

Asymptotic Analysis on the Interaction between Spatial Diversity and Multiuser Diversity in Wireless Networks

Quan Zhou, *Student Member, IEEE*, and Huaiyu Dai^{*}, *Member, IEEE*

Abstract

Spatial diversity and multiuser diversity attract much research interest recently. In this paper, through asymptotic analysis of the average system capacity and scheduling gain, we investigate the cross-layer interaction between these two forms of diversity in wireless networks. Rigorous proofs and necessarily stronger results in terms of convergence are provided for some intuitions in this area. Equally important, explicit expressions of scheduling gain and average system capacity in various circumstances that reveal inter-connections and fundamental tradeoffs among key system parameters are given, which afford us some insights in system design. Our results are general enough to cover many practical scenarios of interest.

Index Terms: Average System Capacity, MIMO, Multiuser Diversity, Spatial Diversity, Scheduling Gain.

^{*} The authors are with the Department of Electrical and Computer Engineering, NC State University, Raleigh, NC 27695-7511. Phone: (919) 513-0299; Fax: (919) 515-7382; Email: Huaiyu_Dai@ncsu.edu. This work was supported in part by the National Science Foundation under Grant CCF-0515164.

I. Introduction

Diversity has long been established as key technology that enables reliable and high-data-rate wireless communications. While diversity can be achieved in many forms, two of them attract much research interest recently. One is spatial diversity realized through employing multiple antennas at either the transmitter or receiver end, or both, the idea of which is not new but interest on which is rekindled with the introduction of multi-input multi-output (MIMO) systems [1][2]. In a multiuser wireless network, there is another form of diversity called multiuser diversity [3][4], which reflects the fact of independent fluctuations of different users' channels. Multiuser diversity can be exploited to increase the system throughput, through intentionally transmitting to the user(s) with good channels at each instant (opportunistic scheduling). Spatial diversity techniques typically reside in the physical (PHY) layer, while multiuser diversity is obtained through user scheduling at the medium-access control (MAC) layer. It is therefore interesting to understand how these two diversity techniques combine to determine overall network performance and how they interact with each other.

There exist some work on joint spatial diversity and multiuser diversity systems. In particular, the capacity analysis for Rayleigh fading channels is given in [5], and in [6] for more general Nakagami fading channels. Some have suggested that spatial diversity can actually diminish the advantages of multiuser diversity [4][7][8][9]. Intuitively, this can be explained by observing that multiuser diversity takes advantage of fading by "riding on the peak", which is unfortunately eliminated by spatial diversity. As noted in [7][6][10], however, this conclusion is valid only for open-loop but not closed-loop spatial diversity schemes, while user scheduling inherently requires feedback.

Our research is different from previous work in the following aspects. First, our study on the interaction between spatial diversity and multiuser diversity focuses on the asymptotic analysis, i.e., by allowing the number of antennas or users or both to go to infinity¹. Besides mathematical tractability, asymptotic analysis also helps reveal some fundamental relationship of key system parameters, which may be concealed in the finite case by random fluctuations and other transient properties of channel

¹ Some asymptotic analysis with respect to the number of users is also pursued in [4][9].

matrices. Moreover in many scenarios (especially with respect to the number of antennas), convergence to the asymptotic limit is rather fast. Secondly, we put emphasis on the scheduling gain in capacity rather than the overall system signal-to-noise ratio (SNR) or capacity, which is the benefit we can really obtain through opportunistic scheduling over the traditional round robin scheduling. The impact of multiple antennas on multiuser wireless networks is increasingly drawing research interest very recently. This work will focus on spatial diversity systems; some pioneer study on spatial multiplexing systems can be found in [25][13].

The main contributions of this paper are summarized below:

1). We derive rigorously explicit expressions for average (ergodic) capacity² of joint spatial diversity and multiuser diversity systems when the number of users goes to infinity while the number of antennas keeps fixed. As expected, the average system capacity and scheduling gain grow with the number of users; and we contribute by providing a rather general asymptotic expression that builds an explicit connection with key system parameters and reveals their interactions, and by providing a strict proof of convergence that is in a stronger sense than what is assumed in previous study. As an application, we confirm that in this scenario, there is a tradeoff between spatial diversity and multiuser diversity for an open-loop spatial diversity system, but the detrimental effect of multiple transmit antennas can be avoided with the closed-loop schemes. We also show that all closed-loop schemes perform similarly in this scenario, in the sense that their differences only occur at the second-order (i.e., $\log \log K$).

2). We show rigorously that the scheduling gain nonetheless diminishes to zero as the size of antenna arrays grows while the number of users keeps fixed, no matter for open-loop or closed-loop spatial diversity systems, through asymptotic study on the mean and variance of the effective link SNR. In this sense, multiuser scheduling is not worthwhile in an antenna-dominant environment. On the other hand, different spatial diversity schemes do make significant difference with respect to system capacity for

² These results obviously also hold for scheduling gain, as the average capacity with round robin scheduling does not depend on the number of users in a homogeneous network, assumed throughout this paper.

round robin scheduling. As a side product, we also solve an open problem regarding the mean-square convergence of the largest eigenvalue of Wishart matrices (see Lemma 3).

3). Since the scheduling gain asymptotically decreases with the number of antennas and increases with the number of users, it's interesting to study the asymptotic trend when both are allowed to grow. We reveal how the scheduling gain behaves depending on the relative growth rate between the two. In particular, we determine a critical point, only beyond which multiuser scheduling is meaningful.

The paper is organized as follows. In Section II, we give our system model with combined spatial diversity and multiuser diversity. Then we provide our asymptotic analysis corresponding to the above three scenarios in Section III, IV and V, respectively, together with some numerical results for illustration purpose. Final conclusions are made in Section VI.

II. Joint Spatial Diversity and Multiuser Diversity System

We consider a homogeneous downlink multiuser MIMO communication scenario, which is envisioned to be of crucial importance for emerging wireless networks. Appropriate spatial diversity techniques are employed for each link. In this paper, we concretize our analysis with three spatial diversity schemes. The first employs well-known space-time block coding at the transmitter and maximum ratio combining at the receiver, coined as STBC/MRC, which does not require channel state information (CSI) at the transmitter end. As user scheduling inherently requires feedback, we further explore two closed-loop diversity schemes. One of them pursues joint maximum ratio transmission and maximum ratio combining (MRT/MRC), which provides the optimal performance reference for MIMO diversity techniques. The other exploits simple antenna selection on both ends (SC/SC), trading performance for complexity. MRT/MRC and SC/SC can be viewed as the two extremes for various hybrid selection combining schemes [15][17]. After diversity combining, the user with the best channel quality, in this case the highest effective link SNR, is chosen for communication in opportunistic scheduling. In contrast, the round robin scheduling simply selects the users in some deterministic order.

