Crosstalk Mitigation in DMT VDSL with Impulse Noise[*]

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Abstract:

Crosstalk and impulse noise are two principal sources of degradation in very-high-rate digital subscriber line (VDSL) transmission systems. The traditional single-user data detector for such systems merges crosstalk into the background noise, which is assumed to be white and Gaussian. Recent research has explored the nature of crosstalk signals and shown the potential benefits of multiuser detection for VDSL signals with strong crosstalkers. Impulse noise is one of the most difficult transmission impairments to suppress and is poorly characterized and understood as well. In DSL transmission impulse noise is typically combated with interleaved forward error correction. However, recent data indicates that a significant minority of impulse noise events are longer than the maximum error correcting capacities of the default interleaved forward error correction (FEC) provided within current ANSI standards. Thus, it is of interest to consider signal processing methods that can jointly mitigate crosstalk and impulsive noise. In this paper, we explore such a technique based on a recently developed robust M-detector structure for multiuser detection in non-Gaussian ambient noise.

Index Terms

Crosstalk, DMT, DSL, Impulse noise, M-estimation, Multiuser detection

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1. Introduction[*]

Digital subscriber line (DSL) technology provides transport of high-bit-rate digital information over telephone subscriber lines. Phone lines, which were originally constructed to carry a single voice signal with a 3.4 kHz bandwidth channel, are actually capable of carrying very high data rates if the narrowband switch in the phone company central office can be avoided. Various DSL techniques (basic rate integrated services digital networks (ISDN), high-bit-rate DSL (HDSL), Asymmetric DSL (ADSL), and very-high-rate DSL (VDSL)) which involve sophisticated digital transmission schemes and extensive signal processing have recently become practical due to advances in microelectronics. The latest in DSL technology is VDSL, which provides tens of megabits per second to those customers who desire broadband entertainment or data services. Asymmetric VDSL is viewed more as a residential service, supporting up to 52 Mb/s downstream and 6.4 Mb/s upstream rates for delivery of digital TV and high definition TV (HDTV) services. Symmetric application of VDSL provides two-way data rates up to 26Mb/s for data network or local area network (LAN) extension, mainly as a business service. At such high rates, signals on twisted pairs can be reliably transmitted at most to a few thousand feet. Thus, VDSL will primarily be used for loops fed from an optical network unit (ONU) or a central office (CO) to a customer's premises, the so-called "last mile" problem. The modulation scheme for VDSL can either be multicarrier-based or single carrier-based, typically discrete multitone (DMT) and carrierless amplitude/phase modulation (CAP)/quadrature amplitude modulation (QAM). The duplexing methods can be either time-division duplexing (TDD) or frequency-division duplexing (FDD).

Typical phone lines that carry VDSL signals are 24- or 26-gauge unshielded twisted pairs (UTP). Multiple telephone pairs may share the same cable. Normally VDSL signals occupy from 300 kHz to 30 MHz of the twisted-pair bandwidth and are separated from plain old telephone system (POTS)/ISDN signals by splitter devices. Noise on phone lines normally occurs because of imperfect balance of the twisted pair. There are many types of noises that couple through imperfect balance into phone lines, the most common of which

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are crosstalk noise, radio noise and impulse noise. Crosstalk is caused by electromagnetic radiation of other phone lines in close proximity, in practice within the same cable. Such coupling increases with frequency and can be caused by signals traveling in the opposite direction, called near-end crosstalk (NEXT), and by signals traveling in the same direction, called far-end crosstalk (FEXT). Radio noise is the remnant of wireless transmission signals coupling into phone lines, particularly AM radio broadcasts and amateur (HAM) operator transmissions. Impulse noise is a nonstationary crosstalk from temporary electromagnetic events (such as the ringing of phones on lines sharing the same binder, and atmospheric electrical surges) that can be narrowband or wideband and that occurs randomly. Impulse noises can be tens of millivolts in amplitude and can last as long as hundreds of microseconds [6], [16].

While the radio noise problem can be solved or at least alleviated by restricting the VDSL transmission in radio bands, crosstalk and impulse noise are two principal sources of degradation in VDSL transmission systems. The traditional single-user detector (SUD) for such systems merges crosstalk into the background noise, which is assumed to be white and Gaussian. Actually, crosstalk is the result of the sum of several filtered discrete data signals. Its distribution deviates from Gaussian, and its power spectral density (PSD) is also significantly greater than that of background added white Gaussian noise (AWGN). Recent research has explored the nature of crosstalk signals and has shown the potential benefits of multiuser detection for VDSL signals with strong crosstalkers [2], [3], [4]. In DSL transmission impulse noise is typically combated with interleaved forward error correction (FEC). However, recent data indicates that a significant minority of impulse noise events are longer than the maximum error correcting capacities of the default interleaved FEC provided within current ANSI standards [9], [10]. Thus, it is of interest to consider signal processing methods that can jointly mitigate crosstalk and impulsive noise. Recent work has examined multiuser detection (MUD) in non-Gaussian ambient environments for wireless code-division multiple-access (CDMA) systems [22]. In particular, this work has shown that standard linear multiuser detectors are not robust to certain types of non-Gaussian ambient noise (particularly impulsive noise), whereas low-complexity nonlinear modifications provide excellent performance in such environments. It is the purpose of this paper to examine similar techniques for crosstalk and impulse noise mitigation in DMT VDSL systems. Note that the crosstalk signals in DSL transmission are of various types and cannot be represented under a uniform framework, to the best of the authors' knowledge. In our application of MUD to signal detection in DSL systems, we deal mainly with NEXT of other types (in contrast to the self NEXT coming from the phone lines carrying the same VDSL service). The reason is given as follows. FEXT experiences the same line attenuation as the desired signal while NEXT does not, which makes NEXT the most detrimental type of interference, especially at high frequencies. Self NEXT can be largely alleviated by duplexing methods that separate the upstream and downstream data in time or frequency. Therefore, the other-type NEXT provides the best opportunity for performance gain. However, the idea of multiuser detection is valid and applicable to mitigation of crosstalk of all types, although modifications of the techniques proposed here may be necessary for each specific situation. Note that we do not consider coding in this paper, but this issue is treated in a sequel [8].

This paper is organized as follows. In Section 2 a signal model for the DMT VDSL communication system, as well as the impulse channel noise model, is described. In Section 3 we propose a robust MUD-based scheme for DMT VDSL signal detection. In order to reduce the receiver complexity while maintaining good performance, a suboptimum receiver is introduced in Section 4, together with its robust version. Simulation results are given in Section 5, and Section 6 concludes the paper.

2. System Model

Figure 1 depicts a basic crosstalking channel with one desired VDSL signal and K-1 crosstalkers. The loop transfer function H of the desired VDSL signal and the crosstalk coupling functions are assumed to be known. At the channel output background noise is added, a model for which will be introduced shortly.

