Joint Tomlinson-Harashima Precoding and Scheduling for Multiuser MIMO with Imperfect Feedback

Quan Zhou, Huaiyu Dai, *Member, IEEE,* and Hongyuan Zhang
Department of Electrical and Computer Engineering,
NC State University
Raleigh, NC 27695

***Abstract*—In this paper, we propose a crosslayer approach that explores Tomlinson-Harashima Precoding (THP) at the physical layer to reduce the multiuser scheduling burden at the MAC layer, and improves the sum rate of the downlink multiuser MIMO system. Our proposed scheme is further evaluated with imperfect feedback, obtained by the long range prediction (LRP) technique. Compared to some existing scheduling schemes, the proposed scheme approaches the performance upper bound in certain scenarios, while incurring much less computation complexity. Significant gains are still maintained with imperfect channel state information (CSI), fed back at a rate much lower than the data rate.**

***Index Terms*—Long-Range Prediction, Multiuser MIMO, Tomlinson-Harashima Precoding.**

I. INTRODUCTION

Crosslayer study has become one of the burgeoning research fields in these few years, based on the realization that the commonly adopted layered protocol architecture, while facilitating the development within each layer, has hindered the optimization of the overall system, especially those utilizing wireless links [1]. At current stage, however, no formal systematic approaches have been built and applications are vastly different. In this paper, we focus on joint physical (PHY) and medium-access control (MAC) layer considerations, which are most relevant to harsh and unstructured wireless medium, and whose design has least interactions against other layers. Our target application is the downlink multiuser MIMO communications, which is envisioned to be of crucial importance to future wireless networks, and is believed to benefit significantly from a crosslayer design. Crosslayer study on multiuser MIMO systems has begun to attract attention only very recently, and very few of them ever explicitly address the details at the PHY layer [2]-[5].

In multiuser communication scenario, multiuser diversity can be exploited through making judicious selections among users with independently faded channels [6]. While choosing the best user is optimal for single antenna systems and is simple to implement, it is decisively suboptimal for multiuser MIMO systems [7] -[9]. To the best of our knowledge, no rules of the thumb are yet known for multiuser MIMO scheduling, especially when perfect instantaneous channel information is not available. In the literature, multiuser scheduling has been considered in the context of channel allocation for a SDMA/TDMA network (e.g., [10]-[12]), but mainly with the uplink and the assumptions that users are equipped with only one antenna or transmit only one data stream. In most of this work, "spatially compatible" users are grouped together in the same time or frequency slot, which is usually measured by channel correlation among users. This approach raises two potential concerns. First, a globally optimal allocation requires a thorough search of all possible choices, and suboptimal or heuristic alternatives induce complexity versus performance tradeoffs. Second, the physical layer details are largely neglected: either (1) the compatibility metric depends solely on the channel and is independent of the underlying transceiver structures; or (2) a conservative view is taken that treats multiuser interference as background noise. Such designs clearly fail to fully exploit the design opportunities at the physical layer. As an alternative, we propose to explore advanced yet feasible signal processing techniques at the physical layer in order to reduce the burden at the MAC layer and enhance overall network performance.

This paper is organized as follows. A joint PHY and MAC design for multiuser MIMO downlink is described in section II, where perfect feedback from the users is assumed. In section III, we exploit the long-range prediction (LRP) technique to effectively reduce feedback and quantify the system throughput loss under imperfect feedback. Simulation results are given and analyzed in section IV. Finally conclusions are presented in section V.

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II. THP AND MULTIUSER SCHEDULING FOR DOWNINK MIMO

Downlink multiuser MIMO forms a vector broadcast channel, whose capacity region is resolved only recently with the dirty paper coding (DPC) approach [7]. As the realization of DPC is rather involved [13][14], we exploit a more feasible PHY approach, Tomlinson-Harashima Precoding [15], which can be viewed as a suboptimal one-dimensional implementation of DPC. As a counterpart of the decision-feedback multiuser detection technique, THP has been employed in DSL systems and more recently in DS-CDMA and multiple-antenna systems to combat ISI and MAI [16] - [18]. In this work, we first propose a THP design for the multiuser MIMO downlink (especially, each user employs multiple antennas and receive multiple data streams), which does not seem to be completely addressed before. Furthermore, such a precoding structure results in interference-free parallel single-user MIMO channels, which greatly reduces the scheduling complexity while simultaneously improves overall system performance.

