5. REFERENCES

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