The VDSL signal studied here uses a DMT transmission system, whose transmitter and receiver are depicted in Fig. 2 and Fig. 3, respectively. The typical twisted pair is an intersymbol-interference (ISI) channel. However, if the number of subchannels is large enough, the continuous transfer function of the channel response can be approximated by discrete subchannel gains, as illustrated in Fig. 4. Then we can effectively decompose the original channel into a set of $\overline{N}$ parallel independent channels with no ISI. For each subchannel in the frequency domain, the output is given by

$$Y_n = H_n \cdot X_n + \sum_{k=2}^{K} C_{n,k} + N_n \quad , \quad n = 1,\ldots,\overline{N} \,, \tag{1}$$

where H_n is the channel gain, X_n is the transmitted (complex) DMT symbol, $C_{n,k}$ is the interference from the *k*th crosstalker, $k = 2$, …, K, and N_n is the background noise at the nth subchannel [5].

Impulse noise is a severe impairment to DSL transmission, especially after long loop attenuation (at a residential location) and in high frequencies (where the DSL signal is more severely attenuated). However, the area of impulse noise modeling remains unsettled. Cook presented an analytical model in [7]. The ADSL standard, however, uses stored representative impulse waveforms, which are measured empirically. Valenti *et al* collected impulse noise and background noise data on ADSL loops at New Jersey residences and did analysis on the data in three ways: as power and energy spectral densities, as probability density functions of the time waveform voltage amplitudes, and as impulse arrival and interarrival time statistics [12], [18], [19]. So far there are no such models for impulse noise in VDSL, but similar results are anticipated [20]. Our key observation from these analyses is: there are significant impulse spikes in the PSD of the measured wideband noise, which is otherwise essentially flat. To model this behavior of the impulse noise we use a two-term Gaussian mixture model in the frequency domain. The first-order probability density function of this noise model has the form

$$(1-\varepsilon)\mathcal{N}(0,\sigma^2) + \varepsilon\mathcal{N}(0,\kappa\sigma^2) \tag{2}$$

with $\sigma > 0$, $0 \le \varepsilon \le 1$, and $\kappa \ge 1$. Here, the $\mathcal{N}(0,\sigma^2)$ term represents the nominal background noise (Gaussian with zero mean and variance σ^2), and the $\mathcal{N}(0,\kappa\sigma^2)$ term represents an impulse component (Gaussian with zero mean and variance $\kappa\sigma^2$), with ε representing the probability that impulses occur [22]. It is assumed that noise samples in disjoint frequency bins are independent.

3. Robust Maximum Likelihood Multiuser Detection Receiver

As we mentioned in Section 1, it is possible to apply multiuser detection to jointly detect the VDSL signal and the crosstalk signals and thus greatly improve the system performance. For simplicity let us assume the background noise to be Gaussian (i.e. (2) with $\varepsilon = 0$) for the moment. We will reintroduce the impulse noise model below. According to the system model given in Fig. 1, the optimal maximum likelihood multiuser detector (ML-MUD) for Gaussian noise is one that estimates the VDSL input and crosstalker inputs in unison so as to minimize the distance between the channel output received signal and all the possible discrete waveform outcomes. It is possible that the crosstalk signals are wrongly estimated, but the probability of erroneous selection of the desired VDSL signal will be less for such a detector than when the crosstalk signals are simply assumed to be Gaussian noise. We will expect a greater improvement in performance if the difference between the PSD level of crosstalk signals and background noise is larger. Generally speaking, crosstalk strength increases with frequency: NEXT with $f^{1.5}$ and FEXT with f^{2}. Fortunately, FEXT experiences the same line attenuation as the desired signal; but unfortunately, NEXT does not. For VDSL systems, high-frequency NEXT is the most detrimental type of crosstalk, but will also be most promising for reduction via MUD. The typical background noise level of VDSL transmission is –140dbm, while the typical NEXT is –90~–110dbm; thus we can expect substantial gain from multiuser detection relative to traditional single user detection in this situation. Besides, in DMT VDSL subchannels where there are substantially stronger crosstalk signals (typically in high frequency bands on long loops), the so-called "near-far" problem in wireless CDMA systems, SUD will fail to work properly while optimal MUD should essentially achieve the single user lower bound.

Let us consider the detection problem for the data model given in (1). The traditional single user detector performs QAM demodulation and detection. On the other hand, joint maximum-likelihood detection of both VDSL and crosstalk signals selects a set of $\overline{N}$ inputs $\{X_n\}$ and the crosstalk sequence $\mathbf{C}_k^{(i)} = \{C_{1,k}{}^{(i)}, C_{2,k}{}^{(i)}, \ldots, C_{\overline{N},k}{}^{(i)}\}$, $k = 2, \ldots, K$, to satisfy

$$X_n = \arg\{ \min_{\{X_n\},\{C_{n,k}{}^{(i)}\}} \sum_{n=1}^{\overline{N}} \left| Y_n - H_n \cdot X_n - \sum_{k=2}^{K} C_{n,k}{}^{(i)} \right|^2 \}, \quad n = 1, \ldots, \overline{N}, \tag{3}$$

where the minimization is searched over the DMT signal alphabet and all possible crosstalk sequences $\mathbf{C}_k = \{\mathbf{C}_k^{(i)}, i = 1,...,|\mathbf{C}_k|\},\ k = 2,\ldots,K,$ that can occur within the VDSL symbol period of interest. The size $|\mathbf{C}_k|,\ k = 2,\ldots,K,$ of the all possible crosstalk sequences set can be large but is always finite when all the crosstalkers are digital signals or are derived from digital signals.

For white Gaussian noise, maximum likelihood detection is the same as least-squares (LS) curve fitting, as can be seen from (3). It is well known from the classic work of Tukey [17] that least-squares estimates are very sensitive to the tail behavior of the probability density of measurement errors (represented here by the additive noise). Its performance depends significantly on the Gaussian assumption, and even a slight deviation of the noise density from the Gaussian distribution can, in principle, cause a substantial degradation of the LS estimate. The LS estimate corresponding to (3) can be robustified by using the class of M-estimators proposed by Huber [11]. Instead of minimizing over a sum of squared residuals, Huber proposed to use a less rapidly increasing penalty function ρ so as to alleviate the effect of the impulses.

$$X_n = \arg\{\min_{\{X_n\},\{C_{n,k}{}^{(i)}\}} \sum_{n=1}^{\overline{N}} \rho(\left|Y_n - H_n \cdot X_n - \sum_{k=2}^{K} C_{n,k}{}^{(i)}\right|)\}, \quad n = 1,...,\overline{N}\,. \tag{4}$$

The usual requirements for the penalty function and its derivative $\psi = \rho'$ are:

(1) ρ is sub-quadratic function for large values of residuals, in order to de-emphasize the error caused by noise "outliers" (in this case caused by impulse noise);

(2) ψ is bounded and continuous;

(3) $\psi(x) \approx kx$ for small x, so as to achieve high efficiency in the Gaussian case;

(4) $E\{\psi(N_j)\} = 0$ to get a consistent estimate; and for symmetric noise densities ψ is usually odd symmetric.