A. THP for multiuser MIMO

We consider a multiuser MIMO system with M antennas at the base station and N_i antennas at the *i*th user (each receive antenna is associated with a data stream), $1 \le i \le K$. A block diagram for the proposed THP scheme is given in Fig. 1, and briefly illustrated here. At the transmitter, we use $\mathbf{x}_i$ to denote an $N_i \times 1$ symbol vector to be transmitted by user i. The backward signal vectors $\mathbf{b}_i$'s from the feedback filter bank $\mathbf{B}$ with size $\sum_{i=1}^{K} N_i \times \sum_{i=1}^{K} N_i$ are added to the intended transmitted vectors $\mathbf{x}_i$'s to pre-eliminate the interference from previous users (users from 1 to $i-1$), and the resultant signals are fed to modulo-operators, which serve to limit the transmit power. The output signals of modulo-operators are then passed through a power control unit before being transformed by feedforward filters $\mathbf{W}_i$'s to further remove the interference from future users (users from $i+1$ to K). Finally, the signals are launched into the MIMO channels. As all interference is taken care of at the transmitter side, the receivers at the mobile users are left with some simple operations including power scaling (which is realized through automatic gain control (AGC) in Fig.1), reverse modulo-operation, and single user detection. The interested reader is referred to [15] for details.

In the following, we focus on the design of feedforward matrices $\{\mathbf{W}_i\}_{i=1}^{K}$ with size $M \times N_i$ and the feedback filter bank $\mathbf{B}$. We assume user channels $\{\mathbf{H}_i\}_{i=1}^{K}$ are quasi-static, whose information is perfectly known at the base station for the moment. The received signals at the receivers are then given as

$$\mathbf{y}_i = \mathbf{H}_i(\mathbf{W}_1\mathbf{s}_1 + \mathbf{W}_2\mathbf{s}_2 + \ldots + \mathbf{W}_K\mathbf{s}_K) + \mathbf{n}_i, \quad 1 \le i \le K, \qquad (1)$$

where $\mathbf{s}_i$ with size $N_i \times 1$ denotes the output signal vector of the modulo operator for user *i*, and $\mathbf{n}_i$ is the circularly symmetric complex Gaussian noise vector with covariance matrix $\sigma^2\mathbf{I}$. Define the overall system channel transfer function as

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1\mathbf{W}_1 & \mathbf{H}_1\mathbf{W}_2 & .. & \mathbf{H}_1\mathbf{W}_K \\ \mathbf{H}_2\mathbf{W}_1 & \mathbf{H}_2\mathbf{W}_2 & .. & \mathbf{H}_2\mathbf{W}_K \\ : & : & : & : \\ \mathbf{H}_K\mathbf{W}_1 & \mathbf{H}_K\mathbf{W}_2 & .. & \mathbf{H}_K\mathbf{W}_K \end{bmatrix}. \qquad (2)$$