It is assumed that the base station has M antennas and each of the K users has N antennas. Throughout the paper, when asymptotic analysis with respect to the size of antenna array is pursued, we

allow both M and N to go to infinity, with their ratio $r = N/M$ fixed. The incorporation of the large M and fixed N scenario is relatively straightforward, and will be briefly discussed as well. We use $\mathbf{H}_k = \{h_{ij}^k\}$ ($1 \leq k \leq K$) to denote the k th user's channel matrix. For simplicity, independent and identically distributed (i.i.d.) Rayleigh fading is mainly considered for $\{\mathbf{H}_k\}_{k=1}^K$, but our analysis can be readily extended to other fading scenarios. As will be seen (in Section III), only the tail behaviors of the relevant probability distributions matter. The background noise is assumed to be white and Gaussian.

Throughout this paper we assume a block-flat fading scenario. Let $s_k(t)$ and $y_k(t)$ be the transmit and receive signal (after diversity combining) at time t for some selected user k respectively, then without loss of generality we have

$$y_k(t) = \sqrt{\gamma_k} s_k(t) + n_k(t), \quad (1)$$

where the noise $n_k(t)$ is assumed to have zero mean and unit variance, the average transmit SNR is $\gamma_t = E|s_k(t)|^2$, and γ_k is the channel gain obtained through diversity combining, which can be interpreted as the normalized effective *link SNR*. Denote the probability density function (PDF) and cumulative distribution function (CDF) of γ_k by $f_\gamma(x)$ and $F_\gamma(x)$, respectively, assumed the same for all users. In the opportunistic scheduling scheme, the base station chooses the user $k^* = \arg \max_k (\gamma_k)_{k=1}^K$. Thus the resultant normalized *system SNR* seen by the base station is γ_{k^*} with PDF

$$f_{\gamma_{k^*}}(x) = K f_\gamma(x) F_\gamma^{K-1}(x). \quad (2)$$

In this paper, we follow the common practice and adopt the information-theoretic spectral efficiency as a performance metric to evaluate the joint spatial diversity and multiuser diversity system. The average system capacity obtained by opportunistic scheduling can be described by a function of K and M as

$$\bar{S}(K, M) = E\left(\log\left(1 + \gamma_t \left(\max_{1 \leq k \leq K} \gamma_k\right)\right)\right) = \int_0^{+\infty} \log(1 + \gamma_t x) f_{\gamma_{k^*}}(x) dx. \quad (3)$$

Similarly, the average system capacity obtained by round-robin scheduling can be defined as a function of M , which is

$$\bar{R}(M) = E(\log(1 + \gamma_t \gamma)) = \int_0^{+\infty} \log(1 + \gamma_t x) f_\gamma(x) dx. \quad (4)$$

Finally, in order to measure the benefit brought by multiuser diversity, we define the scheduling gain $G(K, M)$ as the average capacity gain boosted by opportunistic scheduling from $\bar{R}(M)$:

$$G(K, M) = \bar{S}(K, M) - \bar{R}(M). \quad (5)$$

In the remainder of this paper, we adopt the following notations for the limiting behaviors of two functions $f(x)$ and $g(x)$ with $\lim_{\substack{x \rightarrow \infty \\ \text{or } x \rightarrow 0}} \frac{g(x)}{f(x)} = c$: $g(x) = O(f(x))$ for $0 < |c| < \infty$; $g(x) \sim f(x)$ for $c = 1$; $g(x) = o(f(x))$ for $c = 0$; and $g(x) = \omega(f(x))$ for $c = \infty$. When convergence of a sequence of random variables is involved, shorthand notation “ D ” stands for in distribution, “ P ” for in probability, “ r ” for in r th mean, and “*a.s.*” for almost surely. The user index will be omitted from relevant notations when no ambiguity is incurred.

III. Asymptotic System Capacity and Scheduling Gain as K Goes to Infinity while M Keeps Fixed

In this section, we will examine $\lim_{K \rightarrow \infty} \bar{S}(K, M)$ with M fixed. As a motivation, we first introduce the following lemma from [11][12].

Lemma 1: Let X_1, \dots, X_K be i.i.d. random variables with CDF $F(x)$, with $\Omega(F) = \sup\{x : F(x) < 1\}$.

Suppose there is a real number x_1 such that for all $x_1 \leq x < \Omega(F)$, $f(x) = F'(x)$ and $F''(x)$ exist and $f(x) \neq 0$. Define the growth function

$$g(x) = \frac{1 - F(x)}{f(x)}. \quad (6)$$

If $\lim_{x \rightarrow \Omega(F)} g(x) = c \geq 0$, the following standardized extreme converges in distribution as

$$\frac{\max_{1 \leq k \leq K} X_k - b_K}{a_K} \xrightarrow{D} \Lambda(x) = \exp(-e^{-x}), \quad (7)$$

with

$$b_K = F^{-1}(1 - 1/K) \text{ and } a_K = (Kf(b_K))^{-1}. \quad (8)$$

Remark: From (6) and (8), we can find a connection between the growth function and a_K , i.e.,

$$g(b_K) = \frac{1 - F(b_K)}{f(b_K)} = \frac{1}{Kf(b_K)} = a_K, \text{ therefore if } \lim_{x \rightarrow \Omega(F)} g(x) = c \geq 0, \text{ we can obtain } \lim_{K \rightarrow \infty} a_K = c \geq 0. \text{ Thus}$$

the result of (7) indicates that $\max_{1 \leq k \leq K} X_k$ “grows like” b_K in a coarse sense [4][9], and is widely used in the

study of opportunistic communications involving extreme values and order statistics. This result can

actually be strengthened from existing literature [12]: if $c = 0$ (or $\lim_{K \rightarrow \infty} a_K = 0$), $\max_{1 \leq k \leq K} X_k - b_K \xrightarrow{P} 0$,

otherwise $\max_{1 \leq k \leq K} X_k / b_K \xrightarrow{P} 1$. Nonetheless, our desired outcomes, which are concerned with the

convergence of the expected values of functions of $\max_{1 \leq k \leq K} \gamma_k$, require a yet stronger result as stated below.

Our main contributions in this section lie in providing sufficient conditions for this stronger result to hold, and an explicit expression for the corresponding system capacity (and scheduling gain) that is general enough to include many practical scenarios of interest (see (10)).

Theorem 1: Let $\gamma_1, \dots, \gamma_K$ be i.i.d. positive random variables with absolutely continuous CDF $F_\gamma(x)$ and

PDF $f_\gamma(x)$, as given in Lemma 1 with $\Omega(F_\gamma) = +\infty$. Define $g_\gamma(x) = \frac{1 - F_\gamma(x)}{f_\gamma(x)}$. If $\lim_{x \rightarrow \infty} g_\gamma(x) = c \geq 0$,

$g'_\gamma(x) = O(1/x^{\delta_1})$ with $\delta_1 > 0$, and $b_K = F_\gamma^{-1}(1 - 1/K) = O((\log K)^{\delta_2})$ with $0 < \delta_2 \leq 1$, then

$$\lim_{K \rightarrow \infty} \{ \bar{S}(K, M) - \log(1 + \gamma_i b_K) \} = 0. \quad (9)$$

Proof: See Appendix A³.

