Abstract—Receive antenna selection based on Statistical Channel Knowledge (SCK) is considered for a multiuser uplink system. For spatial multiplexing transmission, we derive the optimal SCK-based selection criteria that maximizes the ergodic capacity in the high signal to noise ratio regime. Two different receiver architectures are considered. The first receiver uses a MMSE front-end to suppress the interference, followed by ML decoding to optimally decode the desired user’s data streams, and the second receiver performs a joint ML detection of all the users’ data streams. In a sufficiently correlated channel, SCK-based selection gives gains close to exact channel knowledge-based selection. We compare the performance of antenna selection with that of full complexity (FC) receiver, that uses all the available antennas for demodulation. We show that, unlike in the single-user system, in a multi-access system, the capacity achieved by a MMSE receiver with antenna selection can be higher than that of a MMSE FC receiver.

I. INTRODUCTION

It is well established that spatial multiplexing substantially improves the system capacity of a communication system, as compared to a single antenna system [1]. However, this comes at the expense of increased hardware and signal processing complexity. Antenna selection, which reduces the hardware costs of a multiple input multiple output (MIMO) system, has gained lot of interest recently. Antenna selection adaptively selects a subset of available antennas at the receiver (transmitter) for down-conversion (up-conversion). This reduces the number of RF chains required by a MIMO system, while maintaining most of the advantages of using multiple antennas.

The antenna selection algorithms proposed in the literature optimize different performance metrics, like, information theoretic capacity [6], [7], [8], [17], probability of error [5], [12], [14], etc. The work on antenna selection can be further classified depending upon whether it uses the exact channel knowledge (ECK) [6], [7], [8], [14], or statistical channel knowledge (SCK) [11], [15], [16], [17]. In [6], [7], Gorokhov, et al., quantify the loss in the capacity due to antenna selection at the receiver when compared with a full complexity system (that uses all the available antennas). They also propose fast near-optimal practical algorithms to compute the best antenna subsets. Based on the ECK, [8] derives the criteria for selecting the optimal subset of transmit antennas to maximize the capacity. A low complexity, ECK-based antenna selection algorithm that minimizes the error probability for a spatial multiplexing system with linear zero-forcing (ZF) receiver is considered in [14]. For a correlated channel with antenna selection based on SCK, the selection criteria that minimizes the probability of error for different receiver architectures for a spatial multiplexing system is derived in [11], [15], [16], [17]. In particular, [11], [15] derives the selection criteria, based on the minimum probability of error, for a maximum likelihood (ML) receiver and a linear ZF receiver.

While most of the previous work on antenna selection has focussed on deriving optimal selection criteria, and on low complexity fast algorithms for a single-user system, little attention has been given to antenna selection for multiuser systems. In [3], [4] the optimal signalling strategy with antenna selection in presence of interference is obtained. For a multi-user system, the optimal selection criteria that maximizes the capacity has not been looked at, to the best of our knowledge.

Catreuex, et al. show in [18] that in an interference-limited environment, the capacity of a MIMO system is not significantly more than that achieved by a receiver employing smart antennas. However, a joint architecture, that cancels interference and detects the desired data stream, can significantly improve the performance of an interference-limited MIMO system [19]. In this work, we derive the optimal receiver antenna selection criteria for a joint design in multiuser uplink environment. We consider two receiver architectures:

1) The receiver (basestation) employs MMSE processing to suppresses the interference, followed by optimal ML decoding to estimate the desired user’s data stream. We refer to this architecture as MMSE-ML receiver.

2) The receiver performs a joint ML estimation to decode all the users’ datastreams. We refer to this as ML receiver.

For these two systems, we derive the criteria, based on SCK, for choosing the selection matrix, such that the post-detection expected mutual information is maximized.

The rest of the paper is organized as follows. Section II introduces the system and the channel model. In Section III and IV, we derive the optimal selection criteria for MMSE-ML receiver and ML receiver, respectively. We compare the performance of antenna selection and full complexity receiver in Section V. Simulation results are presented in Section VI and the conclusions follow in Section VII.
II. SYSTEM AND CHANNEL MODEL

Consider a multi-access MIMO system, in which each of the $K$ users has $N_t$ antennas, and transmits to a common basestation having $N_r$ receive antennas. The received vector, $y$, can be written as

$$y = \sum_{k=1}^{K} \frac{\rho_k}{N_t} H_k x_k + n,$$

where $\rho_k$ is the signal to noise ratio (SNR) of user $k$, $H_k$ is the $N_r \times N_t$ channel matrix of the fading path gains from user $k$ to the receiver, $x_k$ is the data stream transmitted by user $k$ and $n$ is noise vector, with i. i. d. complex Gaussian entries $\sim \mathcal{CN}(0,1)$. The receiver has perfect channel state information (CSI), while the transmitter has no CSI.