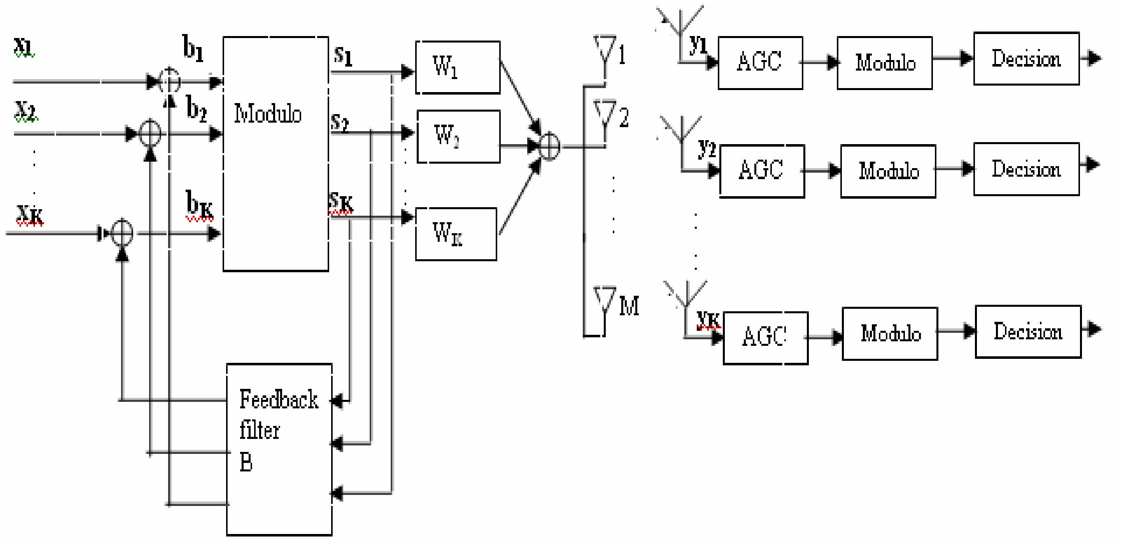

Figure 1. Block diagram of the proposed THP for multi-user MIMO downlink

The design target for feedforward matrices $\{\mathbf{W}_i\}_{i=1}^{K}$ is to make $\mathbf{H}$ a lower triangular one. This can be carried out in two steps. First, we enforce $\mathbf{H}$ to be block lower triangular and next we further enforce each block on the diagonal to be lower triangular. Let $\mathbf{\Gamma}_{i+1}$ denote the matrix of coefficients of orthonormal basis spanning $null(\mathbf{H}_1, \mathbf{H}_2, ..., \mathbf{H}_i)$. The matrix $\mathbf{H}$ would be block lower triangular if we let $\mathbf{W}_i = \mathbf{\Gamma}_i \mathbf{A}_i$, where $\mathbf{A}_i$ is yet to be determined. The matrix $\mathbf{\Gamma}_{i+1}$ can be determined through singular value decomposition (SVD) as follows.

Assume

$$[\mathbf{U}_i, \mathbf{\Sigma}_i, \mathbf{V}_i^*] = svd\left(\begin{bmatrix}\mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_i\end{bmatrix}\right), \quad 1 \le i \le K-1, \tag{3}$$

then the rightmost $M - \sum_{k=1}^{i} N_k$ columns of $\mathbf{V}_i$ compose the joint null space of $\mathbf{H}_1, \mathbf{H}_2, ..., \mathbf{H}_i$, denoted as $\mathbf{\Gamma}_{i+1}$. Now we further enforce $\mathbf{H}_i \mathbf{\Gamma}_i \mathbf{A}_i$ to be lower triangular, which is realized through QR decomposition $(\mathbf{H}_i \mathbf{\Gamma}_i)^H = \mathbf{Q}_i \mathbf{R}_i$, and let $\mathbf{A}_i = \mathbf{Q}_i(:, 1:N_i)$ (the first N_i columns of $\mathbf{Q}_i$)

Once the feedforward matrices $\{\mathbf{W}_i\}_{i=1}^{K}$ are determined as above, the design of the feedback filter bank $\mathbf{B}$ endeavors to eliminate the residual inter-stream and inter-user interference. Decompose $\mathbf{H} = \mathbf{GL}$, where $\mathbf{G}$ is a diagonal matrix that extracts the diagonal elements of $\mathbf{H}$ and $\mathbf{L}$ is a lower triangular matrix with unit diagonal elements, then

$$\mathbf{B} = \mathbf{I} - \mathbf{L}. \tag{4}$$

is what we desire.