According to (9), the system capacity (and scheduling gain) is asymptotically equivalent to $\log(1 + \gamma_i b_K)$ when given conditions are fulfilled. Note that all these conditions involve only the tail behaviors of the distributions of individual link SNR. In the following, we examine a form of special interest, which is general enough to cover common fading models and spatial diversity schemes.

³ In the proof of theorem 1, we generalize Corollary A.1 in [13], as can be seen from Corollary 2 in Appendix A.

Corollary 1: If $f_\gamma(x) \sim \alpha x^p e^{-qx^\nu}$ as $x \rightarrow \infty$ with $\alpha > 0$, $q > 0$, $\nu \geq 1$ and any p , i.e., $f_\gamma(x)$ is tail equivalent to $\alpha x^p e^{-qx^\nu}$, then $\lim_{K \rightarrow \infty} \{\bar{S}(K, M) - \log(1 + \gamma_t b_K)\} = 0$, where (up to the second-order approximation⁴)

$$b_K = \left(\frac{1}{q} \log \tau K \right)^{1/\nu} + \frac{p+1-\nu}{qv^2} \frac{\log \frac{\log \tau K}{q}}{\left(\frac{1}{q} \log \tau K \right)^{(v-1)/\nu}}, \quad (10)$$

with $\tau = \frac{\alpha}{qv}$.

Proof: See Appendix B.

Remark: The parameter α only appears in τ , which is typically not important in large K analysis. In general, a smaller ν and q indicate a better system performance, as seen from the first term of (10). A larger p also helps, though only at the second-order sense.

In the remaining part of this section, we demonstrate the applications of our results, Theorem 1 and Corollary 1, through some representative systems jointly exploiting spatial diversity and multiuser diversity (see Section II). As mentioned before, we assume Raleigh fading for simplicity. The key step lies in examining the tail behavior of the PDF of the corresponding effective link SNR. Once it is verified to take the form given in Corollary 1, we can readily conclude that the corresponding asymptotic system capacity (and scheduling gain) is given by $\log(1 + \gamma_t b_K)$, with b_K given by (10).

STBC/MRC

Without loss of generality, we assume that the adopted space-time block coding scheme achieves the full diversity and rate⁵ and the transmit power is equally allocated among the transmit antennas [8][9]. In this

case, the normalized effective link SNR for a generic user is given by $\gamma = \frac{1}{M} \sum_{i=1}^N \sum_{j=1}^M |h_{i,j}|^2$, whose PDF

admits:

⁴ We define the first order approximation when truncated at $\log K$, and the second order approximation when truncated at $\log \log K$.

⁵ Rigorously speaking, the capacity should be scaled by the coding rate.

$$f_{\gamma}^{STBC/MRC}(x) = \frac{M^{MN}}{(MN-1)!} x^{MN-1} e^{-Mx}, x \geq 0. \quad (11)$$

Clearly Corollary 1 holds with

$$\alpha = \frac{M^{MN}}{(MN-1)!}, p = MN-1, q = M, \text{ and } \nu = 1.$$

So the corresponding asymptotic system capacity is given by $\log(1 + \gamma_i b_K^{STBC/MRC})$ with

$$b_K^{STBC/MRC} = \frac{1}{M} \left(\log \frac{M^{MN-1}}{(MN-1)!} K + (MN-1) \log \log \frac{M^{MN-1}}{(MN-1)!} K \right) + O(\log \log \log K). \quad (12)$$

SC/SC

In this spatial diversity scheme, both the user and the base station choose one optimal antenna such that the resultant channel gain is maximized [15]. Thus the normalized effective link SNR at the receiver is

$\gamma = \max_{1 \leq i \leq N, 1 \leq j \leq M} (|h_{i,j}|^2)$, whose PDF can be easily obtained as

$$f_{\gamma}^{SC/SC}(x) = MN e^{-x} (1 - e^{-x})^{MN-1}, x \geq 0. \quad (13)$$

The PDF in (13) is tail equivalent to $MN e^{-x}$. Again Corollary 1 holds with

$$\alpha = MN, p = 0, q = 1, \text{ and } \nu = 1.$$

So the corresponding asymptotic system capacity is given by $\log(1 + \gamma_i b_K^{SC/SC})$ with⁶

$$b_K^{SC/SC} = \log(MNK). \quad (14)$$

MRT/MRC

In the MIMO MRT/MRC system, the unit-norm principal right singular vector corresponding to the largest singular value σ_{\max} of channel matrix \mathbf{H} , $\mathbf{w}_t \in \mathbb{C}^{M \times 1}$ is applied to the transmitted data at the base station, and at the receiver side the corresponding left singular vector $\mathbf{w}_r \in \mathbb{C}^{N \times 1}$ is employed [16][18].

The CDF of $\gamma = \lambda_{\max}(\mathbf{H}\mathbf{H}^H) = \sigma_{\max}^2(\mathbf{H})$ is given by [19][20]

$$F_{\gamma}^{MRT/MRC}(x) = \frac{|\Psi_c(x)|}{\prod_{k=1}^s \Gamma(t-k+1) \Gamma(s-k+1)}, x \geq 0, \quad (15)$$

⁶ This is one rare accurate expression. Note that in this case, the growth in transmit and receive antennas can be equivalently seen as an increase in the number of users (due to the i.i.d. assumptions).

where $s = \min(M, N)$, $t = \max(M, N)$, and $\Psi_c(x)$ is an $s \times s$ Hankel matrix function with the (i, j) th entry given by $\{\Psi_c(x)\}_{i,j} = \gamma(t - s + i + j - 1, x)$, for $i, j = 1, 2, \dots, s$. Here $\gamma(a, \beta)$ is the incomplete Gamma function defined as $\gamma(a, \beta) = \int_0^\beta e^{-t} t^{a-1} dt$, and $\Gamma(a)$ is the Gamma function defined as $\Gamma(a) = \gamma(a, +\infty) = \int_0^{+\infty} e^{-t} t^{a-1} dt$. The PDF of γ can be derived as

$$f_\gamma^{MRT/MRC}(x) = F_\gamma^{MRT/MRC}(x) \text{tr}(\Psi_c^{-1}(x) \Phi_c(x)), x \in (0, +\infty), \quad (16)$$

where $\Phi_c(x)$ is an $s \times s$ matrix whose (i, j) th entry is given by $\{\Phi_c(x)\}_{i,j} = x^{t-s+i+j-2} e^{-x}$, and $\text{tr}(\cdot)$ denotes the trace operation.

Though the PDF (16) is rather involved, fortunately we are only concerned with its tail behavior, as dictated by the following lemma.

Lemma 2: $f_\gamma^{MRT/MRC}(x)$ is tail equivalent to $\frac{1}{(M-1)!(N-1)!} e^{-x} x^{M+N-2}$.

