

# Joint Antenna Selection and Link Adaptation for MIMO Systems

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**Abstract**—Multi-input multi-output (MIMO) systems, with multiple antennas at both the transmitter and receiver, are anticipated to be widely employed in future wireless networks due to their predicted tremendous system capacity. In order to protect the transmitted data against random channel impairment, it is desirable to consider link adaptation, such as rate adaptation and power control to improve the system performance and guarantee certain quality of service. Based on the observation that link adaptation and antenna selection problems are often coupled, we propose a joint antenna subset selection and link adaptation study for MIMO systems. After the formulation of the multi-dimensional joint optimization problem, the main contribution of this paper lies in the design of efficient algorithms approaching the optimal solution for both uncorrelated and correlated MIMO channels. Specifically, we propose one simplified antenna selection and link adaptation rule based on the expected optimal number of active antennas for uncorrelated MIMO with Rayleigh fading, and one for correlated MIMO channels only based on the slowly varying channel correlation information. Our proposed algorithms are verified through numerical results, demonstrating significant gains over traditional MIMO signaling while feasible for practical implementation.

**Index Terms**— Antenna selection, QR-decomposition, Cholesky-decomposition, MIMO systems

## I. INTRODUCTION

THE use of multiple antennas at both the transmitter and receiver side, so as to form a multiple-input multiple-output (MIMO) antenna system, is an emerging technology that makes building high data rate wireless networks a reality [12][13]. Transmitting independent data streams simultaneously from different antennas through spatial multiplexing (see, e.g., [1]) effectively realizes the high spectral efficiency promised by MIMO systems, but leaves the transmitted data unprotected from random channel impairment. Therefore, it is often desirable to consider link adaptation, such as rate adaptation and power control to improve the system performance and guarantee certain quality of service [5][8][21][24].

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One of the drawbacks with an MIMO system is the increased complexity and hardware cost due to the expensive RF chains required by each active antenna. It is of increasing research interest recently to find a good antenna selection scheme that can significantly reduce such cost while incurring little performance loss. Generally, there are two goals for antenna subset selection in MIMO systems: one aims to maximize the channel capacity [19][25], the other aims to minimize the bit error rate for spatial multiplexing systems when some practical signaling schemes are used [6][7][14][16].

It is interesting to notice that link adaptation and antenna selection problems are actually coupled for MIMO systems, when practical signal processing techniques such as zero-forcing successive interference cancellation (ZF-SIC) (as used in V-BLAST) are employed at the receiver for data decoupling and detection. This is because the decoupled subchannel gains (post-detection signal-to-noise ratio (SNR)) are determined by the active antenna subset, while some weak subchannels are naturally dropped during link adaptation process. Motivated by this fact, we propose a joint antenna subset selection and link adaptation study for MIMO systems.

In a real propagation environment, the capacity of a MIMO system may be lower than what is predicted with rich scattering assumption due to fading correlation [10][26]. Meanwhile, link adaptation and antenna selection are expected to achieve more gains in correlated MIMO channels due to more prominent subchannel discrepancies. Furthermore, fading correlation information varies much more slowly, hence it is feasible and advantageous to implement antenna selection and link adaptation only based on the correlation information rather than on the instantaneous channel information. The author in [9] also proposed some simplified rules for joint antenna selection and link adaptation based on the channel correlation information, aiming to maximize some lower bounds of the minimum post SNR. Therefore the performance of these rules depends on how tight the lower bounds would be. Furthermore, the exhaustive search entailed there might make these rules still complex in implementation.

In this paper, we consider the problem of joint antenna selection and link adaptation for an uncoded spatial multiplexing system with a ZF-SIC receiver, for both uncorrelated and correlated MIMO channels. Our goal is to minimize the bit error rate given a throughput and power constraint. We allow all the available resources, including the number of active transmit antennas, symbol constellation size and transmit power dynamically adapted to the channel

conditions. The main contributions of this paper are summarized below:

1). Both incremental and decremental antenna selection rules with link adaptation are proposed for uncorrelated MIMO systems. Both rules are realized with recursive algorithms, thus greatly reducing the computational complexities and feasible for practical implementation. Rigorously speaking, neither rule provides the optimum solution, but the performance loss is negligible

2). For uncorrelated MIMO channels with independent and identically distributed (i.i.d) Rayleigh fading, we propose an antenna selection rule based on the expectation of the optimal number of active antennas. Based on this rule, the computational complexities can be further reduced while little performance degradation would be incurred. Such computation reduction is especially prominent for large MIMO systems.

3). For correlated MIMO channels, we propose an incremental antenna selection rule with link adaptation based on the slowly varying channel covariance information, which is also implemented in a recursive way to avoid the computational complexity of exhaustive search.

This paper is organized as follows. In Section II, we introduce the MIMO system model with transmit antenna selection, and formulate the problem of joint antenna subset selection and link adaptation. In Section III, we develop incremental and decremental antenna selection rules with link adaptation for uncorrelated MIMO channels. We also propose a simplified rule based on the expected optimal number of active antennas to further reduce complexity. In Section IV, we develop an antenna selection rule with link adaptation for correlated MIMO channels only based on the slowly varying channel correlation information. Simulation results are given and analyzed in Section V. Finally, in Section VI, we make conclusions and propose some future work.

## II. PROBLEM FORMULATION

### A. MIMO System with Transmit Antenna Selection

Without loss of generality, we assume a narrowband MIMO system with total  $K_t$  transmit and  $N_r$  receive antennas, with the channel between  $K_t$  transmit and  $N_r$  receive antennas denoted by  $\mathbf{H}$ . In our study, the antenna selection is only carried out at the transmitter side, and it is easily shown that the best performance is achieved when all receive antennas are active [22]. With  $N_t$  out of  $K_t$  transmit antennas to be chosen, we denote the selected subset of transmit antennas by  $p$  and the channel matrix between the selected  $N_t$  transmit antennas and  $N_r$  receive antennas by  $\mathbf{H}(p)$ , whose columns correspond to the selected antennas. The received signals are then given by

$$\mathbf{y} = \mathbf{H}(p)\mathbf{x} + \mathbf{n}, \quad (1)$$

where  $\mathbf{x} = (x_1, x_2, \dots, x_{N_t})^T$  is the transmitted signal vector,

$\mathbf{y} = (y_1, y_2, \dots, y_{N_r})^T$  is the received signal vector, and

$\mathbf{n} = (n_1, n_2, \dots, n_{N_r})^T$  is assumed to be i.i.d Gaussian with zero mean and variance of  $\sigma_n^2$ . For ease of description, we will drop the index  $p$  in (1) in the following discussion when no ambiguity incurs. All through this paper we assume  $N_r \geq N_t$ .

### B. ZF-SIC with QR Decomposition Interpretation

The zero-forcing successive interference cancellation, widely used in MIMO detection, can be simply interpreted by matrix QR decomposition. With  $\mathbf{H} = \mathbf{Q}\mathbf{R}$ , where  $\mathbf{Q}$  is a unitary matrix and  $\mathbf{R}$  is an upper triangular matrix, we can apply  $\mathbf{Q}^H$  to the received vector to obtain  $\tilde{\mathbf{y}} = \mathbf{Q}^H \mathbf{y} = \mathbf{R}\mathbf{x} + \tilde{\mathbf{n}}$ , detailed as

$$\begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_{N_t} \end{pmatrix} = \begin{pmatrix} r_{1,1} & r_{1,2} & \dots & r_{1,N_t} \\ 0 & r_{2,2} & \dots & r_{2,N_t} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & r_{N_t,N_t} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_t} \end{pmatrix} + \begin{pmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \vdots \\ \tilde{n}_{N_t} \end{pmatrix}, \quad (2)$$

from which the transmitted symbols  $x_{N_t}, x_{N_t-1}, \dots, x_1$  can be detected successively. Assume no error propagation during interference cancellation process<sup>1</sup>, it is clear that QR decomposition decomposes an  $N_r \times N_t$  MIMO channel matrix  $\mathbf{H}$  into  $N_t$  subchannels with  $|r_{i,i}|$  being the gain for the  $i$ -th subchannel.

### C. Joint Antenna and Link Adaptation

As mentioned in the introduction, the link adaptation problem and the antenna selection problem are often coupled for a MIMO system. Furthermore, it is often beneficial to use only a good subset of antennas in MIMO communications to reduce hardware complexity and energy consumption. The induced performance loss is often negligible when judicious antenna selection is made and link adaptation is employed. To this end, we propose to jointly consider the antenna selection and link adaptation for wireless MIMO communications. Antenna selection and link adaptation can be realized either at the transmitter or at the receiver, depending on the availability of channel state information. In the latter case, the receiver will only feed back the selected active antenna subset and corresponding communication modes to the transmitter.

In this paper we assume QAM modulation for illustration purpose. For square  $M$ -ary QAM with average power  $\gamma$ , the minimum Euclidean distance  $d$  is

$$d = \sqrt{\frac{6\gamma}{M-1}}, \quad (3)$$

which is also a good approximation for energy efficient "non-square" QAM in a large range of interest [2].

Assume there are  $N_t$  active antennas in use. For the  $i$ -th subchannel with gain  $|r_{i,i}|$ , the square of the minimum

<sup>1</sup> This assumption is reasonable at sufficiently high SNR regimes and commonly adopted in relevant study to simplify analysis. Our simulation results validate its effectiveness.

Euclidean distance of the output constellation is given as

$$d_{i,out}^2 = \frac{6|r_{i,i}|^2 \gamma_i}{M_i - 1}, i = 1, 2, \dots, N_t, \quad (4)$$

where  $\gamma_i$  and  $M_i$  are the power and constellation size allocated to the  $i$ -th substream. As with many other multi-channel communications, the performance of a spatial multiplexing system is usually limited by the weakest link. Thus the optimization problem can be sensibly formulated as:

$$\max_{(N_t, p, b, \gamma_i)} \left\{ \min_{i \in \{1, \dots, N_t\}} d_{i,out}^2 \right\}, \quad (5)$$

where  $b_i = \log_2 M_i$  is the number of bits allocated to the  $i$ -th subchannel,  $b_T$  and  $\gamma_T$  are the total throughput and power constraints imposed on the system.