We use the widely adopted Kroncker correlation model for the channel, which assumes that the correlation at the receiver and the transmitter is independent of each other. We assume that the columns of $H$ are uncorrelated, i.e., there is no transmitter-side correlation. This is a reasonable assumption because the mobile station is typically located in a rich scattering environment. Then, according to this model, the channel, $H_k$ can be written as

$$H_k = R^L_{k,k} H_{w,k}$$

where the entries of $H_{w,k}$ are i. i. d. complex Gaussian $\sim \mathcal{CN}(0,1)$, and $R_k$ is the receiver correlation matrix for the $k^{th}$ user.

Assume that the receiver has only $L \leq N_r$ demodulator chains, so that the best $L$ out of $N_r$ available antennas need to be selected for down-conversion and subsequent baseband processing. Let $S$ be the selection matrix that selects $L$ rows from $H$. The reduced channel matrix $SH_k$ can be written as

$$SH_k = R^L_{k,k} H_{w,k}$$

where $R_{L,k}$ is a $L \times L$ sub-matrix of $R_k$ formed by removing $N_r - L$ rows and the corresponding columns from $R_k$, and $H_{L,k}$ is $L \times N_t$ matrix with i. i. d. complex Gaussian entries.

Note that since we are considering receiver selection for the uplink system, the same selection matrix is chosen for all the users.

III. ANTENNA SELECTION FOR MMSE-ML RECEIVER

We first consider a receiver architecture where an MMSE front end is used to suppress the interference from other users, followed by ML detection of desired user’s data stream. The ergodic capacity of a full complexity receiver (that uses all the available antennas) is given by

$$C_{\text{MMSE-FC}} = \mathbb{E} \left\{ \log \left| I_L + \frac{\rho_1}{N_t} H_1^H H_1 \right| \right\} \times \left| \mathbb{I} + \frac{1}{N_t} \left( \sum_{i=2}^{K} \frac{\rho_i}{N_t} H_i^H H_i \right)^{-1} \right|^{-1},$$

where user 1 is the user of interest. Here $\mathbb{E}\{\cdot\}$ denotes expectation, and $(.)^\dagger$ denotes the Hermitian of a matrix. When the number of demod chains is less than the number of receive antennas, the ergodic capacity after selection can be written as

$$C_{\text{MMSE-sel}} = \max_S \mathbb{E} \left\{ \log \left| I_L + \frac{\rho_1}{N_t} SH_1 H_1^H S \right| \right\} \times \left| \mathbb{I} + \frac{1}{N_t} \left( \sum_{i=2}^{K} \frac{\rho_i}{N_t} SH_i H_i^H S \right)^{-1} \right|^{-1}. \tag{4}$$

Without loss of generality, assume $K = 2$. Then the maximization problem in (4) becomes

$$C_{\text{MMSE-sel}} = \max_S \mathbb{E} \left\{ \log \left| I_N + \frac{\rho_1}{N_t} H_{L,1}^H R_{L,1}^{-\frac{1}{2}} Q^{-\frac{1}{2}} R_{L,1}^{\frac{1}{2}} H_{L,1} \right| \right\} \tag{5}$$

where $Q$ is defined as

$$Q = I_L + \frac{\rho_1}{N_t} R_{L,2}^{\frac{1}{2}} H_{L,2} H_{L,2}^H R_{L,2}^{-\frac{1}{2}}.$$

Our goal is to find the optimal channel statistics-based selection matrix, $S$, such that (4) is maximized, or equivalently, find the sub-matrices $R_{L,1}$ and $R_{L,2}$ such that (5) is maximized. In general, it is difficult to solve the above optimization problem involving ergodic capacity. Instead, we maximize the following lower bound, which can be derived following the steps in [20]. This bound is shown to be very tight in the high SNR regime [20].

Lemma 1: Let $H = R^L H_w$ be a given such that $R$ is a full rank matrix, and $N_r \leq N_t$. Then, the ergodic capacity of this channel can be lower bounded as

$$C \geq N_r \log \left( 1 + \frac{\rho_1}{N_t} \exp \left( \frac{1}{N_t} \mathbb{E} \left\{ \log \left| HH^\dagger \right| \right\} \right) \right). \tag{6}$$

We make the following assumptions to derive the optimal selection criteria.

Assumptions 1: Assume that $R_{L,1}$ is a full rank matrix and $L \leq N_t$, and that user 1 is operating in the high SNR regime.