A good choice for Gaussian mixture noise is the Huber penalty ρ shown in Fig. 5 together with its derivative ψ. These functions are given explicitly by

$$\rho(x) = \begin{cases} \dfrac{x^2}{2\sigma^2} & |x| \le k\sigma^2 \\ k|x| - \dfrac{k^2\sigma^2}{2} & |x| > k\sigma^2 \end{cases} \tag{5}$$

and

$$\psi(x) = \begin{cases} \dfrac{x}{\sigma^2} & |x| \le k\sigma^2 \\ k\,\mathrm{sgn}(x) & |x| > k\sigma \end{cases}, \tag{6}$$

where k, σ, and ε (see (2)) are related by

$$\frac{\phi(k\sigma)}{k\sigma} - Q(k\sigma) = \frac{\varepsilon}{2(1-\varepsilon)}, \tag{7}$$

with $\phi(x) \equiv \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ and $Q(t) \equiv \frac{1}{\sqrt{2\pi}} \int_t^\infty e^{-x^2/2}\,dx$ (see, [11], [22]).

In this paper, we will consider this particular choice of ρ, and the resulting DMT VDSL detector will be called the robust maximum likelihood multiuser detection receiver (ML-MUD-R). The performance of this detector will be compared with ML-MUD and SUD in Section 5.

4. An Interference Cancellation Multiuser Detector and Its Robust Version

Just as its counterpart in wireless CDMA, the maximum likelihood multiuser detector achieves optimum performance but suffers from very high complexity. A full search in the input domain requires approximately $\overline{N}$ |**C**||**M**| squared error computations, where $\overline{N}$ is the number of subchannels, |**C**| is the number of possible crosstalk sequences, and |**M**| is the average size of the transmitted alphabet. In practice $\overline{N}$ and especially |**C**| can be very large, introducing prohibitive computational complexity. The large number of possible crosstalk sequences also means an exponentially greater number of states, making dynamic programming inappropriate. So we need to consider a simplified receiver structure that maintains satisfactory performance while requiring far less computation complexity.

One lower-complexity approach is to employ a linear multiuser detection technique, such as decorrelating (zero forcing) or minimum-mean-square-error (MMSE) multiuser detection. However, unlike CDMA or space-division multiple-access (SDMA) where linear detection has been effective, there is no identifying signature such as the spreading code for CDMA or the steering vector for SDMA, to aid linear detection in VDSL. Instead, desired signals and crosstalk signals are often of different data format. Another popular approach is to employ interference cancellation, i.e., to attempt removal of the crosstalk from the received signal before making the traditional DMT VDSL signal detection [21]. This is the approach we adopt here. To do so, we need a scheme to detect the crosstalk signals first. As we mentioned before, the crosstalk signals in DSL transmission are of various types and cannot be represented under a uniform framework. The type we examine here is the dominant near-end QAM-like crosstalk (e. g. [3]).

At first glance, it seems quite difficult to detect the crosstalk correctly with reduced computational complexity. After all, it is the huge set of possible crosstalk sequences that complicates the computation in (3). Let us consider the power spectrum of the DMT VDSL signal and the crosstalk signals. As we mentioned before, each subchannel has independent transmitted data and added background noise, so the energy is fairly spread across the frequency domain of interest, although it is not equal everywhere since different bits may be assigned to different subchannels to achieve the optimum performance. In contrast, the PSD of the QAM-like crosstalk signals are often clustered around several relatively narrow spectral components (called "tones" in DMT modulation). A natural idea is to zero these tones in DMT-VDSL transmission, i.e., do not transmit DMT VDSL signals on these tones. This is a form of CDMA where the DMT VDSL signal is “orthogonal” to the crosstalk signals on these tones. Because the data rates of the crosstalk signals are usually low compared to the VDSL signal and the SNRs are excellent (the crosstalk signals are treated as the signals of interest here), only a few zeroed tones are necessary to detect the crosstalk signal fairly well. Thus the computation complexity is greatly reduced to $N_z|\mathbf{C}|$, where N_z is the number of zero tones (e.g., $N_z = 5$**)**, in addition to the nearly trivial conventional demodulation. The choice of tones to be zeroed depends on the knowledge of where the energy of crosstalk signals concentrates, which generally is known. Advances in digital signal processing make tone zeroing easy to implement. Furthermore, spectral compatibility with other DSL transmission and radio broadcast often necessitates some particular tones being zeroed. Finally, zeroing of these tones also leads to a reduction in FEXT on these tones.

Figure 6 gives the structure of this interference cancellation multiuser detector (IC-MUD). The detail of the IC-MUD algorithm is given as follows.

1. Choose the tones to be zeroed based on the knowledge of a specific crosstalk signal;
2. The crosstalk signal is detected and reconstructed in these DMT-symbol-free channels; e.g., for a home phone network of America (HPNA) signal (see [3]), it can be detected via

$$C_{n,k} = \arg\{\min_{\{C_{n,k}\}} \sum_{n \in T_z} \left| Y_n - \sum_{i<k} \hat{C}_{n,i} - C_{n,k} \right|^2 \}, \quad n = 1, \ldots, \overline{N}, \tag{8}$$

   where $\{\hat{C}_{n,i}\}$ are formerly detected crosstalk signals, and T_z is the set of tones being zeroed;
3. The reconstructed crosstalk is subtracted from received signal in all subchannels;
4. Repeat the above process until all crosstalk signals $\hat{\mathbf{C}} = \left\lfloor \hat{\mathbf{C}}_k, k = 2, \ldots, K \right\rfloor$ are estimated, reconstructed, and subtracted (different crosstalk signals may be detected through different methods according to their characteristics);
5. SUD is used for DMT signal detection,

$$X_n = \arg\{\min_{\{X_n\}} \left| Y_n - H_n \cdot X_n - \sum_{k=2}^{K} \hat{C}_{n,k} \right|^2 \}, \quad n = 1, \ldots, \overline{N} \text{ and } n \notin T_z. \tag{9}$$

If the Huber penalty function (5) is used in crosstalk signal decoding for combating the impulse noise, the resulting detector is called the robust interference cancellation multiuser detector (IC-MUD-R). I.e., instead of (8), this detector uses

$$C_{n,k} = \arg\{\min_{\{C_{n,k}\}} \sum_{n \in T_z} \rho(\left| Y_n - \sum_{i<k} \hat{C}_{n,i} - C_{n,k} \right|)\}, \quad n = 1, \ldots, \overline{N}. \tag{10}$$

The interference cancellation scheme is suboptimal since errors may arise in crosstalk detection. However, it is particularly suitable for high SNR channels with power imbalances. Another shortcoming for this suboptimum receiver is the capacity loss due to tone zeroing. Nonetheless, we will show in the following section that it provides a favorable tradeoff of performance and complexity.