Proof: See Appendix C.

Therefore Corollary 1 holds with

$$\alpha = \frac{1}{(M-1)!(N-1)!}, p = M + N - 2, q = 1, \text{ and } \nu = 1.$$

So the corresponding asymptotic system capacity is given by $\log(1 + \gamma_\nu b_K^{MRT/MRC})$ with

$$b_K^{MRT/MRC} = \log \frac{1}{(M-1)!(N-1)!} K + (M + N - 2) \log \log \frac{1}{(M-1)!(N-1)!} K + O(\log \log \log K). \quad (17)$$

Some interesting observations are readily in order. For all the above three schemes we have $\nu = 1$, which simplifies the expressions. From (12), we can observe a tradeoff between spatial diversity and multiuser diversity for an open-loop spatial diversity system (the factor of $1/M$ has a negative role in $b_K^{STBC/MRC}$, which directly determines the asymptotic system capacity $\bar{S}(K, M)$). Here we give a more rigorous proof and reveal how the ultimate capacity is related to M and N . For example, our result does show the positive role of the number of receive antennas N , though in a second-order sense, which is not clear from previous results in literature. It is also observed that the detrimental effect of multiple transmit

antennas can be avoided with the closed-loop spatial diversity schemes, as seen in (14) and (17)⁷. And also from (14) and (17), we can infer that for the general hybrid selection combining schemes, the scaling laws should only have differences in the second order approximations. Numerical results in Fig. 1 verify that $\log(1 + \gamma_i b_K)$ is a good approximation for the average capacity of the STBC/MRC, SC/SC and MRT/MRC systems using the opportunistic scheduler.

Note that Theorem 1 and Corollary 1 can potentially address a larger class of problems with respect to system and channel characteristics than what are presented here. Clearly the more general Nakagami- m distribution takes the form given in Corollary 1. For the Log-normal fading, the transformation $y = \log \gamma$ results in a normal distributed random variable. Ricean fading admits the following distribution

$$f_x(x) = \frac{1}{2\sigma^2} \exp\left(-\frac{x+s^2}{2\sigma^2}\right) I_0\left(\frac{\sqrt{x}s}{\sigma^2}\right) \sim \frac{1}{2\sigma} \frac{1}{\sqrt{2\pi}} (s^2 x)^{-1/4} \exp\left(\frac{-(\sqrt{x}-s)^2}{2\sigma^2}\right), \quad (18)$$

where $s > 0$ is the amplitude of the line-of-sight component, $I_n(x)$ is the n th-order modified Bessel function of the first kind, and the tail-equivalence is due to the fact that for fixed n , $I_n(x) \sim \frac{e^x}{\sqrt{2\pi x}}$. As an example, we can show that for i.i.d. Ricean fading, SC/SC admits

$$b_{K,Ricean}^{SC/SC} = \left(s + \sqrt{2\sigma^2 \log\left(\frac{KMN\sigma}{\sqrt{2\pi}s}\right)} - \frac{\sigma^2}{4} \frac{\log \log \frac{KMN\sigma}{\sqrt{2\pi}s}}{\sqrt{2\sigma^2 \log\left(\frac{KMN}{\sqrt{2\pi}s}\right)}} \right)^2 \quad (19)$$

with an application of Corollary 1.

As seen from above, scheduling gain $G(K, M)$ is an asymptotically increasing function of K . In the next section, we will show that $G(K, M)$ asymptotically decreases with M , no matter for open-loop or closed-loop spatial diversity systems.

⁷ Note that the coefficients of K inside the log functions are not important when K becomes large. In this sense, multiple antennas even help for the MRT/MRC scheme.

IV. Asymptotic Scheduling Gain as M Goes to Infinity while K Keeps Fixed

In this section, we will examine $\lim_{M \rightarrow \infty} G(K, M) = \lim_{M \rightarrow \infty} (\bar{S}(K, M) - \bar{R}(M))$ with K fixed, which complements the study in Section III. The scenario when both M and K go to infinity will be discussed in Section V. We focus on the scenario that $N \rightarrow \infty$ as well, with $r = N/M$ fixed. However, all results apply to the case of fixed N as well, with r taken as 0 when applicable. The following theorem summarizes the main result in this scenario.

Theorem 2: Let μ_M and σ_M be the mean and standard deviation of the normalized effective SNR γ_M for

each individual link⁸. If $\lim_{M \rightarrow \infty} \frac{\sigma_M}{\mu_M} = 0$, then $\lim_{M \rightarrow \infty} \left\{ \frac{\bar{R}(M)}{\log(1 + \gamma_M \mu_M)} \right\} = 1$ and $\lim_{M \rightarrow \infty} \frac{G(K, M)}{\bar{R}(M)} = 0$ when K

keeps fixed. Furthermore, if we have $\left\{ \log \frac{\gamma_M}{\mu_M} \mathbf{I}_{(0,1)} \left(\frac{\gamma_M}{\mu_M} \right) \right\}$ uniformly integrable⁹, then

$\lim_{M \rightarrow \infty} \left\{ \bar{R}(M) - \log(1 + \gamma_M \mu_M) \right\} = 0$, and $\lim_{M \rightarrow \infty} G(K, M) = 0$ when K keeps fixed.

Proof: See Appendix D.

Remark: As shown in the proof, $\lim_{M \rightarrow \infty} \sigma_M / \mu_M = 0$ leads to the conclusion that γ_M / μ_M converges to 1 in

2nd mean (mean square). It is relatively straightforward to show that $\log(1 + \gamma_M / \mu_M) \xrightarrow{2} \log 2$ as γ_M / μ_M is always positive and $\log(1 + x)$ grows slower than x in $(0, \infty)$. The difficulty with $\log \gamma_M / \mu_M$ occurs when the argument falls on $(0, 1)$, which necessitates the condition of uniform integrability. Also, according to (63) of Appendix D, we can see that σ_M / μ_M can be roughly used as a parameter to measure the scheduling gain.

In the following, we apply Theorem 2 on three representative systems. The key steps lie in showing

$\lim_{M \rightarrow \infty} \frac{\sigma_M}{\mu_M} = 0$ and the uniform integrability of $\left\{ \log \frac{\gamma_M}{\mu_M} \mathbf{I}_{(0,1)} \left(\frac{\gamma_M}{\mu_M} \right) \right\}$. Similar to Section III, the cases of

STBC/MRC and SC/SC are relatively easy, while things become much more involved with MRT/MRC.

⁸ We use subscript M to explicitly denote the dependence of corresponding quantities on M .