B. *THP-Aided Scheduling for Multi-user MIMO downlink*

A direct consequence of the THP design is that, each user sees an interference-free MIMO channel, as evidenced from the following relationship (cf. Fig. 1)

$$\mathbf{y} = \mathbf{Hs} + \mathbf{n} = \mathbf{G}(\mathbf{Ls} + \mathbf{G}^{-1}\mathbf{n}) = \mathbf{G}\left[\mathbf{x} + (\mathbf{s} - \mathbf{b})\right] + \mathbf{n}, \tag{5}$$

where $\mathbf{y} = \left[\mathbf{y}_1^T \quad \mathbf{y}_2^T \quad ... \quad \mathbf{y}_K^T\right]^T$, $\mathbf{s} = \left[\mathbf{s}_1^T \quad \mathbf{s}_2^T \quad ... \quad \mathbf{s}_K^T\right]^T$ and $\mathbf{n} = \left[\mathbf{n}_1^T \quad \mathbf{n}_2^T \quad ... \quad \mathbf{n}_K^T\right]^T$ are given as in (1) ; $\mathbf{x} = \left[\mathbf{x}_1^T \quad \mathbf{x}_2^T \quad ... \quad \mathbf{x}_K^T\right]^T$ represents the transmitted data symbols with $\mathrm{tr}(E[\mathbf{x}_i \mathbf{x}_i^H]) \le P_i$ and $\sum_{i=1}^{K} P_i \le P$; and $\mathbf{b} = \left[\mathbf{b}_1^T \quad \mathbf{b}_2^T \quad ... \quad \mathbf{b}_K^T\right]^T$ collects the input vectors to the modulo-operators for all users At the receiver side, a power scaling with $\mathbf{G}_{ii}^{-1}$ ($1 \le i \le K$) followed by the modulo-operator and decision device suffices to recover $\mathbf{x}_i$. Due to the THP design, $\mathbf{H}_i \mathbf{W}_i$ is the equivalent single-user channel matrix for user *i*. In this paper, we follow the common practice in literature and adopt the information-theoretic spectral efficiency (assuming a Gaussian codebook and equal power allocation)

$$R_i = \log_2\left(\left|\mathbf{I} + \frac{P_i}{N_i \sigma^2}(\mathbf{H}_i \mathbf{W}_i)(\mathbf{H}_i \mathbf{W}_i)^H\right|\right), 1 \le i \le K, \tag{6}$$

as the metric[1] for the user channel quality, which can be readily modified to accommodate actual modulation and coding schemes.

Based on the above observations, a simple multiuser scheduling scheme is given below.

Step1. Schedule the first user whose index is $\arg\max_{1 \le i \le K} \log_2\left(\left|\mathbf{I} + \frac{P_i}{N_i \sigma^2}\mathbf{H}_i \mathbf{H}_i^H\right|\right)$.

Step2. Based on the selected user(s), compute the feedforward matrix $\mathbf{W}_i$, then select among the remaining users whose equivalent channel matrix $\mathbf{H}_i \mathbf{W}_i$ will result in the largest contribution (as given in (6)) in sum rate).[2]

Step3. Repeat Step 2 until a given number of users has been selected, or no more users can be added due to channel rank deficiency.

This THP-aided multiuser scheduling scheme will be compared with various sub-optimal and optimal approaches in Section IV to demonstrate its advantages.

III. ANALYSIS OF IMPERFECT CSI FEEDBACK

In previous discussion, perfect CSI is assumed available at the transmit side for PHY and MAC design. In practice, especially for FDD systems, such information is typically measured at the receiver side and fed back to the transmitter with some dedicated channels. The cost of perfect feedback in multiuser MIMO, if ever possible, grows quickly with the number of antennas, users, and system bandwidth, while in real systems the dedicated feedback channels are typically of low rate and prone to errors. In this section, we exploit the LRP technique to effectively reduce feedback and further quantify the system throughput loss under imperfect feedback.

A. *Channel Prediction via LRP*

LRP is a linear prediction method based on autoregressive modeling. With this technique, one can measure and feedback the time-varying CSI at a much lower rate than the data rate. Assume the complex fading process $h(t)$ is sampled at a rate $f_s = 1/T_s$, which is at least twice the maximum Doppler shift f_{dm} but can be much slower than the data rate. The sampled data is represented by $h_n = h(nT_s)$. Then the linear MMSE

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[1] Rigorously speaking, (6) is not the achievable rate of the THP, however, it is shown in [4] that the sum rate achievable using THP can converge to the actual sum rate capacity at high SNR.

