Fast MIMO Transmit Antenna Selection Algorithms: A Geometric Approach

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***Abstract*— Motivated by matrix determinant properties, this letter develops a fast transmit antenna selection algorithm for MIMO systems: the G-circles method. This novel scheme is shown to achieve many advantages over other existing algorithms.**

***Index Terms*— MIMO, Antenna Selection, Fading Correlation**

I. Introduction

MULTIPLE-input multiple-output (MIMO) systems are anticipated to be widely employed to address the ever-increasing capacity demands for wireless communication systems. A major drawback of MIMO systems comes from the increased hardware cost due to multiple RF chains, motivating the investigation of MIMO antenna selection techniques [1], which already made its debut in the emerging high throughput wireless WLAN standard [2].

With channel capacity as the performance criterion, the optimal selection can be obtained through the exhaustive search over all possible antenna subsets [6], while some other algorithms are proposed with good trade-offs between performance and complexity [4][5].

In this letter, motivated by matrix determinant properties, and targeting on maximizing the channel capacity, a geometric approach realizing fast transmit antenna selection, the G-circles algorithm, is explored, based on either instantaneous channel state information (CSI) or channel correlation matrices. This novel algorithm achieves many advantages over existing schemes such as [4].

II. System model and Problem Formulations

Suppose there are N_T transmit and N_R receive antennas in a spatial multiplexing (SM) MIMO system. In a block fading channel model, we select L from N_T transmit antennas and connect them to the L available RF chains. No CSI is available at the transmitter, so the selection is implemented at the receiver, and the selected antenna indices are fed back while the transmitter equally allocates its power among L selected antennas. We denote $\mathbf{H}$ as the $N_R \times N_T$ complete channel matrix, and $\mathbf{H}_{SL}$ the selected $N_R \times L$ sub-matrix. The channel capacity after antenna selection can be expressed as:

$$C = \log_2 \left| \mathbf{I}_L + \frac{\rho}{L} \mathbf{H}_{SL}^H \mathbf{H}_{SL} \right|, \tag{1}$$

Manuscript received April 19 2006. This work was supported in part by the National Science Foundation under Grant CCF-0515164.

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where $\mathbf{I}_L$ is the $L \times L$ identity matrix, ρ is the average signal to noise ratio (SNR) at each receive antenna, and the operator ^{H} represents transpose conjugate. When the transmitter side is subject to fading correlation (e.g. urban outdoor downlink channel), the corresponding channel matrix can be modeled as [7]:

$$\mathbf{H} = \mathbf{H}_W \left(\mathbf{R}_T^{1/2} \right)^H, \tag{2}$$

where $\mathbf{R}_T = E\left[\mathbf{H}^H \mathbf{H}\right]$ is the $N_T \times N_T$ transmitter correlation matrix, and $\mathbf{H}_W$ is an $N_R \times N_T$ normalized white Gaussian matrix. Note that in (2), $\mathbf{H}_W$ models the fast Rayleigh fading, while $\mathbf{R}_T$ is mainly determined by the geometrical structure of the propagation channel and can be assumed to remain unchanged over a much longer time-scale. The fading correlations can be expressed as:

$$[\mathbf{R}_T]_{ik} = \beta \sum_{s=1}^{S} \exp\left[2\pi j (i-k) \frac{\Delta_T}{\lambda} \cos\theta_s \right], \tag{3}$$

where β is a real positive scalar, Δ_T is transmit antenna spacing, λ is the wavelength, S is the number of major far-field scatterers at the transmitter side, and θ_s represents the direction of departure (DOD) for the *s*-th far-field scatterer. Suppose that $S >> rank(\mathbf{H})$, and Δ_T is chosen to be large enough compared with λ, the key factor influencing the channel conditioning is the range of DOD, or the angle spread of transmit scatterers.

Maximizing (1) is equivalent to maximizing $\left| \frac{L}{\rho} \mathbf{I}_L + \mathbf{H}_{SL}^H \mathbf{H}_{SL} \right|$. We can then build a composite matrix:

$$\mathbf{G} = \begin{bmatrix} \mathbf{H} \\ \sqrt{\frac{L}{\rho}} \mathbf{I}_{N_T} \end{bmatrix} \tag{4}$$

and let $\mathbf{G}_{SL}$ be the corresponding sub-matrix after transmit antenna selection, whose first N_R rows form the $N_R \times L$ channel matrix $\mathbf{H}_{SL}$. By further defining

$$\mathbf{Z}_{SL} = \mathbf{G}_{SL}^H \mathbf{G}_{SL}, \tag{5}$$

optimal selection is actually the exhaustive search for L columns in $\mathbf{G}$ that maximize $|\mathbf{Z}_{SL}|$. Assuming that $L \leq rank(\mathbf{H})$, then $\mathbf{Z}_{SL}$ is a Hermitian positive definite square matrix, which contains real positive eigen-values [8]. Also $|\mathbf{Z}_{SL}|$ bears the following property:

$$|\mathbf{Z}_{SL}| = \prod_l \lambda_{\mathbf{Z},l}, \tag{6}$$

where $\{\lambda_{\mathbf{Z},l}\}$ are the eigen-values of $\mathbf{Z}_{SL}$.