⁹ $\mathbf{I}_A(\cdot)$ is the indicator function on the set A , i.e. $\mathbf{I}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A. \end{cases}$

STBC/MRC

In this case, the PDF of $\gamma_M^{STBC/MRC}$ is given in (11), from which it's straightforward to obtain

$\mu_M^{STBC/MRC} = N = rM$ and $\sigma_M^{STBC/MRC} = \sqrt{N/M} = \sqrt{r}$. Clearly we have

$$\lim_{M \rightarrow \infty} \frac{\sigma_M^{STBC/MRC}}{\mu_M^{STBC/MRC}} = \lim_{M \rightarrow \infty} \frac{1}{\sqrt{rM}} = 0. \quad (20)$$

The PDF of $X_M^{STBC/MRC} = \gamma_M^{STBC/MRC} / \mu_M^{STBC/MRC}$ is given by

$$f_M^{STBC/MRC}(x) = \frac{MN^{MN}}{(MN-1)!} x^{MN-1} e^{-MNx} \leq Cx, \text{ when } x < 1, \quad (21)$$

where C is a positive constant, and we can bound the coefficient due to Stirling's formula. Clearly,

$\{\log X_M^{STBC/MRC} \mathbf{I}_{(0,1)}(X_M^{STBC/MRC})\}$ is uniformly integrable. So

$$\lim_{M \rightarrow \infty} \left\{ \bar{R}^{STBC/MRC}(M) - \log(1 + \gamma_t r M) \right\} = 0, \text{ and } \lim_{M \rightarrow \infty} G^{STBC/MRC}(K, M) = 0 \quad (22)$$

with K fixed.

SC/SC

In this case it's easy to obtain $\mu_M^{SC/SC} = \sum_{i=1}^{MN} \frac{1}{i} \rightarrow \log(MN) + C_0$ and $\sigma_M^{SC/SC} = \sqrt{\sum_{i=1}^{MN} \frac{1}{i^2}} \rightarrow \sqrt{\frac{\pi^2}{6}}$ as $M \rightarrow \infty$,

where C_0 is the Euler's constant (see page 298 of [11]). Then we have

$$\lim_{M \rightarrow \infty} \frac{\sigma_M^{SC/SC}}{\mu_M^{SC/SC}} = \lim_{M \rightarrow \infty} \frac{1}{\log(MN)} = 0. \quad (23)$$

When $M \rightarrow \infty$, the single-link SC/SC is equivalent to the corresponding multiuser scheduling scenario when $K \rightarrow \infty$. Results in Section III can be directly applied to get for fixed K

$$\lim_{M \rightarrow \infty} \left\{ \bar{R}^{SC/SC}(M) - \log(1 + \gamma_t \log(MN)) \right\} = 0, \text{ and } \lim_{M \rightarrow \infty} G^{SC/SC}(K, M) = 0. \quad (24)$$

MRT/MRC

The calculation for the MRT/MRC case is more difficult. The closed-form expressions for the $\mu_M^{MRT/MRC}$ and $\sigma_M^{MRT/MRC}$ are unknown. In the asymptotic scenario, it is known that [22]

$$\frac{1}{M} \lambda_{\max} \xrightarrow{a.s.} (1 + \sqrt{r})^2. \quad (25)$$

But surprisingly, $\lim_{M \rightarrow \infty} \frac{1}{M} \mu_M^{MRT/MRC} = \lim_{M \rightarrow \infty} E\left(\frac{1}{M} \lambda_{\max}\right)$ remains open in literature, which is solved here

through the following lemma.

Lemma 3: Let \mathbf{H} be an $N \times M$ matrix with i.i.d. complex entries with $E(h_{ij})=0$, $E(|h_{ij}|^2)=1$, and

$E(|h_{ij}|^4) < \infty$. Define $\lambda_{\max}\left(\frac{1}{M} \mathbf{H} \mathbf{H}^H\right) = \left\| \frac{1}{\sqrt{M}} \mathbf{H} \right\|^2$ ¹⁰. Then

$$\lim_{M \rightarrow \infty} E\left(\lambda_{\max}\left(\frac{1}{M} \mathbf{H} \mathbf{H}^H\right)\right) = (1 + \sqrt{r})^2 \text{ and } \lim_{M \rightarrow \infty} \sigma\left(\lambda_{\max}\left(\frac{1}{M} \mathbf{H} \mathbf{H}^H\right)\right) = 0, \text{ } r = \lim_{M, N \rightarrow \infty} \frac{N}{M}. \quad (26)$$

Proof: See Appendix E.

Remark: A more important conclusion from Lemma 3 is that

$$\lim_{M \rightarrow \infty} \frac{\sigma_M^{MRT/MRC}}{\mu_M^{MRT/MRC}} = 0. \quad (27)$$

The uniform integrability of $\left\{ \log X_M^{MRT/MRC} \mathbf{I}_{(0,1)}\left(X_M^{MRT/MRC}\right) \right\}$ is already verified in Proposition 4.2 of [24].

Therefore, for fixed K ,

$$\lim_{M \rightarrow \infty} \left\{ \bar{R}^{MRT/MRC}(M) - \log\left(1 + \gamma_i (1 + \sqrt{r})^2 M\right) \right\} = 0, \text{ and } \lim_{M \rightarrow \infty} G^{MRT/MRC}(K, M) = 0. \quad (28)$$

Theorem 2 and the above examples indicate that, given the number of users, the scheduling gain will diminish when the number of antennas goes to infinity, if the mean of the link SNR grows at a higher-order rate than its variance. Intuitively, the mean corresponds to what we obtain through round-robin scheduling, while the variance really contributes to the scheduling gain. This is reminiscent of the multiple-antenna channel hardening effect studied in [25]. Therefore, multiuser scheduling is not worthwhile in an antenna-dominant environment. It is also interesting to see the difference in $\lim_{M \rightarrow \infty} \bar{R}(M)$ for different diversity techniques. For STBC/MRC, it achieves a constant unless the number of receive antennas N also grows with M ¹¹. For SC/SC, it grows like $\log \log M$, but less impressive than that achieved by MRT/MRC, $\log M$. Numerical results in Fig. 2 indicate a good match between the

¹⁰ Here $\|\cdot\|$ is the induced spectral norm on matrix, denoting the largest singular value.

¹¹ As we mentioned, the results in (22), (24) and (28) hold for large M and fixed N as well. In this case, rM in (22) should be replaced with N , and r in (28) taken as 0.

simulation results $\bar{R}(M)$ and the approximation results $\log(1 + \gamma_i \mu_M)$ for the above three cases as M grows. Figure 3 verifies through simulations that the scheduling gain will diminish as the number of antennas grows for both the open-loop and closed-loop spatial diversity schemes. Furthermore, based on the above discussion¹² we roughly have $\frac{\sigma_M^{SC/SC}}{\mu_M^{SC/C}} \sim \frac{1}{\log M}$, $\frac{\sigma_M^{STBC/MRC}}{\mu_M^{STBC/MRC}} \sim \frac{1}{M}$, and $\frac{\sigma_M^{MRT/MRC}}{\mu_M^{MRT/MRC}} \sim \frac{1}{M^{2/3}}$, which intuitively explains the different decay rates shown in Fig. 3. The analysis on the mean and variance will also be useful for the discussion in the following section.