[2] Note that existing designs need not be changed, as newly added users are invisible to already selected users.

prediction of the future CSI sample $\hat{h}_n$ based on p previously observed CSI samples[3] $h_{n-1}, h_{n-2}, ..., h_{n-p}$ is

$$\hat{h}_n = \sum_{j=1}^{p} d_j h_{n-j}, \tag{7}$$

where $\{d_j\}$ are the coefficients of the linear prediction filter and p is the prediction order. This approach can also be extended to predict $\tau > 1$ samples ahead. Clearly, multi-step prediction can tolerate more delay in the CSI feedback with some loss in performance. The predicted samples can be interpolated to forecast the fading samples at the same rate as data rate. The reader is referred to [19] for a detailed description of this technique.

B. Actual Achievable Rate under Imperfect CSI

With imperfect feedback, THP design will not be able to completely eliminate all the interference, which necessarily degrades the system performance. Assume the estimated CSI for the *i*th user is $\widehat{\mathbf{H}}_i$ ($1 \le i \le K$), and the designed feedforward matrix based on $\widehat{\mathbf{H}}_i$ ($1 \le i \le K$) is $\widehat{\mathbf{W}}_i$. In the precoding stage, the interference from the previous users for user *i* seen by the transmitter is $\widehat{\mathbf{H}}_i \widehat{\mathbf{W}}_j s_j$ ($1 \le j \le i-1$), while the actual interference is $\mathbf{H}_i \widehat{\mathbf{W}}_j s_j$. Therefore the residual interference from the previous users for the *i*th user is $(\mathbf{H}_i - \widehat{\mathbf{H}}_i)\widehat{\mathbf{W}}_j s_j$ $(1 \le j \le i-1)$. Similarly the interference from the future users is $\mathbf{H}_i \widehat{\mathbf{W}}_j s_j$ ($i+1 \le j \le K$). As for the self-interference due to the imperfect channel estimation, we can decompose $\mathbf{H}_i \widehat{\mathbf{W}}_i$ into three non-overlapping parts representing its lower triangular, diagonal, and upper triangular components as $\mathbf{H}_i \widehat{\mathbf{W}}_i = (\mathbf{H}_i \widehat{\mathbf{W}}_i)_l + (\mathbf{H}_i \widehat{\mathbf{W}}_i)_d + (\mathbf{H}_i \widehat{\mathbf{W}}_i)_u$. Then the interference from user i itself is $((\mathbf{H}_i - \widehat{\mathbf{H}}_i)\widehat{\mathbf{W}}_i)_l + (\mathbf{H}_i \widehat{\mathbf{W}}_i)_u$. In summary for the *i*th user, the actual received signal is given by

$$\begin{aligned}\mathbf{y}_i = {} & \overbrace{(\mathbf{H}_i \widehat{\mathbf{W}}_i)_d \mathbf{s}_i}^{\text{desired signal}} + \overbrace{\left(((\mathbf{H}_i - \widehat{\mathbf{H}}_i)\widehat{\mathbf{W}}_i)_l + (\mathbf{H}_i \widehat{\mathbf{W}}_i)_u\right)\mathbf{s}_i}^{\text{self-interference}} + \\ & \underbrace{\sum_{j=1}^{i-1} (\mathbf{H}_i - \widehat{\mathbf{H}}_i)\widehat{\mathbf{W}}_j \mathbf{s}_j}_{\text{interference from previous users}} + \\ & \underbrace{\sum_{j=i+1}^{K} \mathbf{H}_i \widehat{\mathbf{W}}_j \mathbf{s}_j}_{\text{interference from future users}} + \mathbf{n}_i, 1 \le i \le K.\end{aligned} \tag{8}$$

With $\mathbf{H}_{i,d} \equiv (\mathbf{H}_i \widehat{\mathbf{W}}_i)_d$, $\mathbf{H}_{i,s} \equiv ((\mathbf{H}_i - \widehat{\mathbf{H}}_i)\widehat{\mathbf{W}}_i)_l + (\mathbf{H}_i \widehat{\mathbf{W}}_i)_u$, $\mathbf{H}_{i,j<i} \equiv (\mathbf{H}_i - \widehat{\mathbf{H}}_i)\widehat{\mathbf{W}}_j$, and $\mathbf{H}_{i,j>i} = \mathbf{H}_i \widehat{\mathbf{W}}_j$, the achievable rate for the *k*th substream of user i (assuming Gaussian coding and equal power allocation) is given as