In (5) we want to find an optimal antenna subset together with its optimal bit and power allocation, subject to the total throughput and power constraints. The number of active antennas  $N_t$  can also be an optimization parameter, thus further complicating the problem. To our best knowledge, the global optimal solution is open and often a thorough search has to be resorted to, which is typically infeasible for practical implementations. Therefore we take some effective steps to decouple the original problem into some suboptimal ones, which will be shown to yield excellent performance nonetheless.

First, assuming the set of active antennas and associated bit allocation are given, as the system performance is limited by the worst subchannel, to maximize the aggregate performance we would like to allocate power so as to achieve the same output minimum Euclidean distances for all subchannels, i.e.,  $d_{1,out}^2 = \dots = d_{N_t,out}^2 = d_{(e)}^2$ , given as

$$d_{(e)}^2 = \frac{\gamma_T}{\sum_{j=1}^{N_t} \left( |r_{j,j}|^2 \times \frac{6}{M_j - 1} \right)^{-1}} = \frac{6\gamma_T}{\sum_{j=1}^{N_t} |r_{j,j}|^{-2} \times (M_j - 1)}. \quad (6)$$

Thus our optimization goal is simplified as

$$\min \sum_{j=1}^{N_t} |r_{j,j}|^{-2} \times (M_j - 1) = \min \langle \mathbf{g}(N_t), \mathbf{m}(N_t) \rangle, 1 \leq N_t \leq K_t, \quad (7)$$

subject to  $b_T = \log_2(M_1) + \log_2(M_2) + \dots + \log_2(M_{N_t})$ ,

where  $\mathbf{g}(N_t) = (|r_{1,1}|^{-2}, \dots, |r_{N_t,N_t}|^{-2})^T$  is named the antenna gain vector, while  $\mathbf{m}(N_t) = (M_1 - 1, \dots, M_{N_t} - 1)^T$  is named the bit allocation vector, and  $\langle \cdot \rangle$  denotes the inner product between them. Our target is to find an optimal pair of  $(\mathbf{g}(N_t), \mathbf{m}(N_t))$  for a given  $N_t$ , and further choose the best pair among  $1 \leq N_t \leq K_t$ , when the number of active antennas is not given beforehand.

Given  $N_t$ , the optimal pair of  $(\mathbf{g}(N_t), \mathbf{m}(N_t))$  can be found through a thorough search in principle, which is still not an easy

task when  $N_t$  and  $K_t$  are large. We further decouple the antenna selection and bit allocation problems by exploiting the discrete and finite-alphabet nature of the bit allocation vector  $\mathbf{m}(N_t)$ . When the total throughput and the modulation set are given, the possible choices of the bit allocation vector can be determined in advance by a lookup table. Furthermore, by lemma 1 given in the appendix, in order to minimize (7), only one permutation (decreasing order) of the elements in the bit allocation vector needs to be considered for each possible combination. With this decoupling, the optimization problem is finally approximated as an antenna selection problem to find a suitable  $\mathbf{g}(N_t)$  followed by table lookup to find a matching  $\mathbf{m}(N_t)$ . Some simple recursive algorithms are proposed in the next section to avoid exhaustive search while incurring little performance degradation.

Finally, note that our proposed algorithms can be readily extended to other modulation schemes. For example, when PSK is employed, the minimum Euclidean distance of  $M$ -ary

PSK with power  $\gamma$  is given by  $d = \sqrt{2 \sin^2(\frac{\pi}{M}) \gamma}$ .

Correspondingly, (7) becomes  $\min \sum_{j=1}^{N_t} |r_{j,j}|^{-2} \times \text{csc}^2(\frac{\pi}{M_j})$

subject to the same constraint.

### III. JOINT ANTENNA SELECTION AND LINK ADAPTATION FOR UNCORRELATED MIMO CHANNELS

We first consider the uncorrelated MIMO channels where the channel matrix  $\mathbf{H}$  can be modeled with i.i.d. complex Gaussian entries. Two basic recursive algorithms are proposed to choose the desired antenna gain vector  $\mathbf{g}(N_t)$ : incremental selection means the “desired” antennas are recursively added to an initially empty active antenna set while decremental selection means “undesired” antennas are recursively removed from an initially full antenna set<sup>2</sup>. When  $N_t \ll K_t$ , we can use the incremental selection rule described in III.A, while we can use the decremental rule in III.B when  $N_t$  is close to  $K_t$ . In a general link adaptation problem where  $N_t$  is unknown in advance, we can search over all possible  $1 \leq N_t \leq K_t$  to find the optimal one. To reduce complexity, we propose an adaptive selection rule based on estimation of  $N_t$  in III.C.

#### A. Incremental Selection Rule with Link Adaptation

Intuitively, we want  $|r_{1,1}|, |r_{2,2}|, \dots, |r_{N_t,N_t}|$  as large as possible. Our incremental recursive rule works as follows: starting from a column of  $\mathbf{H}$  ( $N_t \times K_t$ ) which results in maximum  $|r_{1,1}|$  (corresponding to the largest vector norm), we successively choose from the remaining columns of  $\mathbf{H}$  such that the next subchannel gain is maximized. The subchannel

<sup>2</sup> Same notations are used in [19] and other literature with different problem settings.

gain of the newly added antenna can be obtained in a closed-form solution, which is described by the following lemma.

**Lemma2.a** Assume the QR decomposition of a matrix  $\mathbf{H}^{(k)}$  with  $k$  independent columns is  $\mathbf{H}^{(k)} = \mathbf{Q}(k)\mathbf{R}(k)$ . Then for the enhanced matrix  $\mathbf{H}^{(k+1)} = \begin{bmatrix} \mathbf{H}^{(k)} & \mathbf{h} \end{bmatrix}$  with QR decomposition  $\mathbf{H}^{(k+1)} = \mathbf{Q}(k+1)\mathbf{R}(k+1)$ , the first  $k$  diagonal elements of  $\mathbf{R}(k+1)$  keep the same with those of  $\mathbf{R}(k)$ , while the  $(k+1)$ -th one is given by  $\sqrt{\mathbf{h}^H \mathbf{h} - \mathbf{h}^H \mathbf{Q}(k)\mathbf{Q}(k)^H \mathbf{h}}$ ; similarly, the first  $k$  column vectors of  $\mathbf{Q}(k+1)$  keep the same with those of  $\mathbf{Q}(k)$ , while the  $(k+1)$ -th one is given by

$$\mathbf{Q}(:, k+1) = \mathbf{h} - \sum_{l=1}^k \mathbf{Q}(:, l) \mathbf{h}^H \mathbf{Q}(:, l)$$

Proof: see the appendix.

Based on Lemma 2.a, assume in the  $k$ -th step,  $\mathbf{H}^{(k)}$  stores the  $k$  selected columns of  $\mathbf{H}$  and the QR decomposition of  $\mathbf{H}^{(k)}$  is  $\mathbf{Q}(k)\mathbf{R}(k)$ , then in the  $(k+1)$ -th step, we choose the column vector  $\mathbf{h}$  from  $\mathbf{H} \setminus \mathbf{H}^{(k)}$  (which represents the remaining columns of  $\mathbf{H}$ ) in a way such that  $r_{k+1, k+1} = \sqrt{\mathbf{h}^H \mathbf{h} - \mathbf{h}^H \mathbf{Q}(k)\mathbf{Q}(k)^H \mathbf{h}}$  is maximized. Furthermore, it can also be shown as follows that the successively generated antenna gains are already ordered.

**Lemma2.b** In the above incremental selection rule for uncorrelated MIMO,  $|r_{1,1}| \geq |r_{2,2}| \geq \dots \geq |r_{k,k}| \geq \dots \geq |r_{N_t, N_t}|$ .

Proof: see the appendix.

Lemma 2.b shows that the elements in the selected antenna gain vector  $\mathbf{g}(N_t) = (|r_{1,1}|^2, |r_{2,2}|^2, \dots, |r_{N_t, N_t}|^2)^T$  are already in an increasing order. Thus we only need to arrange the elements of candidate bit allocation vectors  $\mathbf{m}(N_t)$  in a decreasing order in the lookup table according to lemma 1, which saves storage space and increases the matching speed for (7). We further assume  $\hat{\mathbf{m}}(N_t)$  is the optimal bit allocation vector that minimizes  $\langle \mathbf{g}(N_t), \mathbf{m}(N_t) \rangle$  for a given  $\mathbf{g}(N_t)$ .

The incremental selection rule with link adaptation for uncorrelated MIMO is summarized in the table I.

Two points are noteworthy for the above algorithm. First, due to Lemma 2.a and 2.b, the antenna selection and link adaptation process is significantly expedited. Secondly, due to the recursive nature of the algorithm, searching an optimal  $N_t$  for a general link adaptation problem does not mean  $K_t$  times of effort (as calculation for  $N_t + 1$  is just one step further based on calculation for  $N_t$ ), but rather the worst-case effort where all  $K_t$  transmit antennas must be deployed. Nonetheless, in case nearly all the  $K_t$  transmit antennas would be deployed, we provide a decremental selection rule for link adaptation, which is described in the following subsection.