Under assumptions 1, we can applying Lemma 1 on (5) to get

$$C_{\text{MMSE-sel}} \geq L \log \left( 1 + \frac{\rho_1}{N_t} \exp \left( \frac{1}{L} \right) \right) \times \mathbb{E} \left\{ \log \left| \frac{R_{L,1} H_{L,1} H_{L,1}^\dagger}{|Q|} \right| \right\} \tag{6}$$

$$= L \log \left( 1 + \frac{\rho_1}{N_t} |R_{L,1}|^{\frac{1}{2}} \beta \exp \left( -\frac{1}{L} \right) \right) \times \mathbb{E} \left\{ \log \left| I_L + \frac{\rho_2}{N_t} R_{L,2} H_{L,2} H_{L,2}^\dagger \right| \right\} \tag{7},$$

where $\beta$ is defined as $\beta = \exp \left( \mathbb{E} \left\{ \log |H_w H_w^\dagger| \right\} \right)$. The expression in (7) can be further lower bounded
Thus the following proposition holds.

Proposition 1: The ergodic capacity of a MMSE-ML receiver, given in (5), is maximized when

\[ \alpha = \frac{|R_{L,1}|}{|I_L + \rho_2 R_{L,2}|} \]

is maximized. Here \( R_{L,1} \) and \( R_{L,2} \) are the receive correlation matrices of user 1 and user 2, corresponding to the rows selected from \( H \).

IV. ANTENNA SELECTION FOR ML RECEIVER

For a full complexity ML receiver, the ergodic capacity is given by

\[ C_{\text{ML-FC}} = \mathbb{E} \left\{ \log |I_L + \frac{\rho_1}{N_t} H_1 H_1^\dagger + \frac{\rho_2}{N_t} H_2 H_2^\dagger| \right\} \]

(10)

When the receiver selects \( L \) out of \( N_r \) antennas, the ergodic capacity can be written as

\[ C_{\text{ML-sel}} = \max_{\mathbf{S}} \mathbb{E} \left\{ \log |I_L + \frac{\rho_1}{N_t} S H_1 H_1^\dagger S^\dagger + \frac{\rho_2}{N_t} S H_2 H_2^\dagger S^\dagger| \right\} \]

(11)

Next, we state a Lemma that will help us simplify (11).

Lemma 2: Consider channel matrices \( H_1 \) and \( H_2 \), such that \( H_1 = R_1 H_{L,1} \) and \( H_2 = R_2 H_{L,2} \). Then \( H_1 H_1^\dagger + H_2 H_2^\dagger \) can be written as

\[ H_1 H_1^\dagger + H_2 H_2^\dagger = R_{\text{eq}} H_w H_w^\dagger \]

where \( R_{\text{eq}} = R_1 + R_2 \) and \( H_w \) has i. d. complex Gaussian entries.

Proof:

Define \( HH^\dagger = H_1 H_1^\dagger + H_2 H_2^\dagger \). Since \( H_1 \) and \( H_2 \) have no transmit correlation, \( H \) can be written as \( H = H_{\text{eq}} H_w \). Then the receive correlation of \( H \) is

\[ R_{\text{eq}} = \mathbb{E}\left\{ HH^\dagger \right\} \]

\[ = \mathbb{E}\left\{ R_1^\dagger H_{L,1} H_{L,1}^\dagger R_1 + R_2^\dagger H_{L,2} H_{L,2}^\dagger R_2 \right\} \]

\[ = R_1 + R_2 \]

(12)

where (12) follows by observing that \( \mathbb{E}\left\{ H_{L,1} H_{L,1}^\dagger \right\} = I \). \( \blacksquare \)

To derive the optimal selection criteria, we make the following assumptions:

Assumptions 2: Assume that \( \left( \frac{\rho_1}{N_t} R_{L,1} + \frac{\rho_2}{N_t} R_{L,2} \right) \) is a full-rank matrix and \( L \leq N_t \), and that the system is operating in the high SNR regime.

Under these assumptions, (11) becomes

\[ C_{\text{ML-sel}} = \max_{\mathbf{S}} \mathbb{E} \left\{ \log |I_L + \frac{\rho_1}{N_t} S H_1 H_1^\dagger S^\dagger + \frac{\rho_2}{N_t} S H_2 H_2^\dagger S^\dagger| \right\} \]

(13)

where (13) follows from Lemma 2 and (14) follows by applying Lemma 1 on (13). Eqn. (14) is maximized when \( \gamma = \frac{\rho_1}{N_t} R_{L,1} + \frac{\rho_2}{N_t} R_{L,2} \) is maximized. Thus the following proposition holds.

Proposition 2: The ergodic capacity of a ML receiver, given in (13), is maximized when

\[ \gamma = \frac{\rho_1}{N_t} R_{L,1} + \frac{\rho_2}{N_t} R_{L,2} \]

is maximized. Here \( R_{L,1} \) and \( R_{L,2} \) are the receive correlation matrices of user 1 and user 2, corresponding to the rows selected from \( H \).