$$\widehat{R}_{i,k} = \log_2\left(1 + \frac{P_i}{N_i \sigma_I^2} \mathbf{H}_{i,d}(k,k) \mathbf{H}_{i,d}(k,k)^*\right). \tag{9}$$

with

$$\begin{aligned}\sigma_I^2 = {} & \sigma^2 + \frac{P_i}{N_i}\mathbf{H}_{i,s}(k,:)\mathbf{H}_{i,s}(k,:)^* + \sum_{j=1}^{i-1}\frac{P_j}{N_j}\mathbf{H}_{i,j<i}(k,:)\mathbf{H}_{i,j<i}(k,:)^* + \\ & \sum_{j=i+1}^{K}\frac{P_j}{N_j}\mathbf{H}_{i,j>i}(k,:)\mathbf{H}_{i,j>i}(k,:)^*.\end{aligned} \tag{10}$$

where (k,k) denotes the *k*th diagonal element of a matrix and $(k,:)$ represents the *k*th row of a matrix. The achievable rate for user i with imperfect feedback is then given by $\widehat{R}_i = \sum_{k=1}^{N_i} \widehat{R}_{i,k}$.

It is observed that the noise plus interference matrix $\sigma^2\mathbf{I} + \frac{P_i}{N_i}\mathbf{H}_{i,s}\mathbf{H}_{i,s}^H + \sum_{j=1}^{i-1}\frac{P_j}{N_j}\mathbf{H}_{i,j<i}\mathbf{H}_{i,j<i}^H + \sum_{j=i+1}^{K}\frac{P_j}{N_j}\mathbf{H}_{i,j>i}\mathbf{H}_{i,j>i}^H$ is typically a diagonally-dominant matrix (a matrix whose diagonal elements are much larger than the off-diagonal ones), therefore the actual achievable rate of user i can be approximated by

$$R_i \approx \log_2\left[\left|\mathbf{I} + \frac{P_i}{N_i}\mathbf{H}_{i,d}\mathbf{H}_{i,d}^*\begin{pmatrix}\sigma^2\mathbf{I} + \frac{P_i}{N_i}\mathbf{H}_{i,s}\mathbf{H}_{i,s}^* + \\ \sum_{j=1}^{i-1}\frac{P_j}{N_j}\mathbf{H}_{i,j<i}\mathbf{H}_{i,j<i}^* + \sum_{j=i+1}^{K}\frac{P_j}{N_j}\mathbf{H}_{i,j>i}\mathbf{H}_{i,j>i}^*\end{pmatrix}^{-1}\right|\right]. \tag{11}$$

IV. NUMERIC RESULTS

To illustrate the effectiveness of our proposed THP based multiuser MIMO downlink scheduling scheme, the total average throughput of our scheduling scheme is compared with that of several well-known schemes in literature, assuming that L users will be scheduled simultaneously and each selected user receive as many streams as its antennas. The HDR approach [6], i.e., serving only the best user at each time, is used as a reference. In contrast, a thorough search based on DPC readily serves as a performance upper bound. Very recently, some preliminary study on feasible multiuser MIMO scheduling schemes is presented in [5], which in some sense bears the same spirit as the "spatially compatible" scheduling mentioned before with a conservative view on the physical layer. In this work, the transmit antennas are partitioned among active users. Each user feedbacks the indices of the N_i antennas it desires together with those of the $\sum_{k \ne i} N_k$ antennas it wants the other users to be assigned to such that his instantaneous channel capacity is maximized. While this scheme is relatively easy to implement and has some throughput advantages over HDR (particularly for large K and moderate SNR), it suffers from two drawbacks as indicated above. One is computational complexity, since optimal

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[3] We assume the samples of the fading channels are perfectly obtained through trainings.

scheduling requires a search of $\binom{M}{N_i}\binom{M-N_i}{\sum\limits_{k\neq i} N_k}$ possibilities at *each user*. Second, as shown in Fig. 2, it is still interference limited at high SNR.