TABLE I  
INCREMENTAL ANTENNA SELECTION RULE WITH LINK ADAPTATION FOR  
UNCORRELATED MIMO

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Set  $\mathbf{I} = \{1, 2, 3, \dots, K_t\}$  and  $\mathbf{p} = \Phi$  (empty set),  $\mathbf{g} = \Phi$  (empty set),  $\mathbf{Q} = \Phi$  (empty set)  
for  $i = 1$  to  $K_t$   
 $\alpha_i = \|\mathbf{H}(:, i)\|^2$ ;  
end  
 $\mathbf{p}(1) = \arg \max_{1 \leq i \leq K_t} \alpha_i$ ,  $r_{1,1}^2 = \max_{1 \leq i \leq K_t} \alpha_i$ ,  $\mathbf{g}(1) = 1/r_{1,1}^2$ ,  $\mathbf{Q}(:, 1) = \mathbf{H}(:, \mathbf{p}(1)) / \|\mathbf{H}(:, \mathbf{p}(1))\|$ ;  
 $\mathbf{I} = \mathbf{I} \setminus \mathbf{p}(1)$ ;  
for  $k = 2$  to  $N_t$   
update  $\alpha_i = \alpha_i - \|\mathbf{H}(:, i)^H \mathbf{Q}(:, k-1)\|^2$ , for all  $i \in \mathbf{I}$ ;  
 $\mathbf{p}(k) = \arg \max_{i \in \mathbf{I}} \alpha_i$ ;  
 $r_{k,k}^2 = \max_{i \in \mathbf{I}} \alpha_i$ ;  
 $\mathbf{g}(k) = 1/r_{k,k}^2$ ;  
 $\mathbf{Q}(:, k) = \mathbf{H}(:, \mathbf{p}(k)) - \sum_{l=1}^{k-1} \mathbf{Q}(:, l) \mathbf{H}(:, \mathbf{p}(k)) \mathbf{Q}(:, l)^H$ ;  
 $\mathbf{Q}(:, k) = \mathbf{Q}(:, k) / \|\mathbf{Q}(:, k)\|$ ;  
 $\mathbf{I} = \mathbf{I} \setminus \mathbf{p}(k)$ ;  
end  
assume  $\hat{\mathbf{m}}(N_t) = \arg \min_{\mathbf{m}(N_t)} \langle \mathbf{g}(N_t), \mathbf{m}(N_t) \rangle$ ;  
return  $\mathbf{p}(1:N_t)$  and  $\hat{\mathbf{m}}(N_t)$ ;

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### B. Decremental Selection Rule with Link Adaptation

Our proposed decremental selection rule is related to the V-BLAST ordering rule first proposed in [20], which successively chooses the antenna (among those not already chosen) that maximizes post-detection SNR under the assumption of perfect feedback. Accordingly, we can successively discard the antenna (among those not already chosen) that minimizes the post-detection SNR under the assumption of perfect feedback. Usually repeated matrix inversion will be involved during the process of discarding, which may introduce much computation complexity and numerical instability. Thanks to the work in [4], we can avoid computing the inversion of the deflated channel matrix by means of a recursive square-root algorithm.

The decremental selection rule with link adaptation for uncorrelated MIMO is summarized in the table II.

TABLE II  
DECREMENTAL ANTENNA SELECTION RULE WITH LINK ADAPTATION FOR  
UNCORRELATED MIMO

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Set  $\mathbf{H}^{(K_t)} = \mathbf{H}$ ;  
Using [4] to find square root of  $(\mathbf{H}^{(K_t)})^H \mathbf{H}^{(K_t)}$ , assume  
 $\mathbf{P}^{(K_t)} = (\mathbf{H}^{(K_t)})^H \mathbf{H}^{(K_t)}^{-1/2}$ ;

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Assume  $\hat{l} = \arg \min_{1 \leq l \leq K_t} \|\mathbf{P}^{(K_t)}(l, :)\|$  (means the length of the  $l$ -th row), discard the  $\hat{l}$ -th column of  $\mathbf{H}^{(K_t)}$ , and assume the deflated matrix to be  $\mathbf{H}^{(K_t-1)}$ ;

for  $k=1$  to  $K_t - (N_t + 1)$

Using [4] to find square root of  $(\mathbf{H}^{(K_t-k)})^H \mathbf{H}^{(K_t-k)}$ , based on  $\mathbf{P}^{(K_t-k+1)}$ ;

assume  $\mathbf{P}^{(K_t-k)} = (\mathbf{H}^{(K_t-k)})^H \mathbf{H}^{(K_t-k)}^{-1/2}$ ;

Discard the  $\hat{l}$ -th column of  $\mathbf{H}^{(K_t-k)}$ , where  $\hat{l} = \arg \min_{1 \leq l \leq K_t} \|\mathbf{P}^{(k)}(l, :)\|$ ;

assume the deflated matrix to be  $\mathbf{H}^{(K_t-k-1)}$ ;

end

Compute the sorted antenna gain vector  $\mathbf{g}(N_t)$  for  $\mathbf{H}^{(N_t)}$ ;

assume  $\hat{\mathbf{m}}(N_t) = \arg \min_{\mathbf{m}(N_t)} \langle \mathbf{g}(N_t), \mathbf{m}(N_t) \rangle$ ;

return  $\mathbf{H}^{(N_t)}$  and  $\hat{\mathbf{m}}(N_t)$ .

### C. Simplified Link Adaptation for Uncorrelated Rayleigh MIMO Channels

In a general link adaptation problem where  $N_t$  is not fixed in advance, we need to test all possibilities  $1 \leq N_t \leq K_t$  to find the optimal one using either the incremental or decremental selection rule. In this subsection, we propose a simplified selection rule based on the estimation of the optimal number of active transmit antennas to further reduce the complexity. With i.i.d. complex Gaussian channel matrix,  $|r_{i,i}|^2$  in (7) is a  $\chi^2$  distributed random variable with  $2 \times (N_r + 1 - i)$  degrees of freedom [3], where the probability density function of the  $\chi^2$  distribution with  $\nu$  degrees of freedom is given as:

$$f(x|\nu) = \frac{x^{(\nu-2)/2} e^{-x/2}}{2^{\nu/2} \Gamma(\nu/2)}, \quad (8)$$

where  $\Gamma(x)$  is the gamma function defined as  $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ .

The expected value of  $1/|r_{i,i}|^2$  can be obtained as

$$\int_0^\infty \frac{1}{x} f(x|\nu) dx = \begin{cases} 1/(\nu-2) & \text{for } \nu > 2 \\ +\infty & \text{for } \nu = 2 \end{cases}, \quad (9)$$

Replacing  $1/|r_{i,i}|^2$  in (7) with their expected values, we have

$$\min E(\langle \mathbf{g}(N_t), \mathbf{m}(N_t) \rangle) = \min \sum_{j=1}^{N_t} \frac{1}{2(N_r - j)} \times (M_j - 1), \quad (10)$$

$1 \leq N_t \leq K_t$ ,

subject to

$$b_r = \log_2(M_1) + \log_2(M_2) + \dots + \log_2(M_{N_t}), \text{ and} \quad (11)$$

$$M_1 \geq M_2 \geq \dots \geq M_{N_t}.$$

Therefore, we can estimate the optimal number of active antennas in a pre-processing stage as follows: for all possible bit allocation vectors  $\mathbf{m}(N_t)$ 's that satisfy (11), find the one that minimizes (10), denoted as  $\hat{\mathbf{m}}(N_t)$ ; then find  $\tilde{N}_t = \arg \min_{1 \leq N_t \leq K_t} E(\langle \mathbf{g}(N_t), \hat{\mathbf{m}}(N_t) \rangle)$ , which is our estimate of the

optimal number of active antennas in i.i.d. Rayleigh fading MIMO channels. Thus we can decide to use either the incremental or decremental selection rule for joint antenna selection and link adaptation for different system settings based on the value of  $\tilde{N}_t$ . Furthermore, we can restrict ourselves to search optimal  $N_t$  only in the range around  $\tilde{N}_t$  to further reduce the computational complexity. Simulations results show that searching  $N_t$  in the range of  $[\tilde{N}_t - 1, \tilde{N}_t + 1]$  and storing only three bit allocation vectors  $\hat{\mathbf{m}}(N_t - 1)$ ,  $\hat{\mathbf{m}}(N_t)$  and  $\hat{\mathbf{m}}(N_t + 1)$  in the bit allocation lookup table incur little performance loss (See Section V).

## IV. JOINT ANTENNA SELECTION AND LINK ADAPTATION FOR CORRELATED MIMO CHANNELS

### A. Correlated MIMO Channels

In this section, we extend the study of joint antenna selection and link adaptation to correlated MIMO channels. We assume correlation only exists at the transmitter side, as described by the "one-ring" model in [10]. This model is feasible, e.g., for the outdoor macrocell situation where the transmitter at the base station is elevated high above the local scattering environment, while there are sufficient local scatters around the mobile receivers. For an  $N_r \times K_t$  MIMO system, the channel can be modeled as  $\mathbf{H} = \mathbf{H}_w \mathbf{A}_T^H$  with  $\mathbf{A}_T^* \mathbf{A}_T^H = \mathbf{R}_T$ , where  $\mathbf{H}_w$  is an  $N_r \times K_t$  matrix containing i.i.d. complex Gaussian random variables and  $\mathbf{R}_T$  is a  $K_t \times K_t$  Hermitian semi-positive definite matrix representing the covariance matrix for each row of  $\mathbf{H}$ .

Again, we assume  $N_t$  out of  $K_t$  antennas are to be selected. As before, the channel matrix between the  $N_t$  transmit antennas and  $N_r$  receive antennas can be described as  $\mathbf{H}(p) = \mathbf{H}_w(p) \mathbf{A}_T^H(p)$ , where  $p$  contains the indices of the selected antennas, and  $\mathbf{A}_T(p) \mathbf{A}_T^H(p) = \mathbf{R}_T(p)$  is the corresponding submatrix of  $\mathbf{R}_T$ .