5. Simulation Results

In this section we examine the behavior and the performance of the proposed multiuser detection receivers for DMT-VDSL signals with crosstalk and impulse noise via computer simulations. Bit-error-rate (BER) is adopted as the performance measure with respect to the average signal-to-noise ratio (SNR), which is defined as

$$\mathbf{SNR} = \frac{\sum_{n=1}^{\bar{N}} \left| H_n \cdot X_n \right|^2}{\sum_{n=1}^{\bar{N}} \left| N_n \right|^2}. \tag{11}$$

In the simulation, the DMT VDSL signal is assumed to occupy 0-25.6 MHz with 256 subchannels in an frequency-division multiplexed (FDM) design. The symbol rate for each VDSL subchannel is 100 kHz. In each subchannel, 2 bits are assigned so the signals are 4-QAM. No bit allocation algorithms are used here, although extension to this case is straightforward. The transfer function of the DMT VDSL signal is simulated by

$$H(e^{j\omega}) = \frac{2 - 4e^{-j\omega} - 2e^{-2j\omega}}{0.965 - 1.50e^{-j\omega} + 0.539e^{-2j\omega}} \times 10^{-3}. \tag{12}$$

We assume one NEXT crosstalk signal with a known coupling function given as

$$F(e^{j\omega}) = K \cdot \omega^{3/4}, \tag{13}$$

where K is a constant used to adjust the PSD of the crosstalk signal. These settings are made to roughly approach the PSD shapes indicated in [1]. We assume that these transfer functions stay fixed for the whole simulation interval, which is reasonable for wireline communications environments. The crosstalk signal is BPSK modulated on 8 MHz carrier frequency with a 1M symbol-per-second rate. Such a situation would arise, for example, due to the coexistence of home-phone LANs and asymmetric DMT VDSL signals in the same cable in the customer premises. Thus, there are 2^{10} possible crosstalk sequences in one VDSL symbol. This number is chosen for simulation simplicity. In reality, this number could be much larger. For IC-MUD and IC-MUD-R, the five zeroed tones are $T_z = \{7.8,\ 7.9,\ 8.0,\ 8.1,\ 8.2\}$ MHz, around which most of the crosstalk energy is concentrated. The impulse noise is assumed to have parameters $\varepsilon = 0.1$ and $\kappa = 100$, which means the impulse spike is 20 dB higher than the background noise floor. The average PSD levels of the crosstalk signal and background noise floor are fixed while that of the desired signal is varied, corresponding to different line length (the signal attenuation is increasing with the line length). In our simulation, the average PSD of the crosstalk is 27 dB higher than that of the background noise floor and the peak PSD of the crosstalk is 40 dB higher. These settings seem to agree roughly with empirical measurements.

In the DSL environment, BER values as low as 10^{-7} are often required. For Monte Carlo (MC) simulation, approximately $10/P_e$ simulation trials are required to have a 95 percent confidence interval of $[2P_e/5,\ \ 8P_e/5]$ [13]. To alieviate this computational burden, we use importance sampling (IS) [13], [14], [15]. The basic idea of importance sampling is to bias the probability density function (pdf) from which the data are generated so that errors of detection are more likely to happen, then weight each error such that an unbiased BER estimate is obtained. Assume an error occurs when the received data R falls within some region Z. Then the BER is given by

$$P_e = \int 1_Z(r) f_R(r) dr\,, \tag{14}$$

where $1_Z(\cdot)$ is the indicator function over Z and $f_R(\cdot)$ is the pdf of R.

The MC estimator of P_e is given by

$$\hat{P}_{MC} = \frac{1}{M}\sum_{i=1}^{M} 1_Z(R_i)\,, \tag{15}$$

where M is the number of trials of the simulation and the R_i 's denote data samples. When the data samples are independent and identical distributed (i.i.d.), $\hat{P}_{MC}$ is an unbiased estimator with variance

$$\mathrm{var}(\hat{P}_{MC}) = \frac{P_e(1-P_e)}{M}\,. \tag{16}$$

The IS estimator of P_e is given by

$$\hat{P}_{IS} = \frac{1}{M}\sum_{i=1}^{M} 1_Z(R_i^*) W(R_i^*) \tag{17}$$

with

$$W(r) = \frac{f_R(r)}{f_{R^*}(r)}, \tag{18}$$

where R_i^* is the ith data sample from biased density $f_{R^*}(\cdot)$ and $W(\cdot)$ is the weight function. If the new generated data are i.i.d., $\hat{P}_{IS}$ is an unbiased estimator with variance

$$\mathrm{var}(\hat{P}_{IS}) = \frac{\overline{W} - P_e^2}{M}, \tag{19}$$

where $\overline{W}$ is defined as

$$\overline{W} = \int_Z W(r) f_R(r) dr. \tag{20}$$

When $f_{R^*}(\cdot)$ is appropriately selected, the variance of the IS estimator will be far less than that of the MC estimator. Thus the number of trials needed for a given estimator variance is greatly reduced for the IS estimator compared to the MC estimator. The optimal bias distribution is given by

$$f_{R^*_{opt}}(r) = \frac{1_Z(r) f_R(r)}{P_e}, \tag{21}$$

which achieves zero estimation variance but is degenerative since it requires the knowledge of P_e. A widely used method of designing suboptimal $f_{R^*}(\cdot)$ is mean translation (MT). This class of biased density functions is of the form

$$f_{R^*}(r^*) = f_R(r^* + T), \tag{22}$$

where T is chosen to be the mode (at which maximum value of a pdf is achieved) of $f_{R^*_{opt}}(\cdot)$. For the multiuser communication system of (1), let $\underline{\rho} = (X, C_2, \ldots, C_K)$, impose the restriction $f_{\underline{\rho}^*}(\cdot) = f_{\underline{\rho}}(\cdot)$ and conditionally shift the mean of the noise

$$f_{N^*|\underline{\rho}^*}(n^*) = f_N(n^* + m(\underline{\rho}^*)) = f_N(n^* + H \cdot X^* + \sum_{k=2}^{K} C_k^*). \tag{23}$$

The IS estimator of BER is then given by

$$\hat{P}_{IS} = \frac{1}{M} \sum_{i=1}^{M} \mathrm{I}(|\hat{X}_i^* - X_i^*|) W(\underline{\rho}_i^*, N_i^*) = \frac{1}{M} \sum_{i=1}^{M} \mathrm{I}(|\hat{X}_i^* - X_i^*|) \frac{f_N(N_i^*)}{f_{N^*|\underline{\rho}^*}(N_i^*)}, \tag{24}$$

where we assume the independence of $\underline{\rho}$ and N, $\hat{X}_i^*$ is the detected data of X_i^* with the original decision rule, and

$$\mathrm{I}(x) = \begin{cases} 1 & x > 0 \\ 0 & x \le 0 \end{cases}. \tag{25}$$

When the near-far problem occurs, i.e.,

$$\mathrm{sgn}(m(\underline{\rho}^*)) = -\mathrm{sgn}(H \cdot X^*), \tag{26}$$

we need to adjust the IS error estimator as follows:

$$\hat{P}_{IS} = \frac{1}{M}\sum_{i=1}^{M}\left(\mathrm{I}(|\,\hat{X}_i^* - X_i^*\,|) + (1 - \mathrm{I}(|\,\hat{X}_i^* - X_i^*\,|))(1 - \frac{f_N(N_i^*)}{f_{N^*|\underline{\rho}^*}(N_i^*)}) \right). \tag{27}$$

Note that in this situation, the IS technique is used to count correct detections (which happen with small probability), which then gives (see (24))

$$\hat{P}_{IS-correct} = \frac{1}{M}\sum_{i=1}^{M}\mathrm{I}(|\,\hat{X}_i^* - X_i^*\,|)\frac{f_N(N_i^*)}{f_{N^*|\underline{\rho}^*}(N_i^*)}. \tag{28}$$

The quantity of (27) is then obtained through $\hat{P}_{IS} = 1 - \hat{P}_{IS-correct}$. In our simulations, the IS technique is uniformly better than the MC technique. It achieves great variance reduction for optimal detection (maximum likelihood) methods and also gets substantial gains for others.