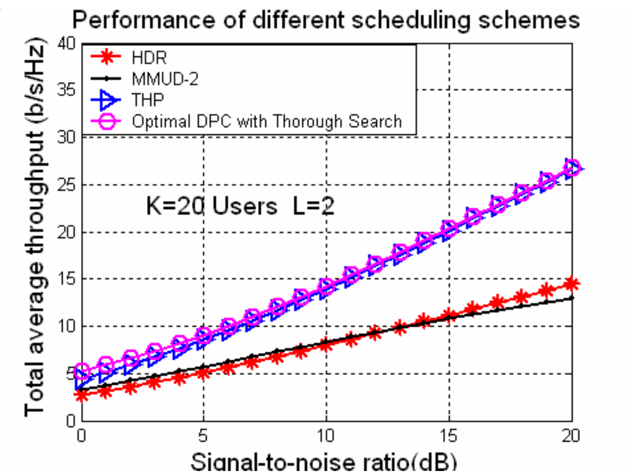

Figure 2. Performance comparison of different scheduling schemes

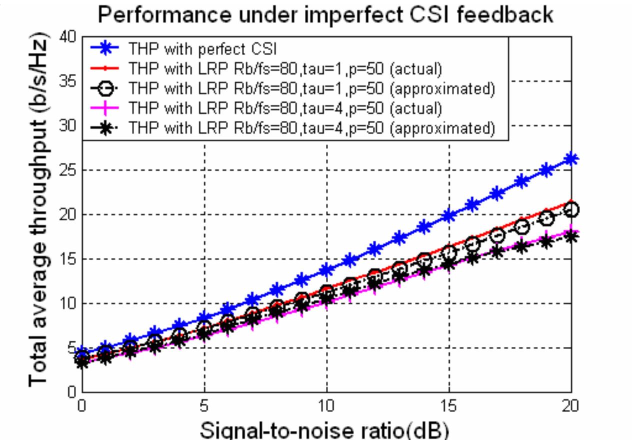

Figure 3. Performance under imperfect CSI feedback

In Fig. 2, we compare the system throughputs of different scheduling schemes as a function of total transmit SNR. We consider a symmetric multiuser MIMO downlink system with $M = 4$ antennas at the BS and $N_i = 2$ antennas at each of $K = 20$ mobiles, and schedule at most $L = 2$ users in each time instance. It can be seen that our joint THP and scheduling scheme performs much better than the HDR and the MMUD-L scheduling scheme in [5], and approaches the performance of dirty paper coding (a thorough search for *two* best users), while with much affordable computational complexity.

In Fig. 3, we re-evaluate our scheduling scheme with imperfect CSI feedback. A flat Rayleigh fading channel with Jakes' model with a Doppler frequency of 200 Hz (corresponding to a vehicular speed of 65 mi/h at a carrier frequency of 2 GHz) is considered. We use a channel sampling frequency of $f_s = 1600\text{Hz}$ and the prediction order in (7) is taken as $p = 50$, while the transmit data rate of each user is $R_b = 128\text{k}$ bps. So the channel is measured and fed back once every 80 symbols. We choose two different prediction steps for evaluation $\tau = 1$ and $\tau = 4$ (which corresponds to a prediction of 320 data symbols ahead).

From Fig. 3, we can see that even with imperfect CSI and slower feedback rate through LRP, our scheme still has a significant gain over the MMUD-L scheme and single best user scheduling scheme. Our simulation also indicates the good match between the true value and the approximate value of the total average rate under imperfect feedback.

V. CONCLUSION AND FUTURE WORK

In this paper, we proposed a joint THP and scheduling scheme for multi-user MIMO downlink. Compared to some existing scheduling schemes, the proposed scheme greatly reduces the scheduling complexity while simultaneously improves overall system performance. Strictly speaking, the proposed THP-aided multiuser scheduling is not necessarily globally optimal, but we conjecture that the loss in optimality is negligible (when the number of scheduled users is given) and plan to undertake a comprehensive analysis of this approach. Another note is that, there is an inherent limitation on the maximum number of users that can be simultaneously supported for our proposed scheme, due to the geometric structure revealed in II. A. Whether there exists an optimal number for the scheduled users, in terms of system throughput averaged over all channel realizations, remains open and constitutes our future work.

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