V. Scheduling Gain When both M and K Go to Infinity

In the previous two sections we have given rigorous asymptotic results when either M or K grows. An interesting question naturally arises: when both the number of users and antennas are allowed to grow simultaneously, how will $G(K, M)$ behave? Intuitively, this depends on the relative growth rate of M and K ¹³. Our goal is to find a critical point $K = O(f(M))$, only beyond which multiuser scheduling is meaningful.

We again facilitate the study through asymptotic analysis. Since the number of antennas also grows to infinity, the results derived in Section III do not apply directly. Let $\gamma(k, M)$ be the effective link SNR for the k th user. Following the idea introduced in Lemma 1 (see (7)), we can also introduce two norming constants p_M and q_M to form $\frac{\gamma(k, M) - q_M}{p_M}$, whose distribution is asymptotically independent of M as

$M \rightarrow \infty$, i.e., $\frac{\gamma(k, M) - q_M}{p_M} \xrightarrow{D} w_k$ for some random variable w_k . Desirably, the PDF of w_k takes the form

given in Corollary 1. Then we can apply the results in Section III to obtain the scaling law for $\max\{w_k\}_{k=1}^K$,

denoted as b_K , which leads us to approximate the scaling law for $\max\{\gamma(k, M)\}_{k=1}^K$ by $q_M + p_M b_K$ when

both M and K grow. Finally combining this result with Theorem 2, we can approximate the asymptotic

¹² For MRT/MRC, as will be seen in (32) and (33), we can roughly consider $\sigma_M^{MRT/MRC} \sim M^{1/3}$.

¹³ A similar study is conducted in [13] for spatial multiplexing systems to guarantee that the system throughput can still scale linearly with M .

scheduling gain as $\log\left(\frac{q_M + p_M b_K}{\mu_M}\right)$. As will be seen, most often $q_M = \mu_M$; our approach thus nicely combine the effect of multiple antennas (p_M and q_M (or μ_M)) and multiple users (b_K) for convenience of analysis. Now the problem boils down to the determination of the dominant factor between the two. Note that such approach was also taken in the relevant study of [25], where MIMO capacity is shown to be asymptotically Gaussian when the number of antennas grows.

In the following, we demonstrate this approach through examining the asymptotic scheduling gain for STBC/MRC, SC/SC and MRT/MRC when both the number of antennas and users go to infinity, with $r = N/M$ fixed. The case of fixed N follows the same line and will also be briefly discussed in Appendix F.

STBC/MRC

Choose $p_M = \sqrt{N/M} = \sqrt{r}$ and $q_M = \mu_M = N = rM$. By Central Limit Theorem, we can get

$\frac{\gamma(k, M) - q_M}{p_M} \xrightarrow{D} w_k$, whose PDF is the standard normal distribution function $\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$. From Corollary

1, we can obtain $b_K \sim \sqrt{2 \log K}$. According to our approach the asymptotic scheduling gain is given

by $\log\left(1 + \frac{\sqrt{2r \log K}}{rM}\right)$, which admits

$$G^{STBC/MRC}(K, M) \rightarrow \begin{cases} 0, & \text{when } \log K = o(M^2) \\ c, & \text{when } \log K = O(M^2) \\ +\infty, & \text{when } \log K = \omega(M^2). \end{cases} \quad (29)$$

SC/SC

According to the results in Section III, we know if we choose $q_M = \log(MN) \sim \mu_M$, $p_M = 1$, then

$\gamma(k, M) - q_M \xrightarrow{D} w_k$, whose CDF is $\exp(-e^{-x})$. We can then obtain $b_K \sim \log K$ through Corollary 1 and

the asymptotic scheduling gain is given by $\log\left(1 + \frac{\log K}{\log(rM^2)}\right)$. Therefore

$$G^{SC/SC}(K, M) \rightarrow \begin{cases} 0, & \text{when } \log K = o(\log M) \\ c, & \text{when } \log K = O(\log M) \\ +\infty, & \text{when } \log K = \omega(\log M). \end{cases} \quad (30)$$

MRT/MRC

In this scenario, the norming constants p_M and q_M have already been obtained in [26] as

$$q_M = (\sqrt{M} + \sqrt{N})^2 = M(1 + \sqrt{r})^2 \sim \mu_M, \quad (31)$$

$$p_M = (\sqrt{M} + \sqrt{N}) \left(\frac{1}{\sqrt{M}} + \frac{1}{\sqrt{N}} \right)^{1/3} = M^{1/3} \frac{(1 + \sqrt{r})^{4/3}}{\sqrt{r}}. \quad (32)$$

Further

$$\frac{\gamma(k, M) - q_M^D}{p_M} \rightarrow w_k, \quad (33)$$

where w_k is a random variable whose distribution follows the Tracy-Widom law of order 2. This distribution is defined by

$$F_2(s) = \exp\left\{ \int_s^\infty (x-s)q^2(x)dx \right\}, s \in \mathfrak{R}, \quad (34)$$

where $q(x)$ solves the nonlinear Painleve II differential equation

$$q''(x) = xq(x) + 2q^3(x), \quad (35)$$

and $q(x) \sim 2^{-1} \pi^{-1/2} s^{-1/4} e^{\frac{-2}{3}s^{3/2}}$, as $x \rightarrow \infty$. From (34), we can obtain

$$f_2(s) = dF_2(s)/ds = F_2(s) \cdot \int_s^\infty q^2(x)dx \sim \int_s^\infty q^2(x)dx \sim (8\pi s)^{-1} e^{\frac{-4}{3}s^{3/2}}. \quad (36)$$

We can check that $f_2(x)$ given in (36) satisfies the necessary conditions given in Corollary 1, which

leads to $b_K \sim (\log K)^{2/3}$. The asymptotic scheduling gain is thus given by $\log\left(1 + \left(\frac{\log K}{M}\right)^{2/3}\right)$ and

$$G^{MRT/MRC}(K, M) \rightarrow \begin{cases} 0, & \text{when } \log K = o(M) \\ c, & \text{when } \log K = O(M) \\ +\infty, & \text{when } \log K = \omega(M). \end{cases} \quad (37)$$

From (29), (30) and (37), we can observe that for STBC/MRC, K should at least grow like e^{M^2} to maintain the scheduling gain, while for SC/SC and MRT/MRC, K should grow at least as fast as

M and e^M , respectively. To verify the above results (when both M and N are large), we consider two relative growth rate between K and M , with $N/M = 1$. First, we assume $K = M$ in Fig. 4, which shows that the scheduling gain for SC/SC has a tendency towards saturation; while for MRT/MRC and STBC/MRC, the scheduling gain asymptotically decreases. Then in Fig. 5, we assume $K = e^M$, which shows the scheduling gain for MRT/MRC almost saturates as M grows, while scheduling gain asymptotically increases for SC/SC and decreases for STBC/MRC. Due to the computation constraint, it's difficult to simulate the scenario $K = e^{M^2}$, but our results already show that for STBC/MRC, much more users are required to make opportunistic scheduling beneficial.