Figure 7 shows the performance of various detectors for DMT VDSL systems with one crosstalker and impulse noise. As we can see, there is a significant gap between the performance of the traditional single user detector and the single user lower bound (corresponding to a crosstalk-free channel), indicating the ineffectiveness of the single-user detector. While the maximum likelihood multiuser detector essentially achieves the single user lower bound, it suffers from prohibitive complexity. The interference cancellation multiuser detector offers a favorable performance and complexity tradeoff compared with the single-user and ML multiuser detectors.

Figure 8 shows the performance of the M-estimator-based robust detectors in the crosstalk and impulse noise environment. While there is not much difference between the ML multiuser detector and its robust version, both of which approximate the single user lower bound, there is significant improvement for the robust interference cancellation multiuser detection compared with its Gaussian-based counterpart. The crosstalk detection errors are 3.42×10^{-4} for IC-MUD, 6.10×10^{-5} for IC-MUD-R and almost 0 for ML-MUD and ML-MUD-R. It should be noted that the expected improvement from using M-estimators is due to better estimation of the crosstalk signals in the DMT VDSL case. The desired DMT VDSL signals in different subchannels are independent while the crosstalk signals are correlated in the frequency domain, which means that M-estimators are especially applicable to impulse-noise-contaminated DMT VDSL systems with crosstalk signals strongly correlated in the frequency domain. However, more crosstalk errors do not necessarily mean worse performance, especially for the ML joint detection scheme. This is because the whole set of possible crosstalk sequences is usually divided into many small subsets. While the corresponding sequences of a subset can be largely different, their spectral components are similar. In fact, we found from our simulations that the IC scheme is much more sensitive to crosstalk detection errors than is the ML scheme. For example, if we lower the power of the crosstalk by 15 dB (which can be thought of as a FEXT) while keeping the other settings unchanged, the crosstalk detection errors are 1.63×10^{-1} for IC-MUD, 3.65×10^{-2} for IC-MUD-R, 3.91×10^{-2} for ML-MUD and 1.95×10^{-4} for ML-MUD-R. But ML-MUD still almost approaches the single user lower bound, which can be seen from Fig. 9. Since the crosstalk signals are estimated first in only a few "tone-free" subchannels for IC-MUD, more gain of IC-MUD-R over IC-MUD is achieved as compared with the gain of ML-MUD-R over ML-MUD.

Finally, Fig. 10 shows that, for the interference cancellation multiuser detector, strong crosstalk actually improves the situation. We lower the strength of the crosstalk 12dB and compare the performances of the traditional single user detector and robust interference cancellation multiuser detector applied to the two different crosstalk environments. The impulse noise settings remain unchanged. It is seen that IC-MUD-R performs better with the stronger crosstalk. This is no surprise, since for successive cancellation, strong interference is almost as good as no interference. These results also indicate that for crosstalk without significantly greater PSD level than that of the background noise, at high SNR, the IC scheme does not get much gain over SUD.

6. Conclusions

In this paper we have shown the potential benefits of multiuser detection for crosstalk mitigation in DMT VDSL systems subject to impulse noise. We see that ML-MUD can essentially eliminate crosstalk signals in DMT systems at a cost of high complexity. As a tradeoff, IC-MUD can significantly outperform SUD, with lower complexity than ML-MUD. We have also shown the effectiveness of the M-estimator in combating the impulse noise.

There are some issues overlooked in this paper, which might be of interest for further study. For example, we have assumed knowledge of the line transfer function and the crosstalk coupling functions. In reality, however, channel identification is needed. Also, we have not considered the issue of optimal bit allocation to subchannels with different SNRs. Finally, in our simulation, only one crosstalk signal is assumed. The treatment of multiple crosstalk signals follows straightforward, although higher complexity is inevitable.

In future work, we plan to study other crosstalk applications where multiuser detection techniques can be applied more directly (e.g. combating self-FEXT). We admit that in reality crosstalk signals vary widely in modulation formats and data rates and so far there is no uniform framework for mitigation of crosstalk in DSL. What we address here is the combating of a special class of crosstalk signal (QAM-modulated signals), but we believe that the general idea of multiuser detection is a promising technique for crosstalk mitigation in DSL. Also, iterative (Turbo style) joint decoding and multiuser detection (see [23]) is an attractive technique whose application on DSL is of interest.

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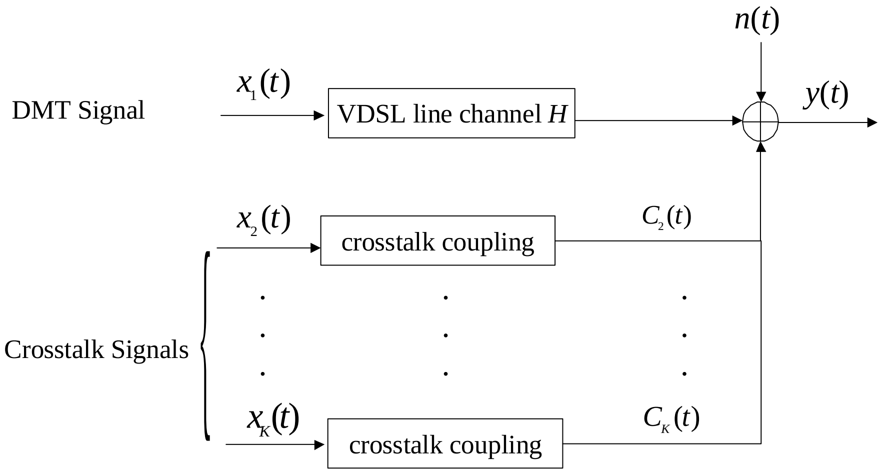

Fig. 1. System model for DMT VDSL.

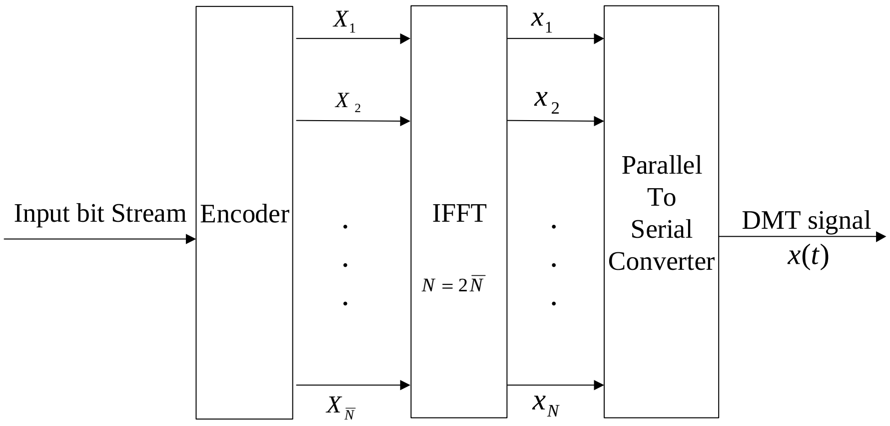

Fig. 2. DMT transmitter.