III. The Gerschgorin Circles (G-circles) Based Algorithm

From (6), to maximize the capacity, L transmit antennas with the largest $\prod_l \lambda_{\mathbf{Z}\_l}$ are to be selected. Denoting $\{\lambda_{\mathbf{Z}\_l}\}$ as the eigen-values of the square matrix $\mathbf{T}^{(L)} = \mathbf{H}_{SL}^H \mathbf{H}_{SL}$, it is easy to prove that $\lambda_{\mathbf{Z}\_l} = \lambda_l + \frac{L}{\rho}$. Therefore, we shall select $\mathbf{H}_{SL}$ with L eigen-values as large as possible to maximize $\prod_l \lambda_{\mathbf{Z}\_l}$. As in [4][5], we develop a successive selection strategy, in which one antenna is selected at each step. Denote $\mathbf{T}^{(l)} = (\mathbf{H}^{(l)})^H \mathbf{H}^{(l)}$, where $\mathbf{H}^{(l)} = \left[\mathbf{h}_{(1)}, \mathbf{h}_{(2)}, \cdots, \mathbf{h}_{(l)}\right]$ with (l) the selected antenna index in step l, and $\mathbf{H}_{SL} = \mathbf{H}^{(L)}$. Let $\lambda_1^{(l)} \geq \lambda_2^{(l)} \geq \cdots \geq \lambda_l^{(l)}$ be the eigen-values of $\mathbf{T}^{(l)}$. We have [8]:

$$\lambda_1^{(l)} \geq \lambda_1^{(l-1)} \geq \lambda_2^{(l)} \geq \lambda_2^{(l-1)} \geq \cdots \geq \lambda_{l-1}^{(l-1)} \geq \lambda_l^{(l)}. \qquad (7)$$

Therefore $\lambda_{max}\left(\mathbf{H}_{SL}^H \mathbf{H}_{SL}\right)$ is upper bounded by $\lambda_{max}\left(\mathbf{H}^H \mathbf{H}\right)$, and a large $\lambda_{min}\left(\mathbf{H}_{SL}^H \mathbf{H}_{SL}\right)$ basically results in a small channel conditioning number: $\kappa = \sqrt{\lambda_{max}/\lambda_{min}}$, desirable for capacity maximization, especially at high SNR. To avoid investigating the eigen-values of all possible antenna subsets, we will focus on the approximation of $\lambda_{min}\left(\mathbf{H}_{SL}^H \mathbf{H}_{SL}\right)$. It can be seen from (7) that λ_{min} is decreased after each selection. Our strategy is: *in each step we select one column in* $\mathbf{H}$ *so that the decrease of* λ_{min} *can be approximately minimized*. The G-circles theorem gives us an approximation of the eigen-value distributions [8]:

Theorem III.1(G-circles): The L *eigen-values of* $\mathbf{T}^{(L)} = \mathbf{H}_{SL}^H \mathbf{H}_{SL}$ *are trapped in the circles centered at* $\left[T^{(L)}\right]_{ll}$ *with radii given by the following expression*:

$$\begin{aligned} \left|\lambda_l - \left[\mathbf{T}^{(L)}\right]_{ll}\right| &\leq r_l \\ &= \min\left(\sum_{k\neq l}\left|\left[\mathbf{T}^{(L)}\right]_{lk}\right|, \sum_{k\neq l}\left|\left[\mathbf{T}^{(L)}\right]_{kl}\right|\right). \end{aligned} \qquad (8)$$

Since $\mathbf{T}^{(L)}$ is Hermitian, we get $\sum_{k\neq l}\left|\left[\mathbf{T}^{(L)}\right]_{lk}\right| = \sum_{k\neq l}\left|\left[\mathbf{T}^{(L)}\right]_{kl}\right|$. From the definition of $\mathbf{T}^{(L)}$, we can further simplify (8) by:

$$\left|\lambda_l - \|\mathbf{h}_{(l)}\|^2\right| \leq r_l = \sum_{k\neq l}\left|\mathbf{h}_{(k)}^H \mathbf{h}_{(l)}\right|. \qquad (9)$$