We assume uniform linear arrays at both the transmitter and receiver, with antenna spacing  $\Delta_T$  (relative to the carrier wavelength). We also assume there are  $L$  clusters of scatterers in the environment and the angle of departure for the  $l$ -th path cluster is Gaussian distributed as  $\theta_l \sim N(\bar{\theta}_l, \sigma_l^2)$ . Then the  $(i, j)$ -th entry of the transmit covariance matrix contributed by the  $l$ -th scattering cluster can be shown to be approximated as [9][16]-[18]:

$$[\mathbf{R}_{T,l}]_{i,j} \approx e^{-j2\pi(i-j)\Delta_T \cos(\bar{\theta}_l)} e^{-\frac{1}{2}(2\pi(i-j)\Delta_T \sin(\bar{\theta}_l)\sigma_l)^2}. \quad (12)$$

For a narrowband system, the net correlation matrix can be obtained by summing the covariance matrices contributed by the  $L$  clusters weighted by the fraction of power in the corresponding cluster. As a counterpart to (1), the received signal in correlated MIMO can be written as:

$$\mathbf{y} = \mathbf{H}_w(p)\mathbf{A}_T^H(p)\mathbf{x} + \mathbf{n}. \quad (13)$$

Clearly, joint antenna selection and link adaptation algorithms described in the previous section can be readily applied to correlated MIMO channels and are expected to achieve more substantial gains. However it is noteworthy that for correlated MIMO, the elements of  $\mathbf{A}_T^H(p)$  vary much more slowly than those of  $\mathbf{H}_w(p)$ , which is mainly determined by the local physical parameters, such as antenna spacing and angle spread. Since these parameters are relatively static and can be measured more accurately than instantaneous channel information, antenna selection and link adaptation based on  $\mathbf{A}_T^H(p)$  is more attractive than that based on  $\mathbf{H}_w(p)\mathbf{A}_T^H(p)$ . Targeting on this goal, in the next subsection, we will describe a joint antenna selection and link adaptation algorithm for correlated MIMO only based on the channel correlation information.

### B. Antenna Selection and Link Adaptation Only Based on Channel Correlation Information

By applying QR decomposition successively to the correlation matrix  $\mathbf{A}_T^H(p) = \mathbf{Q}_1\mathbf{R}_1$  and  $\mathbf{H}_w(p)\mathbf{Q}_1 = \mathbf{Q}_2\mathbf{R}_2$ , (13) becomes

$$\mathbf{y} = \mathbf{Q}_2\mathbf{R}_2\mathbf{R}_1\mathbf{x} + \mathbf{n}, \quad (14)$$

Apply  $\mathbf{Q}_2^H$  to the received vector, we have

$$\tilde{\mathbf{y}} = \mathbf{Q}_2^H\mathbf{y} = \mathbf{R}_2\mathbf{R}_1\mathbf{x} + \tilde{\mathbf{n}}, \quad (15)$$

The optimization goal for correlated MIMO channels is given as (cf. (2) and (7)):

$$\min \sum_{j=1}^{N_r} |\mathbf{R}_1(j, j)\mathbf{R}_2(j, j)|^{-2} \times (M_j - 1) = \min \langle \mathbf{g}'(N_t), \mathbf{m}(N_t) \rangle, \quad (16)$$

subject to  $b_r = \log_2(M_1) + \log_2(M_2) + \dots + \log_2(M_{N_r})$ ,

with

$$\mathbf{g}'(N_t) = \left[ |\mathbf{R}_1(1,1)\mathbf{R}_2(1,1)|^{-2} \quad \dots \quad |\mathbf{R}_1(N_t, N_t)\mathbf{R}_2(N_t, N_t)|^{-2} \right]^T \quad (17)$$

and

$$\mathbf{m}(N_t) = (M_1 - 1, M_2 - 1, \dots, M_{N_r} - 1)^T$$

the corresponding antenna gain vector and bit allocation vector for correlated MIMO.

Since the distribution of  $\mathbf{H}_w\mathbf{Q}_1$  is the same as  $\mathbf{H}_w$ ,  $|\mathbf{R}_2(j, j)|^2$  is still  $\chi^2$  distributed with degree of freedom  $2 \times (N_r + 1 - j)$ . In order to derive an antenna selection and link adaptation rule only based on  $\mathbf{R}_T$ , we replace  $|\mathbf{R}_2(j, j)|^{-2}$  in (17) with their expected values (see (9)) to get

$$\bar{\mathbf{g}}'(N_t) = \left[ \frac{|\mathbf{R}_1(1,1)|^{-2}}{2(N_r - 1)} \quad \dots \quad \frac{|\mathbf{R}_1(N_t, N_t)|^{-2}}{2(N_r - N_t)} \right]^T, \quad (18)$$

Hence (16) is turned into:

$$\min \sum_{j=1}^{N_r} \frac{|\mathbf{R}_1(j, j)|^{-2}}{2(N_r - j)} \times (M_j - 1) = \min \langle \bar{\mathbf{g}}'(N_t), \mathbf{m}(N_t) \rangle, \quad (19)$$

subject to.  $b_r = \log_2(M_1) + \log_2(M_2) + \dots + \log_2(M_{N_r})$ .

In recognition of  $\mathbf{R}_1^H * \mathbf{R}_1 = \mathbf{A}_T(p)\mathbf{A}_T^H(p) = \mathbf{R}_T(p)$ , for correlated MIMO, our goal is to find a submatrix of  $\mathbf{R}_T$  whose Cholesky factor will provide a close-to-minimization result of (19).

Similar to III.A, we decouple the antenna selection and link adaptation problem and present an incremental selection rule as follows. Starting from an empty set, in each step we would like to choose from the remaining components of  $\mathbf{R}_T$  such that the next subchannel gain is maximized. This process is expedited by the following lemmas.

**Lemma3.a** Assume matrix  $\mathbf{R}_T^{(k)}$  is Hermitian positive definite with size  $k$ , whose Cholesky decomposition is given by  $\mathbf{R}_T^{(k)} = \mathbf{R}(k)\mathbf{R}^H(k)$ , then for the enhanced matrix

$$\mathbf{R}_T^{(k+1)} = \begin{bmatrix} \mathbf{R}_T^{(k)} & \mathbf{v} \\ \mathbf{v}^H & 1 \end{bmatrix} \quad \text{with Cholesky decomposition}$$

$\mathbf{R}_T^{(k+1)} = \mathbf{R}(k+1)\mathbf{R}^H(k+1)$ , the first  $k$  diagonal elements of  $\mathbf{R}(k+1)$   $\{r_{i,i}\}_{i=1}^k$  keep the same with those of  $\mathbf{R}(k)$ , while the

$(k+1)$ -th one is given by  $r_{k+1,k+1} = \sqrt{1 - \mathbf{v}^H (\mathbf{R}_T^{(k)})^{-1} \mathbf{v}}$ .

Proof: see the appendix.

Based on Lemma 3.a, assume in step  $k$ , there are  $k$  selected transmit antennas, and  $\mathbf{R}_T^{(k)}$  is the  $k \times k$  covariance matrix for those  $k$  selected transmit antennas, which is guaranteed to be invertible according to our selection rule, then in step  $k+1$ , we will choose the antenna whose covariance vector  $\mathbf{v}$  will

maximize  $r_{k+1,k+1} = \sqrt{1 - \mathbf{v}^H (\mathbf{R}_T^{(k)})^{-1} \mathbf{v}}$ . Note that the diagonal elements of the covariance matrix  $\mathbf{R}_T$  are all 1's, thus  $r_{1,1}$  is always 1 no matter which antenna is selected first. However, we can determine the first and second active antennas jointly by means of maximizing  $r_{2,2}$ , i.e., choose the first two active antennas whose corresponding Cholesky decomposition will result in maximization of  $r_{2,2}$ . Also note that the condition number of  $\mathbf{R}_T$  is high, hence we can set a positive threshold value  $C_0$  in practice to discard those essentially zero-gain subchannels.

Similar to Lemma 2.b, the following lemma facilitates the optimization of (19).

**Lemma3.b** In the above incremental selection rule for correlated MIMO,  $r_{1,1} \geq \dots r_{k,k} \geq r_{k+1,k+1}$ .

Lemma3.b shows the elements in  $\bar{\mathbf{g}}'(N_t)$  are already in an increasing order. Thus we only need to arrange the elements of candidate bit allocation vectors  $\mathbf{m}(N_t)$  in a decreasing order according to Lemma 1. In summary, the incremental selection rule with link adaptation for correlated MIMO is described in Table III. For correlated MIMO, a decremental selection rule is usually not necessary, since the ill-conditioning of the channel matrix typically results in much fewer antennas being selected

as opposed to the uncorrelated MIMO of the same size. Furthermore, antenna selection and link adaptation modes need to be updated only when the channel covariance matrix changes, which happens far less frequently compared to that based on the instantaneous channel fading.

TABLE III  
INCREMENTAL ANTENNA SELECTION RULE WITH LINK ADAPTATION FOR CORRELATED MIMO

---

Set  $\mathbf{I} := \{1, 2, 3, \dots, K_t\}$  and  $\mathbf{p} := \Phi$  (empty set),  $\mathbf{T} := \Phi$  (empty set)

$\mathbf{p}(1:2) = \arg \max_{i,j} (1 - \mathbf{v}_i^H \mathbf{v}_j)$ ,  $\mathbf{T}(1) := 1$ ,  $\mathbf{T}(2) := \max_{i,j} (1 - \mathbf{v}_i^H \mathbf{v}_j)$ , where

$\mathbf{v}_i = \mathbf{R}_t(i, j)$  and  $i \leq j$ ;

$\mathbf{I} := \mathbf{I} - \mathbf{p}(1) - \mathbf{p}(2)$ ;

for  $k = 3$  to  $N_t$

$\mathbf{A}_{k-1} := \mathbf{R}_t(\mathbf{p}(1:k-1))$  (Submatrix of  $\mathbf{R}_t$  designated by  $\mathbf{p}(1:k-1)$ );

$\mathbf{T}(k) = \max_{j \in \mathbf{I}} (1 - \mathbf{v}_{k-1}^H \mathbf{A}_{k-1}^{-1} \mathbf{v}_{k-1})$ ;

$\mathbf{p}(k) = \arg \max_{j \in \mathbf{I}} (1 - \mathbf{v}_{k-1}^H \mathbf{A}_{k-1}^{-1} \mathbf{v}_{k-1})$ , where

$\mathbf{v}_{k-1} = \mathbf{R}_t(\mathbf{p}(1:k-1), \mathbf{p}(j))$  (The column vector in  $\mathbf{p}(j)$  column and  $\mathbf{p}(1:k-1)$  rows of  $\mathbf{R}_t$ );

$\mathbf{I} := \mathbf{I} - \mathbf{p}(k)$ ;

if  $\frac{1}{N_t - k} \left\langle \frac{1}{\mathbf{T}(1:k)}, \hat{\mathbf{m}}(k) \right\rangle > \frac{1}{N_t - (k-1)} \left\langle \frac{1}{\mathbf{T}(1:k-1)}, \hat{\mathbf{m}}(k-1) \right\rangle$

    return  $\mathbf{p}(1:k-1)$  and  $\hat{\mathbf{m}}(k-1)$

else if  $\mathbf{T}(k) \leq C_0$  ( $C_0$  is a threshold value)

    break;

else if  $k = N_t$

    return  $\mathbf{p}(1:N_t)$  and  $\hat{\mathbf{m}}(N_t)$

end

---

Similarly, in a general link adaptation problem where  $N_t$  is unknown in advance, we can search over all possible  $1 \leq N_t \leq K_t$  to find the optimal one.