One of the interesting observations from this section is that, when the number of antennas grows, generally we need even greater (sometimes exponentially greater) users to maintain the scheduling gain.

VI. Conclusions

In this paper, we present asymptotic analysis on the interaction between spatial diversity and multiuser diversity in wireless networks. Rigorous proofs and necessarily stronger results in terms of convergence are provided for some intuitions in this area. Equally important, explicit expressions of scheduling gain and average system capacity in various scenarios that reveal inter-connections and fundamental tradeoffs among key system parameters are given, which afford us some insights in system design.

A point worth noting here is that, asymptotic analysis with respect to the number of users K , rooted from order statistics and extreme-value theory, focuses on the tail behavior of relevant distributions and may raise concerns on practical feasibility. However, this approach is by far still the most effective one for theoretical analysis on multiuser diversity, valuable at least in the following two aspects: 1) just as large M analysis, it provides good approximations in practice when K is large (though with slower convergence as compared to asymptotes in M); 2) it facilitates the study on the interplay between multiple antennas and multiple users. Also note that our results potentially allow the exploitation of other forms of channel distributions and system models.

Our future work includes study of the interaction between spatial diversity and multiuser diversity in a correlated fading scenario, and extension to the situations when users' channels are heterogeneous,

together with the associated fairness issues. The interaction of multiuser diversity and the diversity-multiplexing tradeoff in MIMO systems also deserves further study.

Acknowledgements

The authors thank Professor J. W. Silverstein and Professor Z. D. Bai for helping with the proof of Lemma 3 in Appendix E.

Appendix A: Proof of Theorem 1

A.1. Preliminary:

In order to prove Theorem 1, we first provide some preliminary results in [14] in Lemma 4. Based on this lemma, Corollary 2 follows, which is key to deriving a tight lower bound for $\bar{S}(K, M)$.

Lemma 4: Let X_1, \dots, X_K be i.i.d. random variables as given in Lemma 1. If $\lim_{x \rightarrow \infty} g(x) = c \geq 0$, then the asymptotic expansion of $\log[-\log F^K(b_K + xg(b_K))]$ at b_K is given by

$$\begin{aligned} \log[-\log F^K(b_K + xg(b_K))] &= \left(-x + \frac{x^2}{2!} g'(b_K) + \frac{x^3}{3!} \left[g(b_K)g^{(2)}(b_K) - 2(g'(b_K))^2 \right] \dots + \dots \right) \\ &\quad + \left(\frac{e^{-x}}{2K} + \frac{5e^{-2x}}{24K^2} + \dots - \frac{1}{8K^3} e^{-3x} + \dots + \dots \right), \end{aligned} \quad (38)$$

where $b_K = F^{-1}(1 - 1/K)$. Furthermore if $\lim_{K \rightarrow \infty} [K \cdot g'(b_K)] = \infty$, the terms in the last group of (38) starting with the term $e^{-x}/2K$ are negligibly small compared to the terms in the first group, i.e.,

$$\log[-\log F^K(b_K + xg(b_K))] \sim -x + \frac{x^2}{2!} g'(b_K) + \frac{x^3}{3!} \left[g(b_K)g^{(2)}(b_K) - 2(g'(b_K))^2 \right] \dots + \dots. \quad (39)$$

Corollary 2: Let X_1, \dots, X_K be i.i.d. random variables as given in Lemma 1. If $\lim_{x \rightarrow \infty} g(x) = c \geq 0$, $g'(x) = O(1/x^{\delta_1})$ with $\delta_1 > 0$, and $b_K = O((\log K)^{\delta_2})$ with $0 < \delta_2 \leq 1$, then there exists a $\kappa > 0$, such that

$$\mathbb{P} \left\{ -\kappa \log \log K \leq \left(\max_{1 \leq k \leq K} X_k \right) - b_K \leq \kappa \log \log K \right\} \geq 1 - O\left(\frac{1}{\log K} \right). \quad (40)$$

Remark: First we can check that $\lim_{K \rightarrow \infty} [K \cdot g'(b_K)] = \infty$, therefore (39) holds. The proof follows readily from Lemma 4 with $x = \log \log K$. This result has slightly generalized Corollary A.1. in [13], where it is assumed that $c > 0$ (in this case we can let $\kappa = c$), $g^{(m)}(x) = O(1/x^m)$ and $b_K = O(\log K)$.

A.2. Tight Lower Bound for $\bar{S}(K, M)$

Apply an extension of Markov's inequality, we have (for sufficiently large K)

$$\begin{aligned} \bar{S}(K, M) &= E \left(\log \left(1 + \gamma_t \left(\max_{1 \leq k \leq K} \gamma_k \right) \right) \right) \geq P \left(\max_{1 \leq k \leq K} \gamma_k \geq b_K - \kappa \log \log K \right) \times \log \left(1 + \gamma_t (b_K - \kappa \log \log K) \right) \\ &\geq \left(1 - O \left(\frac{1}{\log K} \right) \right) \times \log \left(1 + \gamma_t (b_K - \kappa \log \log K) \right) \geq \log \left(1 + \gamma_t b_K \right) - o(1), \end{aligned} \quad (41)$$

where the second inequality follows from Corollary 2, and the last one follows from $b_K = O((\log K)^{\delta_2})$ with $0 < \delta_2 \leq 1$.

A.3. Tight Upper Bound for $\bar{S}(K, M)$

As a final step, we provide an upper bound for $\bar{S}(K, M)$, which coincides with the lower bound asymptotically. Let $S(K, M) = \log\left(1 + \gamma_t \left(\max_{1 \leq k \leq K} \gamma_k\right)\right)$, which is positive with probability 1, then [21]

$$\begin{aligned} \bar{S}(K, M) &= \int_0^\infty P(S(K, M) > x) dx = \int_0^{\log(1+\gamma_t b_K)} P(S(K, M) > x) dx + \int_{\log(1+\gamma_t b_K)}^{+\infty} P(S(K, M) > x) dx \\ &\leq \log(1 + \gamma_t b_K) + \int_{\log(1+\gamma_t b_K)}^{+\infty} P(S(K, M) > x) dx. \end{aligned} \quad (42)$$

In the following, we show that the second term above diminishes as $K \rightarrow \infty$. First note

$$P(S(K, M) > x) = 1 - P(S(K, M) \leq x) = 1 - F_\gamma^K\left(\frac{e^x - 1}{\gamma_t}\right). \quad (43)$$

Since $\lim_{x \rightarrow \infty} \frac{1 - F_\gamma(x)}{f_\gamma(x)} = c \geq 0$, we can find positive constants c_2 and x_0 , such that $1 - F_\gamma(x) < c_2 f_\gamma(x)$,

when $x > x_0$. Thus for sufficiently large x we have

$$1 - F_\gamma^K\left(\frac{e^x - 1}{\gamma_t}\right) = \left(1 - F_\gamma\left(\frac{e^x - 1}{\gamma_t}\right)\right) \left(1 + F_\gamma\left(\frac{e^x - 1}{\gamma_t}\right) + \dots + F_\gamma^{K-1}\left(\frac{e^x - 1}{\gamma_t}\right)\right) \leq K c_2 f_\gamma\left(\frac{e^x - 1}{\gamma_t}\right). \quad (44)$$