Since eigen-values are all real positive numbers, they are actually distributed on the real axis within the range of G-circles. From (9), a large center of the *l*-th G-circle represents a large channel gain of transmit antenna (l), while a small radius means that antenna (l) has low correlations with all the other selected antennas. A lower bound of λ_{min} is found from (9):

$$\lambda_{min} \geq \min_{k,l=1\cdots L}\left(\|\mathbf{h}_{(l)}\|^2 - \sum_{k\neq l}\left|\mathbf{h}_{(k)}^H \mathbf{h}_{(l)}\right|\right) \qquad (10)$$

which is the left most point among all the L G-circles. To approximately minimize the decrease of λ_{min} in each step, we maximize the lower bound of λ_{min} in (10), motivating the following algorithm, where $\Gamma^{(l)}$ and $\Psi^{(l)}$ represent selected and remaining antenna sets at step l respectively:

Algorithm III.1 (G-circles):
Select $(1) = \arg\max_{k=1\cdots N_T} \|\mathbf{h}_k\|$,
$\Gamma^{(l)} = \{(1)\}$,
$\Psi^{(l)} = \{i\}_{i\neq(1)}$
For $l = 2 : L$
  For $i \in \Gamma^{(l-1)}$
    $\Gamma_{temp} = \{\Gamma^{(l-1)}, i\}$
    $LB_i = \min_{j,k\in\Psi_{temp}}\left(\|\mathbf{h}_{(k)}\|^2 - \sum_{j\neq k}\left|\mathbf{h}_{(j)}^H \mathbf{h}_{(k)}\right|\right)$
  End
  Select $(l) = \arg\min_i LB_i$
  $\Gamma^{(l)} \leftarrow \{\Gamma^{(l-1)}, (l)\}, \Psi^{(l)} \leftarrow \Psi^{(l-1)} - \{(l)\}$
End

Initially, we select the antenna with the largest channel gain. In the following steps, selecting one more antenna results in adding one more G-circle and the expansion of the radii of existing G-circles. From a geometric viewpoint, the maximization of (10) requires selecting one antenna with large norm and small fading correlations with all the other selected antennas. G-circles method is simple compared with existing algorithms, because only vector multiplication and scalar additions are involved. In particular, it is also simpler than correlation-based selection (CBS) in [4], which requires the calculations of the correlation between any two possible transmit antenna candidates.

On the other hand, in fast fading channels, instantaneous CSI-based transmit antenna selections are hard to implement, with the concerns of both channel variation and training overhead [2]. Therefore it is desirable to develop antenna selection algorithms based on slowly varying channel statistics. At high SNR, we can approximate (1) by the equation $\left|\frac{L}{\rho}\mathbf{I}_L + \mathbf{H}_{SL}^H \mathbf{H}_{SL}\right| \simeq \left|\mathbf{H}_{SL}^H \mathbf{H}_{SL}\right|$. If only fading correlation information is available for antenna selection, we need to maximize $\left|\mathbf{H}_{SL}^H \mathbf{H}_{SL}\right|$ based on the knowledge of $\mathbf{R}_T$. The selected channel is then $\mathbf{H}_{SL} = \mathbf{H}_W\left(\mathbf{R}_{T\_SL}^{1/2}\right)^H$, where $\mathbf{R}_{T\_SL}$ is a certain principle minor of $\mathbf{R}_T$ corresponding to the selected antennas. From matrix determinant property [8], we can get the following expression:

$$\begin{aligned} \left|\mathbf{H}_{SL}^H \mathbf{H}_{SL}\right| &= \left|\mathbf{R}_{T\_SL}^{1/2} \mathbf{H}_W^H \mathbf{H}_W \left(\mathbf{R}_{T\_SL}^{1/2}\right)^H\right| \\ &= \left|\left(\mathbf{R}_{T\_SL}^{1/2}\right)^H \mathbf{R}_{T\_SL}^{1/2} \mathbf{H}_W^H \mathbf{H}_W\right| \\ &= \left|\mathbf{R}_{T\_SL}\right| \left|\mathbf{H}_W^H \mathbf{H}_W\right| \end{aligned} \qquad (11)$$

When only $\mathbf{R}_T$ is available for antenna selection to maximize (11), it is natural to get the following fact, which is also formally proved in [3]:

Fact III.1: To maximize (11) *if only* $\boldsymbol{R}_T$ *is available,* L *transmit antennas are chosen such that* $|\boldsymbol{R}_{T\_SL}|$ *is maximized.*

The G-circles method is also applicable in this scenario.