### V. NUMERICAL RESULTS

In this section, we evaluate the performance of our proposed joint antenna selection and link adaptation algorithms for both uncorrelated and correlated MIMO channels through several representative examples. Square QAM modulation is employed in all simulations with 256-QAM the largest constellation to be used.

**Example 1.** In this example, we demonstrate that both antenna selection gain and link adaptation gain are obtained through our algorithm. Consider a  $3 \times 6$  MIMO ( $N_r = 3, K_t = 6$ ) with i.i.d Rayleigh fading. The number of active transmit antennas to be chosen is  $N_t = 3$  and the target throughput is 12 bits/s/Hz. For performance evaluation, we consider the following three systems: the first one is V-BLAST (i.e., equal power and rate allocation) with random antenna

selection; the second one is V-BLAST with a selected transmit antenna subset obtained through the incremental selection rule; and the last one is our proposed incremental antenna subset selection with link adaptation given in Table I. Link adaptation based on singular value decomposition (SVD) of the channel matrix [23] is also included, which can be viewed as a performance upper bound since the decomposed subchannels are truly interference free. From Fig.1, we can see that antenna selection gain is dominant, while link adaptation gain is also significant especially at high SNR (by observing the difference of the slope of the third curve (diversity gain) from that of the second one, and its similarity with that of the upper bound curve).

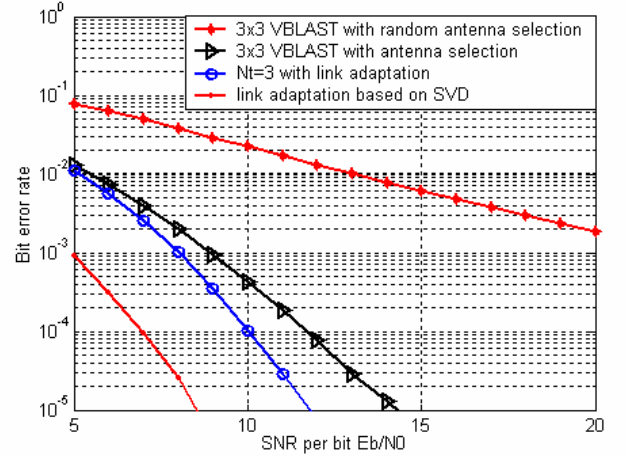


Fig.1. Antenna selection gain and link adaptation gain for  $3 \times 6$  MIMO with throughput 12bits/s/hz.

In the next example, we compare the performances of the incremental and decremental selection rules for a general link adaptation problem where the number of active antennas is not given beforehand, based on both full-size lookup tables and reduced-size lookup tables (see III.C).

**Example 2.** Here we consider a  $6 \times 6$  MIMO with uncorrelated Rayleigh fading, and the target throughput is 12 bits/s/Hz.

The full-size lookup table is shown in (20) with each row representing the bit allocation for a certain number of active antennas. By Lemma 1 and Lemma 2.b, only the decreasing order of each possible combination is listed.

From (10), the estimated optimal number of active antennas is  $\tilde{N}_t = 4$ , so we only need to store the *optimal* bit allocation vectors  $\hat{\mathbf{m}}(N_t)$  for  $N_t = \{3, 4, 5\}$  active antennas in (21), which incurs almost no performance loss as shown in Fig. 2. It is also shown that the incremental and decremental selection rules achieve almost the same performance and approach the SVD

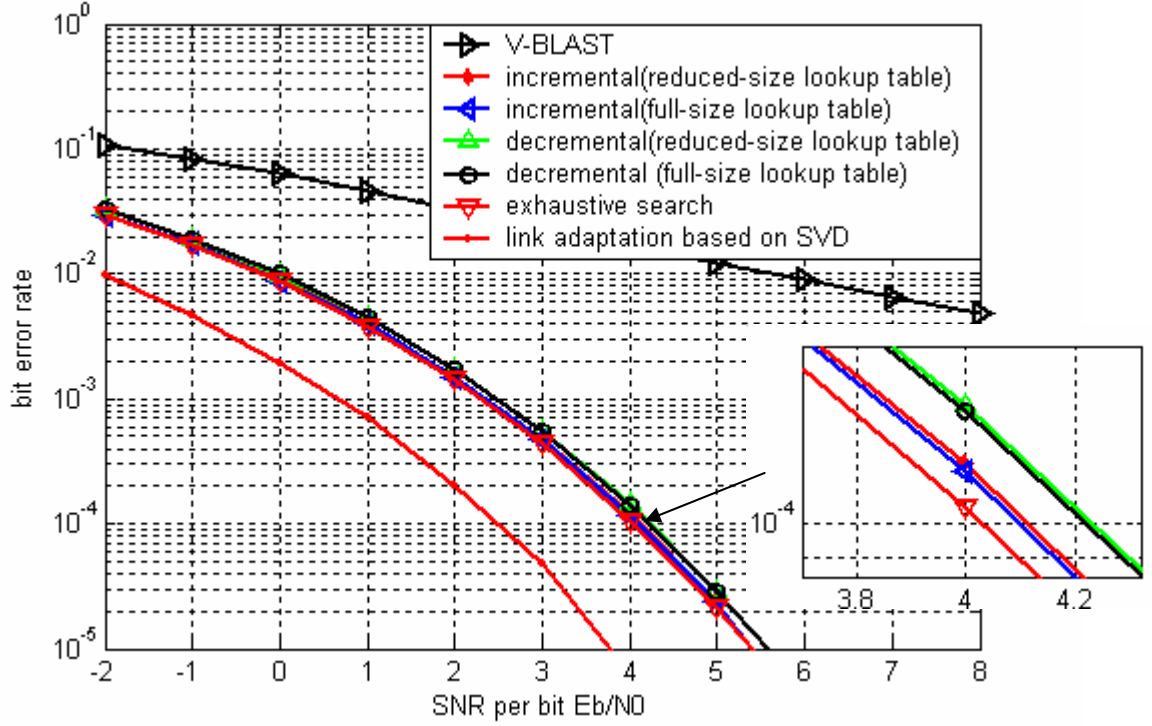


Fig.2. Performance comparison of the proposed joint antenna and link adaptation algorithms in 6x6 MIMO with throughput 12bits/s/hz (the smaller figure is the exploded view at BER around  $10^{-4}$ ).

upper bound quite closely. (Note that the four curves for joint antenna selection and link adaptation are almost indistinguishable in Fig. 2.)

$$Bits\_table\_full = \begin{bmatrix} 8 & 4 & 0 & 0 & 0 & 0 \\ 6 & 6 & 0 & 0 & 0 & 0 \\ 8 & 2 & 2 & 0 & 0 & 0 \\ 6 & 4 & 2 & 0 & 0 & 0 \\ 4 & 4 & 4 & 0 & 0 & 0 \\ 6 & 2 & 2 & 2 & 0 & 0 \\ 4 & 4 & 2 & 2 & 0 & 0 \\ 4 & 2 & 2 & 2 & 2 & 0 \\ 2 & 2 & 2 & 2 & 2 & 2 \end{bmatrix}. \quad (20)$$

$$Bits\_table\_reduced = \begin{bmatrix} 4 & 4 & 4 & 0 & 0 & 0 \\ 4 & 4 & 2 & 2 & 0 & 0 \\ 4 & 2 & 2 & 2 & 2 & 0 \end{bmatrix}. \quad (21)$$

To verify the effectiveness of our approach in decoupling the antenna selection and link adaptation problems, exhaustive search (among all the possible combinations of the transmit antennas) is conducted to find the optimal pair of  $(\mathbf{g}(N_t), \mathbf{m}(N_t))$  for (7). As observed in Fig. 2, our algorithms incur negligible performance degradation.

**Example 3.** This example demonstrates the application of our proposed algorithms on a large MIMO systems, emphasizing the demand of complexity reduction. The MIMO systems we consider is of size  $16 \times 16$ , and the target throughput is 32 bits/s/Hz.

Clearly with this system, any exhaustive search will lead

to tremendous computational complexity. Our proposed algorithms exhibit their simplicity advantage while closely approaching the SVD performance upper bound, as shown in Fig. 3. By using the reduced-size lookup table, the computation complexity is significantly reduced. From (10) the estimated optimal number of active antennas is  $\tilde{N}_t = 12$ , hence in the lookup table, we only need to store the optimal bits pattern for  $N_t = \{11, 12, 13\}$ , which is shown below:

$$Bits\_table\_reduced = \begin{bmatrix} 4 & 4 & 4 & 4 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 \\ 4 & 4 & 4 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 0 & 0 \\ 4 & 4 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 0 \end{bmatrix}. \quad (22)$$

We do not provide the full size lookup table here because of its large size. From Fig. 3, we can see that little performance loss is incurred when using the reduced-size lookup table.