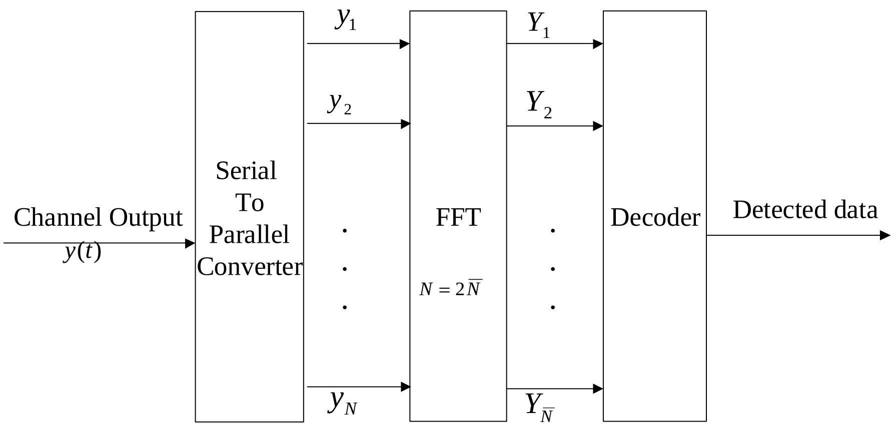

Fig. 3. DMT receiver.

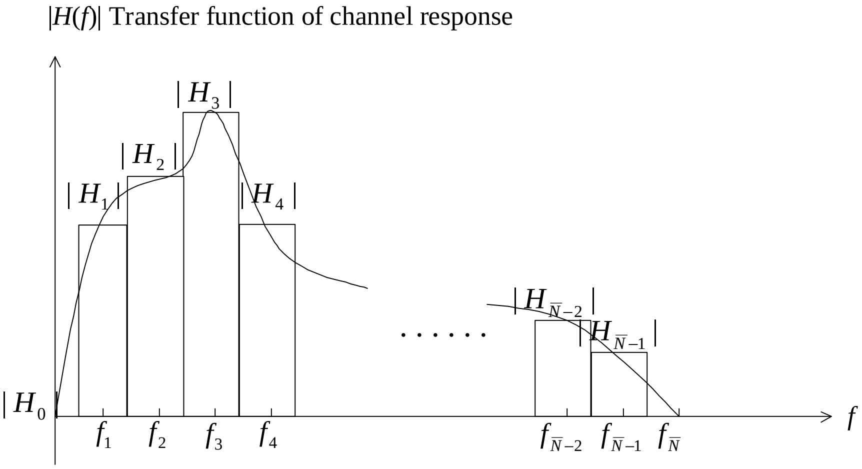

Fig. 4. Multichannel decomposition of a channel response.

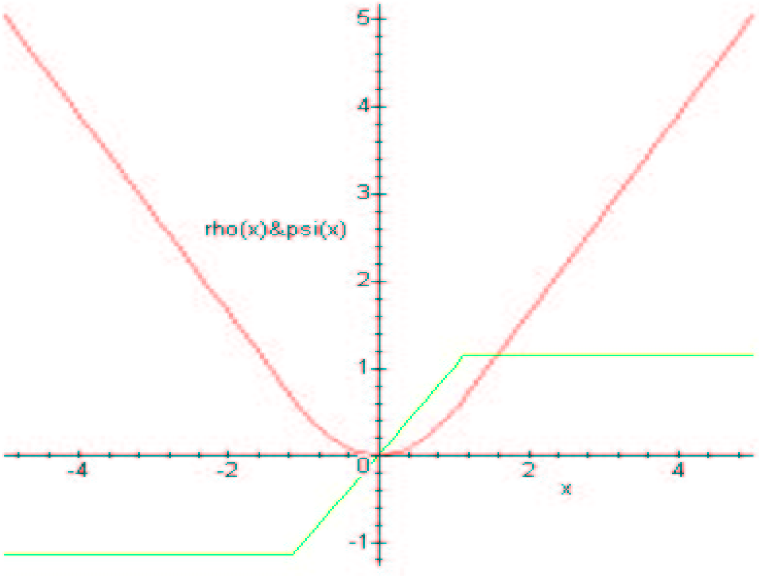

Fig. 5. Huber penalty function and its derivative for the Gaussian mixture model used in this paper. $\varepsilon = 0.1$, $\kappa = 100$, $\sigma^2 = 1$, $k = 1.14$.

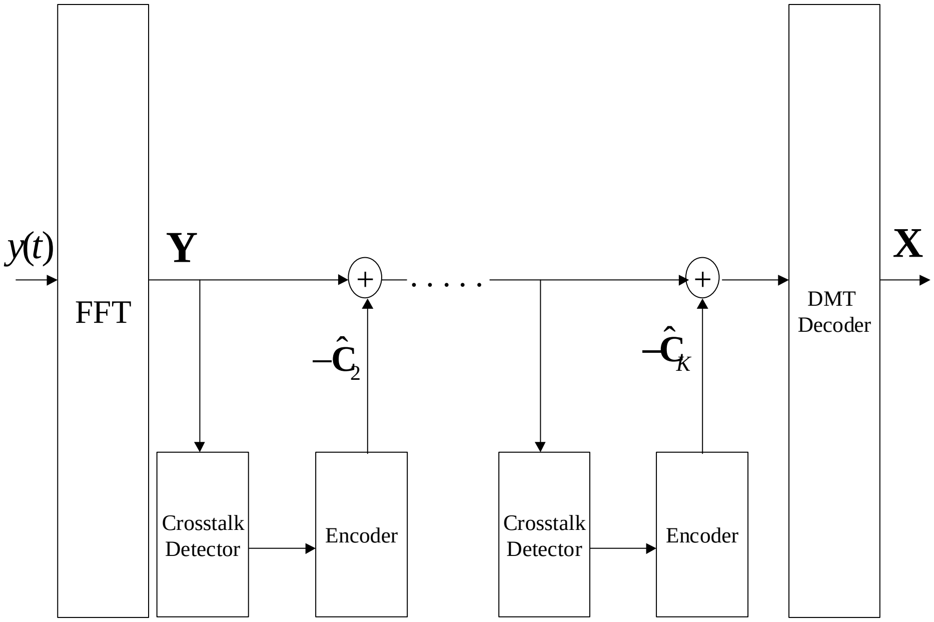

Fig. 6. Interference cancellation multiuser detector for DMT VDSL system with crosstalks.