V. RELATION BETWEEN \( C_{\text{SEL}} \) AND \( C_{\text{FC}} \)

In this section, we show the impact of selection on the ergodic capacity of multi-access systems using ML receiver and MMSE-ML receiver. Let \( e_i(\mathbf{A} \mathbf{A}^\dagger) \) denote the \( i^{\text{th}} \) largest eigenvalue of \( \mathbf{A} \). Then the capacity of full complexity ML receiver can be written as

\[ C_{\text{ML-FC}} = \sum_{i=1}^{N_r} \log \left( 1 + e_1 \left( \frac{\rho_1}{N_t} H_1 H_1^\dagger + \frac{\rho_2}{N_t} H_2 H_2^\dagger \right) \right) \]

(15)

Using the property that the \( i^{\text{th}} \) largest eigenvalue of a matrix is greater than the \( i^{\text{th}} \) largest eigenvalue of its sub-matrix [21], we get the following lower bound on \( C_{\text{ML-FC}} \):

\[ C_{\text{ML-FC}} \geq \sum_{i=1}^{L} \log \left( 1 + e_1 \left( \mathbf{S} \left( \frac{\rho_1}{N_t} H_1 H_1^\dagger + \frac{\rho_2}{N_t} H_2 H_2^\dagger \right) \mathbf{S}^\dagger \right) \right) \]

(16)

Noting that the right-hand side of the inequality in (16) is \( C_{\text{ML-sel}} \), we conclude that \( C_{\text{ML-FC}} \) serves as an upper bound for \( C_{\text{ML-sel}} \), i.e. selection necessarily reduces the capacity of multi-user ML receiver.

For a MMSE-ML receiver, such an inference cannot be made. The full complexity MMSE receiver employs a linear front-end to mitigate interference. \( C_{\text{MMSE-FC}} \) serves as an upper bound to the capacity achieved by any linear front-end detector. However, selection is a non-linear operation, and by appropriately choosing the antenna subset, the net interference seen by the desired data stream can be reduced.
Therefore, in a multi-user system, the link capacity of MMSE-ML receiver can increase by processing the signal from a subset of antennas. This is in contrast to a single-user system, where selection at the receiver necessarily reduces the capacity.

VI. SIMULATION RESULTS

For the purpose of simulations, we assume a uniform linear array (ULA) with antenna spacing $d = 0.4\lambda$, where $\lambda$ is the wavelength. The angle of arrival (AoA) of user $k$ follows normal distribution $N(\theta_{rk}, \sigma_{rk}^2)$, where $\theta_{rk}$ and $\sigma_{rk}$ are the mean AoA and the RMS angle spread for user $k$. We use the characterization of $\mathbf{R}$ given in [22]. We compare the performance of various selection matrices for $N_t = 2$, $N_r = 3$ antennas and $L = 2$ demod chains. The simulation parameters are chosen as follows: $\theta_{r1} = 60^\circ$, $\theta_{r2} = 75^\circ$, $\sigma_{r1} = 20^\circ$, $\sigma_{r2} = 20^\circ$, $\rho_1 = 10$ dB and $\rho_2 = 10$ dB.

A. MMSE Detector

Table I compares the ergodic capacity for different subsets of antennas. Also shown are the values of $\alpha$ for these subsets. We see that the optimal antenna subset (1,3) gives a 0.4 bits/s/Hz selection gain. We define selection gain as the difference in the ergodic capacity between the best and the worst antenna subsets. Note that optimal SCK-based selection comes within 0.03 bits/s/Hz of the ECK-based selection.

From Table I, we also see that optimal antenna selection gives a 0.2 bits/s/Hz advantage over full complexity receiver, which uses the signals received at all the antennas. This conforms with the intuition given in Section V that $C_{\text{MMSE-ML}}$ can be greater than $C_{\text{MMSE-FC}}$.

We also plot the CDF of the capacity (Fig. 1), which provides a complete characterization of capacity, unlike ergodic capacity, which provides information on only the first moment.

B. ML Detector

Table II compares the ergodic capacity and $\gamma$ for different subsets of antennas for ML receiver. We see that we get significant selection gain of about 1 bits/s/Hz. Note that the capacity of FC receiver is greater than that of antenna selection. Figure 2 plots the CDF of capacity for ML receiver.

We see that the curves for optimal SCK-based selection and ECK-based selection are indistinguishably close.

VII. CONCLUSIONS

In this paper, we have derived the optimal receiver antenna selection criteria for a multiple access system. We considered two receiver architecture, which employ different multiuser detectors, and showed that the optimal channel statistics-based selection gives significant selection gains. For a sufficiently correlated channel, we showed that the statistics-based solution achieves capacity close to that of exact channel knowledge-based selection. When MMSE front end is employed to suppress multi-user interference, optimally selecting a subset of antennas at the receiver can increase the capacity of the multi-user system as compared to a full complexity receiver that uses all the available antennas. This is the main difference between the behavior of multi-user antenna selection and single-user antenna selection.

REFERENCES

Fig. 2. CDF of capacity for ML receiver