Note that even though matrix inversion is mostly eliminated in the decremental antenna selection rule by means of square root algorithm, it is still inevitable in the initial step, which incurs substantial computational complexity. Furthermore, while the antenna gain vector  $\mathbf{g}$  is readily obtained through Lemma 2.a in the incremental antenna selection, explicit QR decomposition is required for the decremental selection. One can check the computational complexity in antenna selection for incremental method is  $O(K_t N_r N_t)$ , while for decremental method, the computation complexity is dominated by the initial matrix inversion step, which is  $O(K_t N_r^2 + K_t^2 N_r)$ . Therefore for MIMO systems of small to medium size (such as

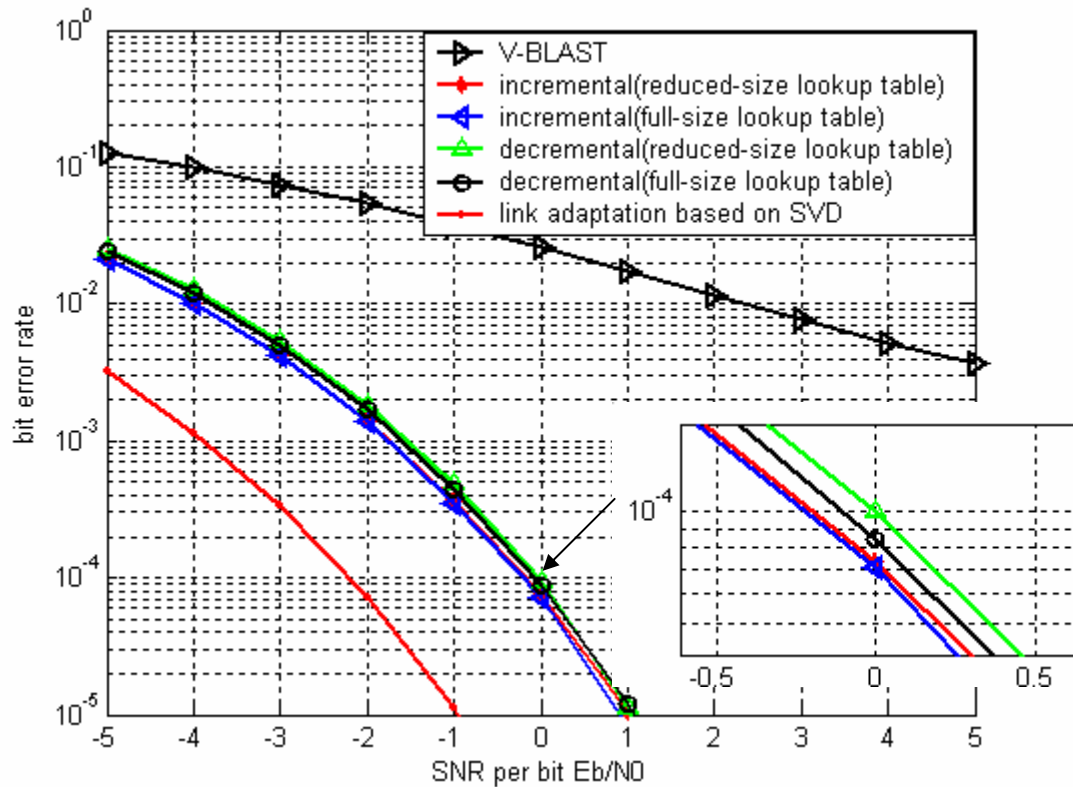


Fig.3. Performance comparison of the proposed joint antenna and link adaptation algorithms in 16x16 MIMO with throughput 32bits/s/hz (the smaller figure is the exploded view at BER around  $10^{-4}$ ).

in Example 2), the incremental rule is preferred, while decremental rule is favored only for very large MIMO systems where the estimated optimal number  $\tilde{N}_i$  is very close to  $K_i$  (due to reduced complexity in link adaptation).

**Example 4.** In the above examples, we assume perfect CSI is available for antenna selection and link adaptation. In actual vehicular based communications, the channels may vary too fast to allow timely feedback. In this example, we re-evaluate our algorithms in fast fading channels with limited feedback, and explore the long range prediction (LRP) technique [27] as a remedy. LRP is essentially a linear prediction method based on autoregressive modeling. With this technique, one can measure and feedback the time-varying CSI to the transmitter at a much lower rate than the data rate. The transmitter will make compensations through prediction and interpolation, and then determine the active antenna set and modulation modes based on the predicted and interpolated CSI. The reader is referred to [27] for a detailed description of this technique.

Fig. 4 shows the performance losses of our joint antenna selection and link adaptation algorithms with different feedback delays. Clearly there is a tradeoff between feedback delays and dedicated feedback channel bandwidth. For the LRP technique, the longer the feedback delays, the larger the prediction steps should be taken. In our simulation, we assume a Rayleigh fading channel with Jakes' model with the Doppler shift of  $f_d$  (thus the coherence time is  $\tau_0 \approx 1/f_d$  seconds). The channel sampling rate is  $8f_d$ , while the data rate is  $640f_d$ , so

the channel is measured and fed back once every 80 symbols. The prediction order of LRP is set as 50. It is observed that the proposed algorithms, in conjunction with the LRP technique, have a fairly graceful degradation in performance with increase of feedback delays.

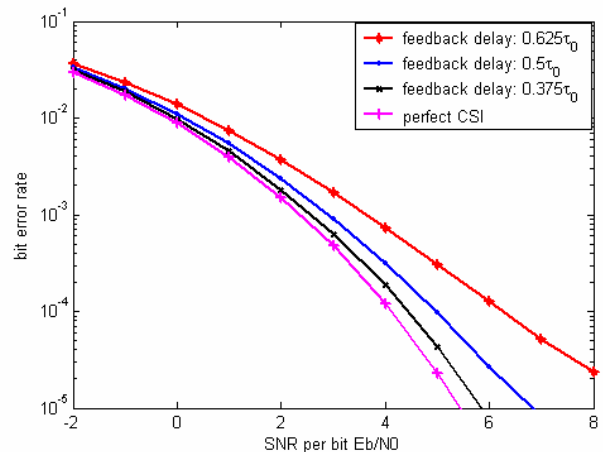


Fig.4. (a) Performance with different feedback delays for incremental methods.

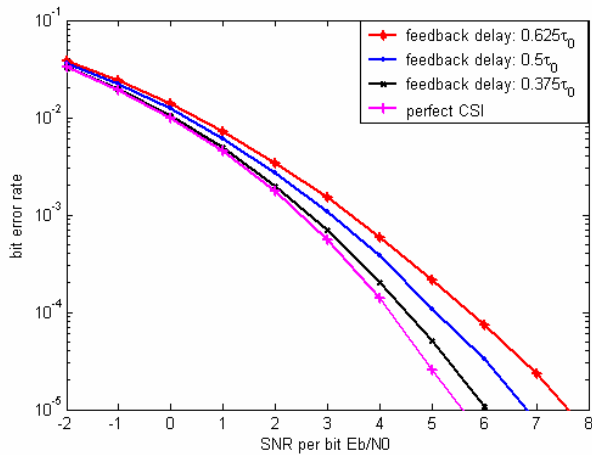


Fig.4.(b) Performance with different feedback delays for decremental methods.

**Example5.** Finally, this numerical example demonstrates the performance of link adaptation only based on correlation information, compared with that based on the full channel information. Consider a  $6 \times 6$  correlated MIMO with correlation matrix generated as in (12), and the target throughput is 12 bits/s/Hz. We consider three correlated fading scenarios as listed in Table IV with an increasing order of fading correlation, all assuming there is only one transmit cluster in the communication environments.

TABLE IV  
FADING CORRELATION SCENARIOS

Scenario 1.	$\bar{\theta} = \frac{\pi}{6}, \sigma = \frac{\pi}{10}$
Scenario 2.	$\bar{\theta} = \frac{\pi}{6}, \sigma = \frac{\pi}{15}$
Scenario 3.	$\bar{\theta} = \frac{\pi}{10}, \sigma = \frac{\pi}{30}$

In the upper part of Fig.5 - Fig.7, we compare the BER performance among link adaptation only based on the correlation information, link adaptation based on the full channel information and the conventional V-BLAST. From the simulation results, we can see the performance of the traditional V-BLAST degrades significantly in the correlated MIMO channels. On the other hand, antenna selection and link adaptation achieves more substantial gains for correlated MIMO than for uncorrelated MIMO, and the performance gap between link adaptation only based on channel correlated information and link adaptation based on the full channel information decreases as the degree of correlation increases.

In the lower part of Fig.5 - Fig.7 (generated for the link adaptation based on the full channel information) we plot the histograms of the number of active antennas for the three fading correlation scenarios. From the histograms, we can see that the number of active antennas also decreases when the correlation increases.

Table V below illustrates the active transmit antenna index and the constellation carried by each active antenna using our link adaptation only based on channel correlation information. In contrast to that based on the full channel information, this

configuration only depends on channel physical characteristics, and is invariant to instantaneous channel realizations. It's interesting to see that the first two active antennas are always antenna 1 and antenna 6, which accords with the practical situation: antenna 1 and antenna 6 have the largest distance, so their correlation is the smallest.

TABLE V  
ACTIVE ANTENNA INDEX AND CONSTELLATION CARRIED BY EACH ACTIVE ANTENNA

Scenario 1	Antenna 1(64QAM), Antenna 6(16QAM), Antenna 3.(4QAM)
Scenario 2	Antenna 1(16QAM), Antenna 6(16QAM), Antenna 3(16QAM)
Scenario 3	Antenna 1(256QAM), Antenna 6(4QAM)

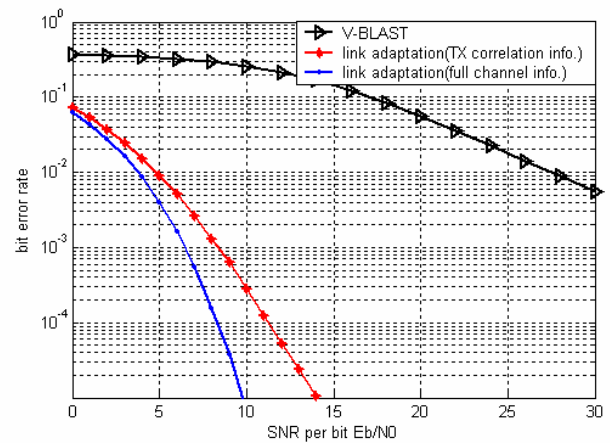


Fig.5. (a) Joint antenna selection and link adaptation for fading correlation scenario 1.