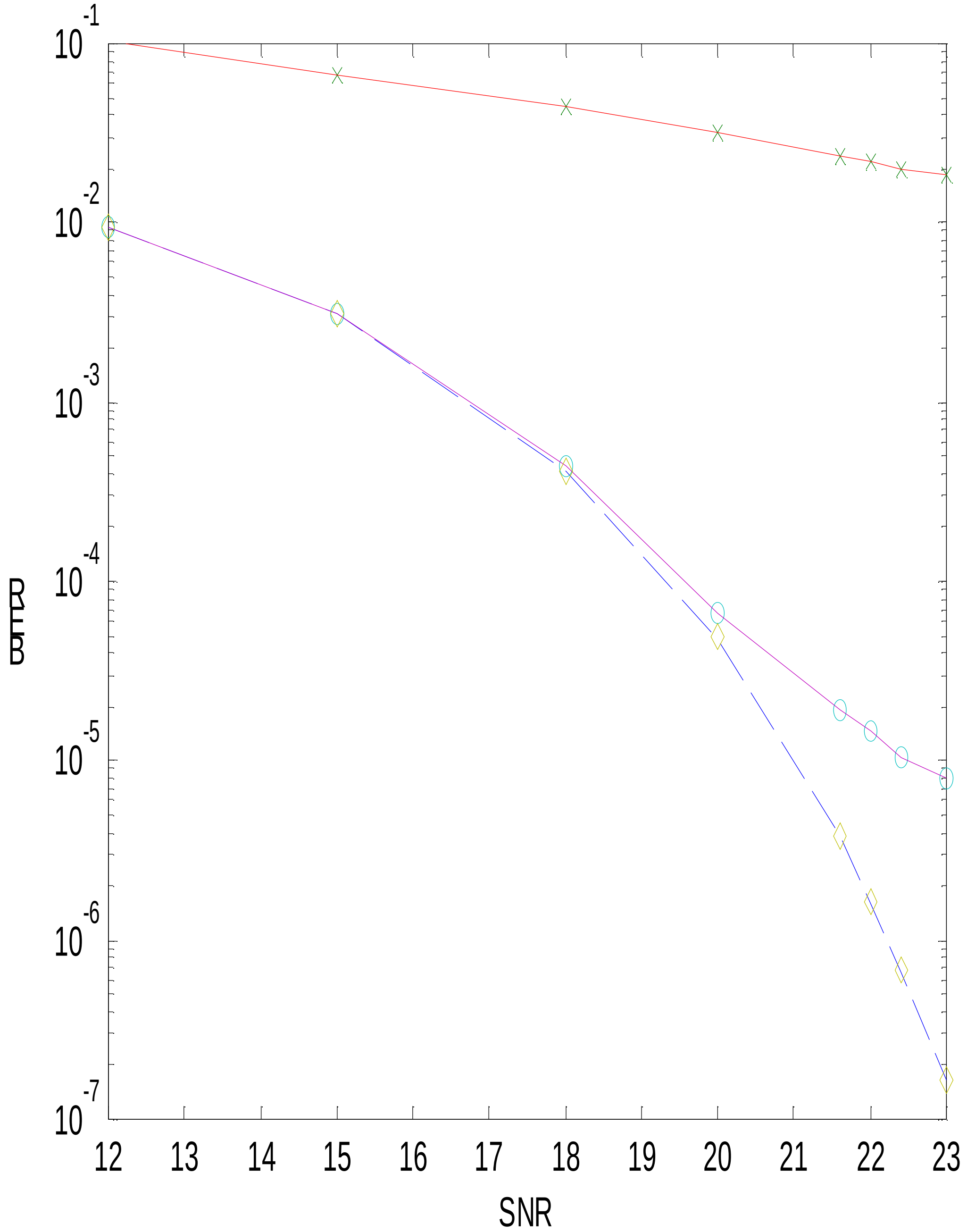

Fig. 7. Bit error rate (BER) versus signal-to-noise ratio (SNR) for different detectors (x-mark: SUD, circle: IC-MUD, diamond: ML-MUD, dashed: single user lower bound).

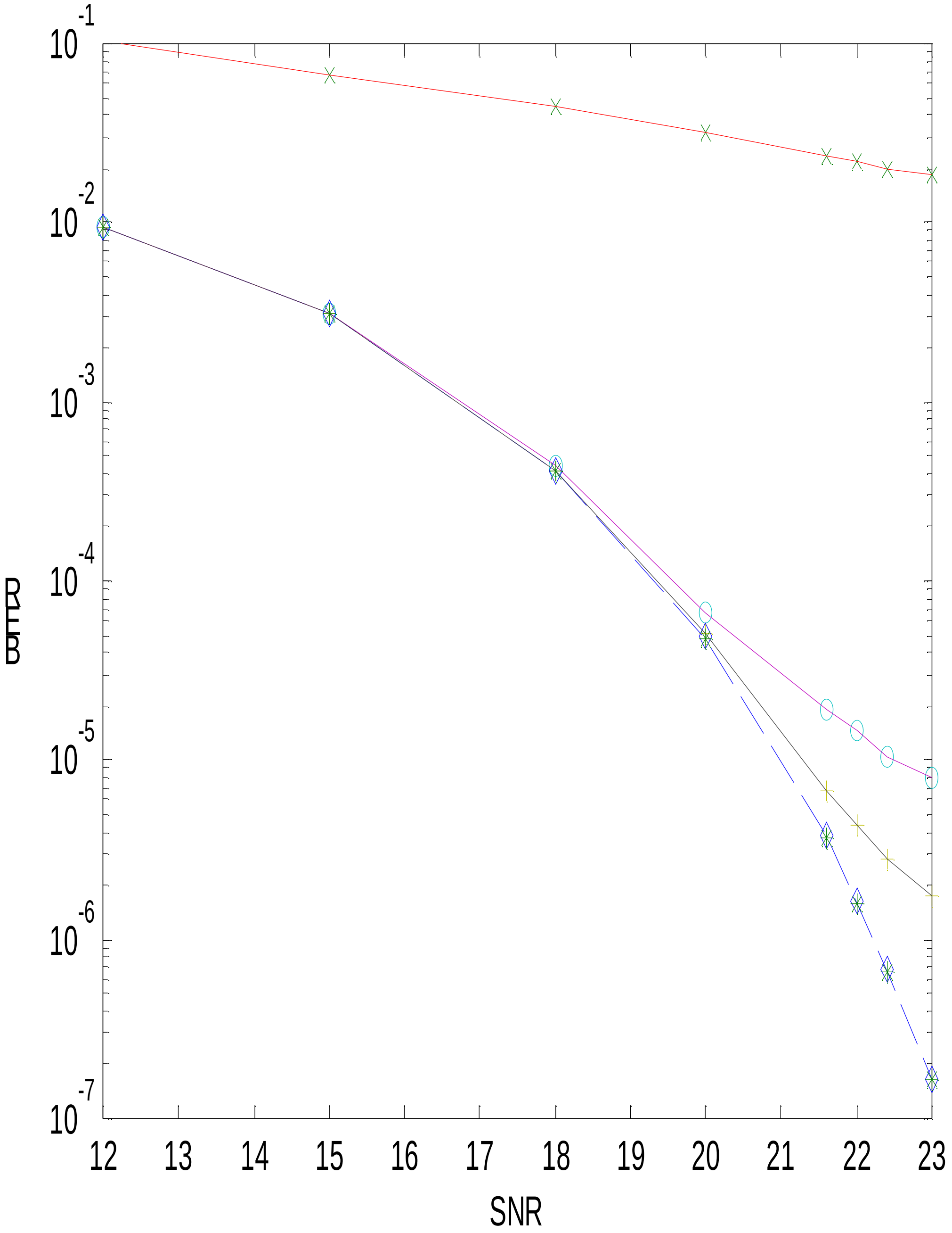

Fig. 8. Bit error rate (BER) versus signal-to-noise ratio (SNR) for different detectors (x-mark: SUD, circle: IC-MUD, plus: IC-MUD-R, diamond: ML-MUD, star: ML-MUD-R, dashed: single user lower bound).