From Fact III.1 and the property $|\mathbf{R}_{T\_SL}| = \prod_l \lambda_{\mathbf{R}_{T\_l}}$, one possible method for capacity maximization is to maximize $\lambda_{min}(\mathbf{R}_{T\_SL})$ by the exhaustive search over all possible $L \times L$ principle minors of $\mathbf{R}_T$. To avoid such exhaustive search, we further simplify it by the G-circles of $\mathbf{R}_{SL}$:

Algorithm III.2 (G-circles-$\mathbf{R}_T$):

$a = [\mathbf{R}_T]_{ii}$, (idnetical diagonals in $\mathbf{R}_T$);

Select $\mathbf{h}_{(1)}$ and $\mathbf{h}_{(2)}$ such that $\left|[\mathbf{R}_T]_{(1)(2)}\right|$ is the minimum;

Follow the same procedure as Algorithm III. 1, to select antennas $(3)\cdots(L)$, using $\left|[\mathbf{R}_T]_{(j)(k)}\right|$ instead of $\mathbf{h}_{(j)}^H\mathbf{h}_{(k)}$, and a instead of $\|\mathbf{h}_{(k)}\|^2$ in (10).

IV. Numerical Results

We simulate a MIMO system with $N_R = 3$, $N_T = 9$ and $L = 3$. In (3), we assume that $\Delta_T = 2\lambda$, $S = 10$, and $\theta_1 \cdots \theta_S$ are uniformly distributed in the range $(-\Delta\theta/2,\ \Delta\theta/2)$. Therefore, the channel conditioning number largely depends on $\Delta\theta$.

In the first simulation (Figure 1), capacities achieved by instantaneous CSI-based antenna selections are investigated. Two extreme conditions: $\Delta\theta = 180^o$, and $\Delta\theta = 15^o$, are simulated representing well- and ill- conditioned channels, respectively. We see that the G-circles method is near optimal for well-conditioned channels. For ill-conditioned channels, the G-circles method preserves the spatial multiplexing gain, and presents a reasonable performance loss (about 1dB). It also yields uniformly better performances over the CBS in [4], especially for well-conditioned channels.

In the second simulation, we investigate $\mathbf{R}_T$-based antenna selection algorithms. The transmit SNR is fixed to be 20dB and the capacity curves are drawn with respect to $\Delta\theta$. Also, we simulate the selection scheme that makes the exhaustive search for maximum $|\mathbf{R}_{T\_SL}|$ in (11), which in general represents the optimal $\mathbf{R}_T$-based antenna selection for capacity maximization (c.f. Fact III.1). From Figure 2, we see that both $\mathbf{R}_T$-based algorithms perform near optimal for ill-conditioned channels. The $\mathbf{R}_T$-based G-circles method even outperform its counterpart based on instantaneous CSI for channels with $\Delta\theta < 20^o$, since for ill-conditioned channels $|\mathbf{R}_{T\_SL}|$ dominates the eigen-value distributions in $\mathbf{H}_{SL}^H\mathbf{H}_{SL}$.

V. Conclusion

In this paper, motivated by matrix determinant properties, we develop fast MIMO transmit antenna selection algorithms based on geometric analysis. The simple G-circles method reduces the complexity significantly with reasonable performance loss, which is shown to be smaller than other algorithms with similar complexity. It can also be effectively deployed in correlation matrix-based antenna selections.

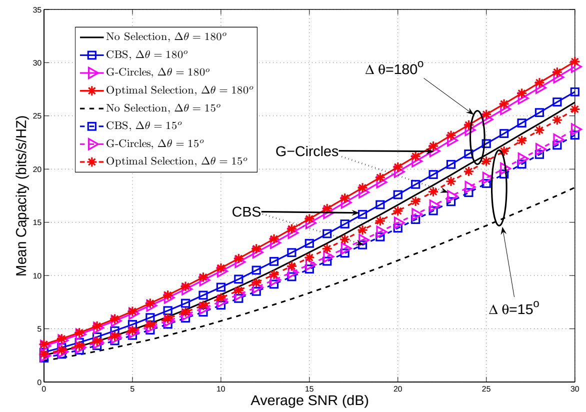

Fig. 1. Capacity performances for well- and ill-conditioned channels

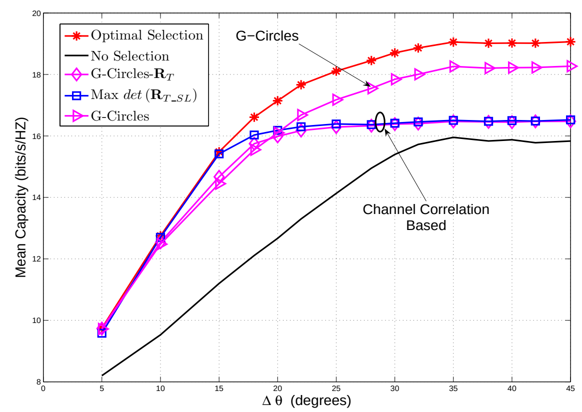

Fig. 2. Capacities of $\mathbf{R}_T$-based selections w.r.t. different $\Delta\theta$

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