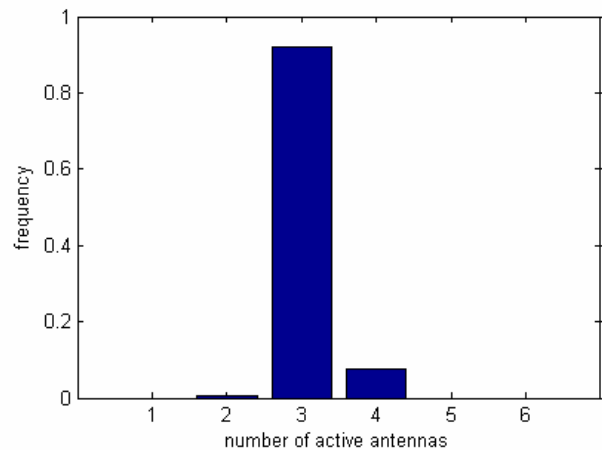


Fig.5. (b)Histogram of the number of active antennas for fading correlation scenario 1.

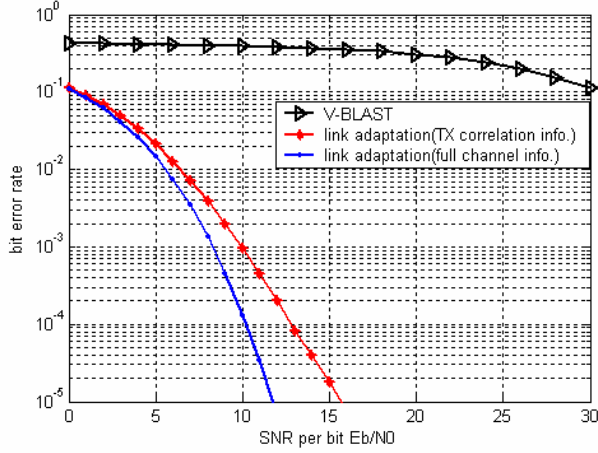


Fig.6. (a) Joint antenna selection and link adaptation for fading correlation scenario 2.

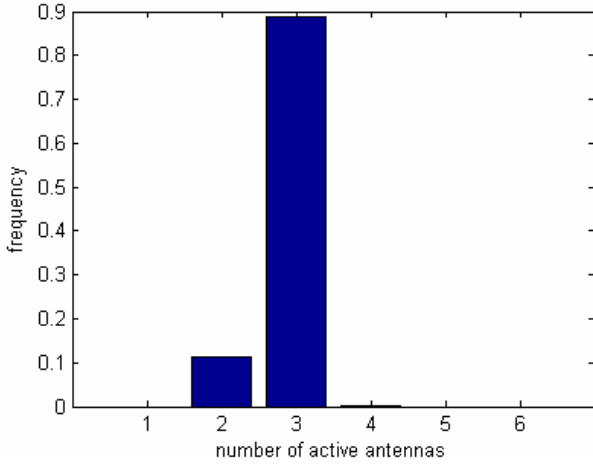


Fig.6. (b)Histogram of the number of active antennas for fading correlation scenario 2.

I. CONCLUSION

In this paper, we propose joint antenna selection and link adaptation algorithms for both uncorrelated and correlated MIMO channels. Simulation results show that in most situations, significant performance gains are achieved compared with traditional equal power and equal rate V-BLAST. We also propose a simplified link adaptation algorithm based on the estimation of optimal number of active transmit antennas for Rayleigh i.i.d. MIMO channels. For correlated MIMO, we propose a link adaptation algorithm only based on channel correlation information, which is more practical in realization than that based on the instantaneous channel information, while approaching the latter in performance as the fading correlation increases. Finally, our antenna selection and link adaptation algorithms can be readily extended to other antenna selection applications, such as capacity maximization for both uncorrelated and correlated MIMO systems.

Future research topics include more detailed computational complexity analysis of the “incremental” and “decremental” selection rules. The theoretical study on the performance gap between our proposed methods and the optimal one is also of

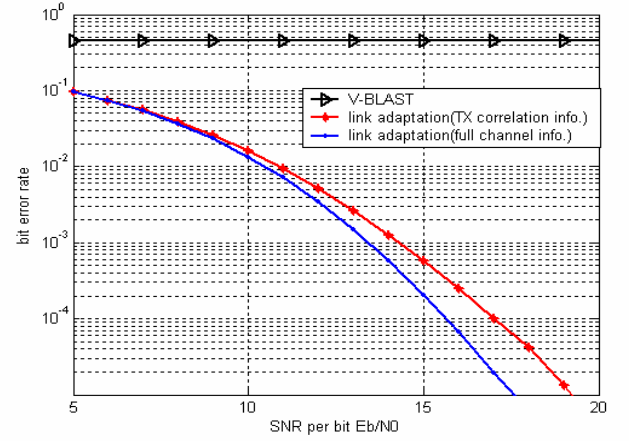


Fig.7. (a) Joint antenna selection and link adaptation for fading correlation scenario 3.

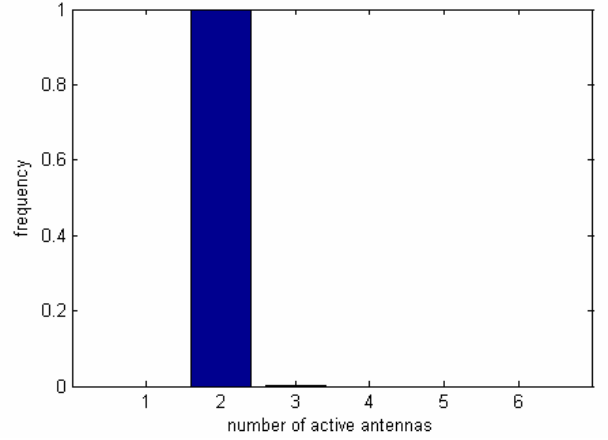


Fig.7. (b) Histogram of the number of active antennas for fading correlation scenario 3.

our interest

APPENDIX

**Lemma1.** For two ordered real sequence  $\{a_i\}_{i=1}^{i=n}$  and  $\{b_i\}_{i=1}^{i=n}$  such that  $a_1 \leq a_2 \leq a_3 \leq \dots \leq a_n$  and  $b_1 \leq b_2 \leq b_3 \leq \dots \leq b_n$ , if  $c_1, c_2, \dots, c_n$  is any permutation of  $b_1, b_2, \dots, b_n$ , then  $\sum_i a_i b_i \geq \sum_i a_i c_i \geq \sum_i a_i b_{n-i+1}$ .

Proof: for any ordered multiplication, if  $i > j$ ,  $c_i \geq c_j$ , consider

$$\begin{aligned} T &= a_1 c_1 + a_2 c_2 + \dots + a_j c_i + \dots + a_i c_j + \dots + a_n c_n, \\ S &= a_1 c_1 + a_2 c_2 + \dots + a_j c_j + \dots + a_i c_i + \dots + a_n c_n, \\ S - T &= a_i c_i + a_j c_j - a_i c_j - a_j c_i = (a_i - a_j)(c_i - c_j) \geq 0. \end{aligned}$$

By induction, we can see if we sort  $b_i$  in an ascending order, the corresponding summation will be maximized.

**Lemma2.a** Proof: From [11], we can see that matrix QR decomposition is related to Gram-Schmidt orthogonalization, hence it is not difficult to see that  $\mathbf{R}(k+1)$  shares the same first  $k$  diagonal elements with  $\mathbf{R}(k)$  and  $\mathbf{Q}(k+1)$  shares the same first  $k$  columns with  $\mathbf{Q}(k)$  while the  $(k+1)$ -th one given by

$$\mathbf{Q}(:, k+1) = \mathbf{h} - \sum_{l=1}^k \mathbf{Q}(:, l)^H \mathbf{h} \mathbf{Q}(:, l).$$

Assume the QR decomposition of  $\mathbf{H}^{(k)}$  is  $\mathbf{Q}(k)\mathbf{R}(k)$ , then

$$\det((\mathbf{H}^{(k)})^H \mathbf{H}^{(k)}) = \det(\mathbf{R}(k)^H \mathbf{R}(k)) = \prod_{i=1}^k |r_{i,i}|^2. \quad (23)$$

In the  $(k+1)$ -th step, assume the QR decomposition of  $\mathbf{H}^{(k+1)}$  is  $\mathbf{Q}(k+1)\mathbf{R}(k+1)$ , then,

$$\begin{aligned} \det((\mathbf{H}^{(k+1)})^H \mathbf{H}^{(k+1)}) &= \det \begin{bmatrix} (\mathbf{H}^{(k)})^H \mathbf{H}^{(k)} & \mathbf{H}^{(k)} \mathbf{h} \\ \mathbf{h}^H \mathbf{H}^{(k)} & \mathbf{h}^H \mathbf{h} \end{bmatrix} \\ (24) \quad &= \det \begin{bmatrix} (\mathbf{H}^{(k)})^H \mathbf{H}^{(k)} & \mathbf{H}^{(k)} \mathbf{h} \\ \mathbf{0} & \mathbf{h}^H \mathbf{h} - \mathbf{h}^H \mathbf{H}^{(k)} ((\mathbf{H}^{(k)})^H \mathbf{H}^{(k)})^{-1} (\mathbf{H}^{(k)})^H \mathbf{h} \end{bmatrix} \\ &= (\mathbf{h}^H \mathbf{h} - \mathbf{h}^H \mathbf{H}^{(k)} ((\mathbf{H}^{(k)})^H \mathbf{H}^{(k)})^{-1} (\mathbf{H}^{(k)})^H \mathbf{h}) \det((\mathbf{H}^{(k)})^H \mathbf{H}^{(k)}) \\ &= \det(\mathbf{R}(k+1)^H \mathbf{R}(k+1)) \end{aligned}$$

hence

$$\begin{aligned} &\frac{\det(\mathbf{R}(k+1)^H \mathbf{R}(k+1))}{\det(\mathbf{R}(k)^H \mathbf{R}(k))} \\ &= \mathbf{h}^H \mathbf{h} - \mathbf{h}^H \mathbf{H}^{(k)} ((\mathbf{H}^{(k)})^H \mathbf{H}^{(k)})^{-1} (\mathbf{H}^{(k)})^H \mathbf{h} \quad (25) \\ &= \mathbf{h}^H \mathbf{h} - \mathbf{h}^H \mathbf{Q}(k) \mathbf{Q}(k)^H \mathbf{h} \end{aligned}$$