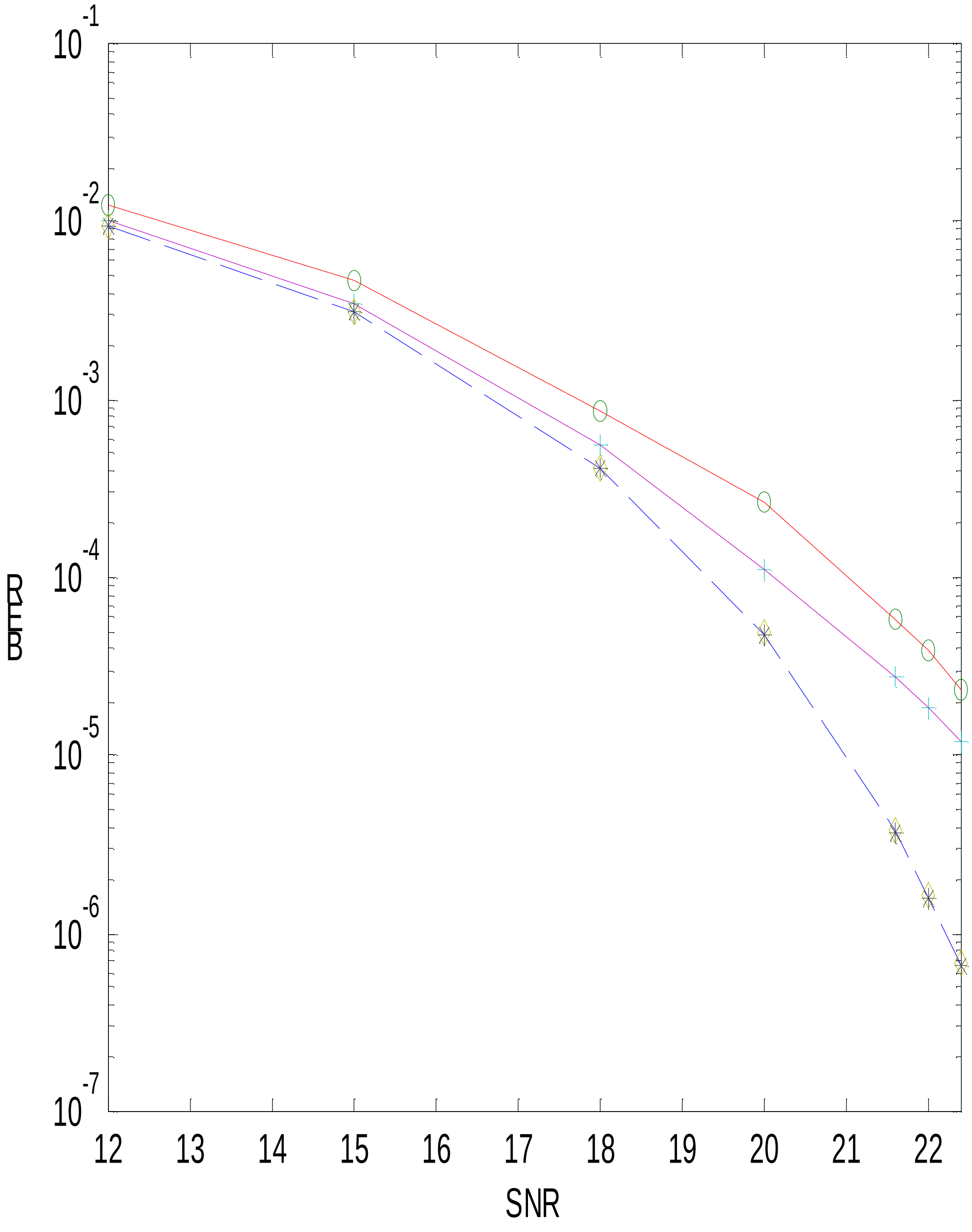

Fig. 9. Bit error rate (BER) versus signal-to-noise ratio (SNR) for different detectors with 15 dB weaker crosstalk (circle: IC-MUD, plus: IC-MUD-R, diamond: ML-MUD, star: ML-MUD-R, dashed: single user lower bound).

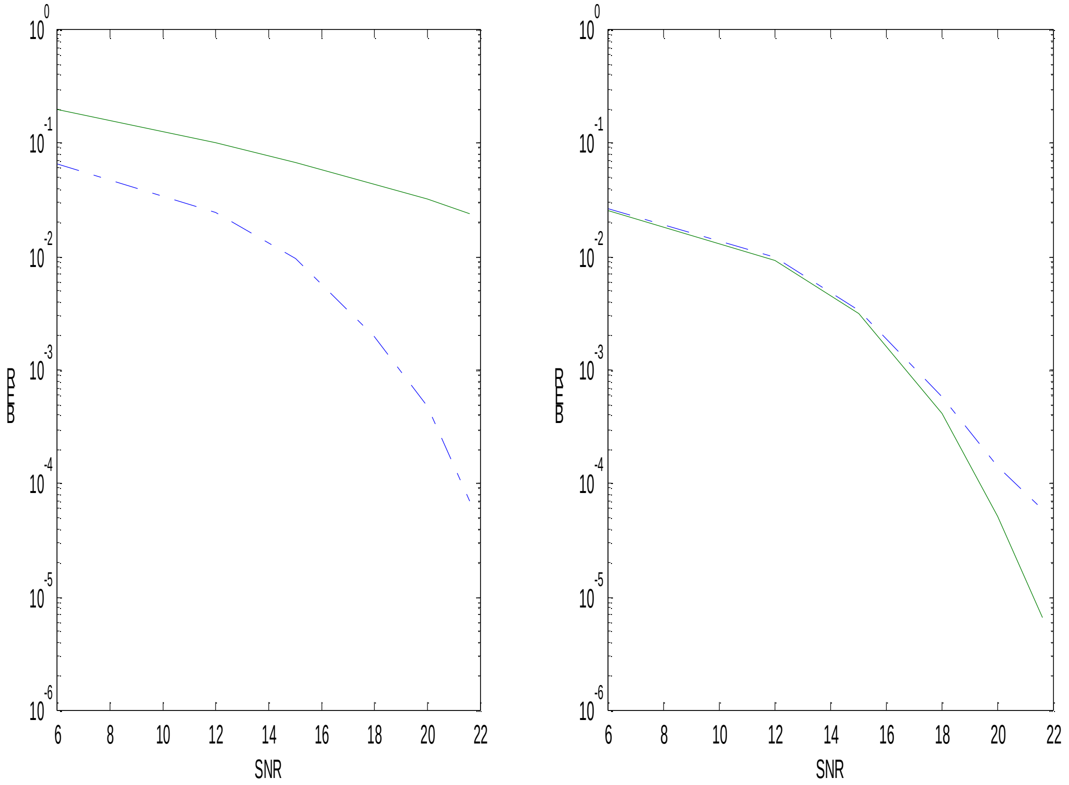

Fig. 10. Effect of crosstalk strength for traditional and robust interference cancellation multiuser detection. left: SUD; right: IC-MUD-R (solid: stronger crosstalk; dash dot: (12 dB) weaker crosstalk).