Hence the amplitude of the  $(k+1)$ -th diagonal element of  $\mathbf{R}(k+1)$  is given by (25), which is

$$|r_{k+1,k+1}| = \sqrt{\mathbf{h}^H \mathbf{h} - \mathbf{h}^H \mathbf{Q}(k) \mathbf{Q}(k)^H \mathbf{h}}. \quad (26)$$

**Lemma2.b** Proof: In the  $(k+1)$ -th step, assume column  $\mathbf{h}(k+1)$  is selected. Partition  $\mathbf{Q}(k)$  as  $\mathbf{Q}(k) = [\mathbf{Q}(k-1) \ \mathbf{q}(k)]$ , where  $\mathbf{q}(k)$  is the rightmost column of  $\mathbf{Q}(k)$ . From (26), we get

$$\begin{aligned} |r_{k+1,k+1}|^2 &= \mathbf{h}^H (k+1) \mathbf{h}(k+1) - \mathbf{h}^H (k+1) \mathbf{Q}(k) \mathbf{Q}(k)^H (k) \mathbf{h}(k+1) \\ &= \mathbf{h}^H (k+1) \mathbf{h}(k+1) - \\ &\quad \mathbf{h}^H (k+1) \mathbf{Q}(k-1) \mathbf{Q}^H (k-1) \mathbf{h}(k+1) - |\mathbf{h}^H (k+1) \mathbf{q}(k)|^2 \\ &\leq \mathbf{h}^H (k+1) \mathbf{h}(k+1) - \mathbf{h}^H (k+1) \mathbf{Q}(k-1) \mathbf{Q}^H (k-1) \mathbf{h}(k+1) \end{aligned} \quad (27)$$

According to our selection criterion, we know in the  $k$ -th step,  $\mathbf{h}(k)$  is the selected column vector in the remaining of  $\mathbf{H}$  such that  $f(\mathbf{x}) = \mathbf{x}^H \mathbf{x} - \mathbf{x}^H \mathbf{Q}(k-1) \mathbf{Q}^H (k-1) \mathbf{x}$  is maximized. Thus,

$$\begin{aligned} |r_{k,k}|^2 &= \mathbf{h}^H (k) \mathbf{h}(k) - \mathbf{h}^H (k) \mathbf{Q}(k-1) \mathbf{Q}^H (k-1) \mathbf{h}(k) \\ &= \max f(\mathbf{h}(k)) \geq f(\mathbf{h}(k+1)) \\ &= \mathbf{h}^H (k+1) \mathbf{h}(k+1) - \mathbf{h}^H (k+1) \mathbf{Q}(k-1) \mathbf{Q}^H (k-1) \mathbf{h}(k+1) \geq |r_{k+1,k+1}|^2 \end{aligned} \quad (28)$$

**Lemma 3.a** Proof: The Cholesky factor of  $\mathbf{R}_T^{(k+1)} = \begin{bmatrix} \mathbf{R}_T^{(k)} & \mathbf{v} \\ \mathbf{v}^H & 1 \end{bmatrix}$

can be partitioned as [11]

$$\mathbf{R}(k+1) = \begin{bmatrix} \mathbf{R}(k) & \boldsymbol{\eta} \\ \mathbf{0}^H & r_{k+1,k+1} \end{bmatrix}, \quad (29)$$

where  $\mathbf{R}(k)$  is the Cholesky factor of  $\mathbf{R}_T^{(k)}$ ,  $\boldsymbol{\eta}$  is a  $k \times 1$  vector,  $\mathbf{0}$  is a  $k \times 1$  vector comprising all zeros elements, and  $r_{k+1,k+1}$  is the scalar we are interested in. Writing out the Cholesky decomposition of  $\mathbf{R}_T^{(k+1)}$ :

$$\mathbf{R}_T^{(k+1)} = \begin{bmatrix} \mathbf{R}(k)^H \mathbf{R}(k) & \mathbf{R}(k)^H \boldsymbol{\eta} \\ \boldsymbol{\eta}^H \mathbf{R}(k) & r_{k+1,k+1}^2 + \boldsymbol{\eta}^H \boldsymbol{\eta} \end{bmatrix}, \quad (30)$$

$$\text{we have } \begin{cases} \mathbf{R}(k)^H \boldsymbol{\eta} = \mathbf{v} \\ r_{k+1,k+1}^2 + \boldsymbol{\eta}^H \boldsymbol{\eta} = 1 \end{cases}. \quad (31)$$

From (31), we can get

$$r_{k+1,k+1} = \sqrt{1 - \boldsymbol{\eta}^H \boldsymbol{\eta}} = \sqrt{1 - \mathbf{v}^H (\mathbf{R}_T^{(k)})^{-1} \mathbf{v}}. \quad (32)$$

**Lemma3.b** Proof: Assume  $\mathbf{R}_T^{(k-1)}$ ,  $\mathbf{R}_T^{(k)}$  and  $\mathbf{R}_T^{(k+1)}$  are the covariance matrix for the  $k-1$ ,  $k$  and  $k+1$  selected antennas respectively. Also we assume  $\mathbf{R}(k-1)$  and  $\mathbf{R}(k)$  are the Cholesky factors of  $\mathbf{R}_T^{(k-1)}$ ,  $\mathbf{R}_T^{(k)}$  respectively. According to our recursive selection rule,  $\mathbf{R}_T^{(k+1)}$  is an ‘‘enhanced matrix’’ based on  $\mathbf{R}_T^{(k)}$  and  $\mathbf{R}_T^{(k)}$  is an ‘‘enhanced matrix’’ based on  $\mathbf{R}_T^{(k-1)}$ .

$$\text{Assume } \mathbf{R}_T^{(k)} = \begin{bmatrix} \mathbf{R}_T^{(k-1)} & \mathbf{v}_{k-1} \\ \mathbf{v}_{k-1}^H & 1 \end{bmatrix}, \text{ where } \mathbf{v}_{k-1} \text{ is the } (k-1) \times 1$$

covariance vector between the  $k$ -th selected antenna and the previous  $(k-1)$  selected antennas. Then according to (32),

$$\begin{aligned} r_{k,k} &= \sqrt{1 - \mathbf{v}_{k-1}^H (\mathbf{R}_T^{(k-1)})^{-1} \mathbf{v}_{k-1}} \\ &= \sqrt{1 - \mathbf{v}_{k-1}^H \mathbf{R}(k-1)^{-1} (\mathbf{R}(k-1)^{-1})^H \mathbf{v}_{k-1}} \end{aligned} \quad (33)$$

Similarly,

$$r_{k+1,k+1} = \sqrt{1 - \mathbf{v}_k^H \mathbf{R}(k)^{-1} (\mathbf{R}(k)^{-1})^H \mathbf{v}_k}. \quad (34)$$

$$\text{Note } \mathbf{R}(k)^{-1} = \begin{bmatrix} \mathbf{R}(k-1)^{-1} & \times \\ \mathbf{0}^H & \times \end{bmatrix}, \text{ where ‘‘}\times\text{’’ denotes}$$

irrelevant entries. Assuming the rightmost column of  $\mathbf{R}(k)^{-1}$  is  $\mathbf{b}_k$ , then

$$\begin{aligned} r_{k+1,k+1} &= \sqrt{1 - \mathbf{v}_k^H \mathbf{R}(k)^{-1} (\mathbf{R}(k)^{-1})^H \mathbf{v}_k} \\ &= \sqrt{1 - \mathbf{v}_k^H \begin{bmatrix} \mathbf{R}(k-1)^{-1} \\ \mathbf{0}^H \end{bmatrix} \begin{bmatrix} (\mathbf{R}(k-1)^{-1})^H & \mathbf{0} \end{bmatrix} \mathbf{v}_k - |\mathbf{v}_k^H \mathbf{b}_k|^2} \quad (35) \\ &\leq \sqrt{1 - \mathbf{v}_k^H \begin{bmatrix} \mathbf{R}(k-1)^{-1} \\ \mathbf{0}^H \end{bmatrix} \begin{bmatrix} (\mathbf{R}(k-1)^{-1})^H & \mathbf{0} \end{bmatrix} \mathbf{v}_k} \end{aligned}$$

Assuming  $\tilde{\mathbf{v}}_k$  is a  $(k-1) \times 1$  vector truncated from  $\mathbf{v}_k$  by discarding the last element of  $\mathbf{v}_k$ . Thus (35)

becomes  $r_{k+1,k+1} \leq \sqrt{1 - \tilde{\mathbf{v}}_k^H (\mathbf{R}_T^{(k-1)})^{-1} \tilde{\mathbf{v}}_k}$ . According to our selection rule, in the  $k$ -th step, we choose the  $(k-1) \times 1$  vector  $\mathbf{v}_{k-1}$  such that  $r_{k,k} = \sqrt{1 - \mathbf{v}_{k-1}^H (\mathbf{R}_T^{(k-1)})^{-1} \mathbf{v}_{k-1}}$  is

maximized, hence  $r_{k+1,k+1} \leq \sqrt{1 - \tilde{\mathbf{v}}_k^H ((\mathbf{R}_T^{(k-1)})^{-1} \tilde{\mathbf{v}}_k)} \leq r_{k,k}$ .

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