Therefore for sufficiently large K and $b_K = O((\log K)^{\delta_2})$,

$$\begin{aligned} \int_{\log(1+\gamma_t b_K)}^{+\infty} P(S(K, M) \geq x) dx &\leq \int_{\log(1+\gamma_t b_K)}^{+\infty} K c_2 f\left(\frac{e^x - 1}{\gamma_t}\right) dx = \int_{b_K}^{+\infty} K c_2 f(x) \frac{\gamma_t}{1 + x \gamma_t} dx \\ &\leq K \frac{c_2 \gamma_t}{1 + \gamma_t b_K} \int_{b_K}^{+\infty} f(x) dx = \frac{K c_2 \gamma_t}{1 + \gamma_t b_K} (1 - F(b_K)) = O\left(\frac{1}{(\log K)^{\delta_2}}\right), \end{aligned} \quad (45)$$

where the last equality uses the fact $(1 - F(b_K)) = 1/K$.

Based on (41), (42) and (45) we can conclude

$$\lim_{K \rightarrow \infty} \left\{ E\left(\log\left(1 + \gamma_t \left(\max_{1 \leq k \leq K} \gamma_k\right)\right)\right) - \log(1 + \gamma_t b_K) \right\} = 0. \quad (46)$$

Appendix B: Proof of Corollary 1

First we can check that

$$\lim_{x \rightarrow \infty} \left(qv x^{v-1} \frac{1 - F_\gamma(x)}{f_\gamma(x)} \right) = 1, \quad (47)$$

when $f_\gamma(x) \sim \alpha x^p e^{-qx^v}$. This leads to the conclusion that $\lim_{x \rightarrow \infty} g_\gamma(x) = c \geq 0$ ($c = 0$ when $v > 1$), and $g'_\gamma(x) = O(1/x^v)$, therefore by Theorem 1, we are left to verify (10).

It can also be referred from (47) that $\lim_{x \rightarrow \infty} \frac{1 - F_\gamma(x)}{\varphi(x)} = 1$, where $\varphi(x) = \alpha x^{p+1-v} e^{-qx^v} / qv$. Therefore, we

only need to solve $\varphi(b_K) = 1/K$, i.e.,

$$b_K^v = \frac{1}{q} \log \frac{K\alpha}{qv} + \frac{p+1-v}{q} \log b_K. \quad (48)$$

Assume $\tau = \frac{\alpha}{qv}$, the first order approximation for b_K is readily given by $\left(\frac{1}{q} \log \tau K \right)^{1/v}$. To obtain the

second order approximation, we just replace b_K on the right hand side of (48) with $\left(\frac{1}{q} \log \tau K \right)^{1/v}$ to get

$$b_K^v = \frac{1}{q} \log \tau K + \frac{p+1-v}{qv} \log \log \tau K, \quad (49)$$

which leads to

$$b_K = \left(\frac{1}{q} \log \tau K \right)^{1/v} \left(1 + \frac{p+1-v}{qv} \frac{\log \log \tau K}{\frac{1}{q} \log \tau K} \right)^{1/v} = \left(\frac{1}{q} \log \tau K \right)^{1/v} + \frac{p+1-v}{qv^2} \frac{\log \log \tau K}{\left(\frac{1}{q} \log \tau K \right)^{(v-1)/v}}. \quad (50)$$

Appendix C: Proof of Lemma 2

When $x \rightarrow +\infty$, $F_\gamma^{MRT/MRC}(x) \rightarrow 1$, therefore

$$f_\gamma^{MRT/MRC}(x) \sim \lim_{x \rightarrow \infty} \text{tr}(\Psi_c^{-1}(x) \Phi_c(x)). \quad (51)$$

As $\lim_{x \rightarrow \infty} \{\Psi_c(x)\}_{i,j} = \lim_{x \rightarrow \infty} \gamma(t-s+i+j-1, x) = (t-s+i+j-2)!$, letting $\lambda = t-s$, we have

$$\Psi_c(+\infty) = \begin{pmatrix} \lambda! & \dots & (\lambda + s - 1)! \\ \vdots & \vdots & \vdots \\ (\lambda + s - 1)! & \dots & (\lambda + 2s - 2)! \end{pmatrix} \quad (52)$$

Meanwhile

$$\Phi_c(x) = \begin{pmatrix} x^\lambda e^{-x} & x^{\lambda+1} e^{-x} & \dots & x^{\lambda+s-1} e^{-x} \\ x^{\lambda+1} e^{-x} & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ x^{\lambda+s+1} e^{-x} & \vdots & \vdots & x^{\lambda+2s-2} e^{-x} \end{pmatrix} = \begin{pmatrix} 1 & x & \dots & x^{s-1} \\ x & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ x^{s-1} & \vdots & \vdots & x^{2s-2} \end{pmatrix} x^\lambda e^{-x}. \quad (53)$$

Therefore

$$\begin{aligned} \text{tr}(\Psi_c^{-1}(+\infty)\Phi_c(x)) &= e^{-x} x^\lambda [a_1 x^{2s-2} + a_2 x^{2s-3} + \dots + a_{2s-2} x + a_{2s-1}] \\ &= e^{-x} x^{\lambda+2s-2} [a_1 + O(1/x)], \end{aligned} \quad (54)$$

where the coefficients $\{a_i\}$ come from linear combinations of elements in $\Psi_c^{-1}(+\infty)$. We are only concerned with the dominant term, whose coefficient can be obtained as follows.

$$\begin{aligned} a_1 &= \frac{\left| \begin{pmatrix} \lambda! & \dots & (\lambda + s - 2)! \\ \vdots & \vdots & \vdots \\ (\lambda + s - 2)! & \dots & (\lambda + 2s - 4)! \end{pmatrix} \right|}{|\Psi_c(+\infty)|} \\ &= \frac{\prod_{k=1}^{s-1} (t-k-1)!(s-k-1)!}{\prod_{k=1}^s (t-k)!(s-k)!} = \frac{1}{(t-1)!(s-1)!} = \frac{1}{(M-1)!(N-1)!}. \end{aligned} \quad (55)$$

Appendix D: Proof of Theorem 2

Let $X_M = \frac{\gamma_M}{\mu_M}$, $\lim_{M \rightarrow \infty} \frac{\sigma_M}{\mu_M} = 0$ indicates that $X_M \xrightarrow{2} 1$ as $M \rightarrow \infty$.

First we show the weaker conclusion regarding the relative mean capacity and scheduling gain. Fix $\delta > 0$. By Markov's inequality, we have

$$E(\log(1 + \gamma_i \gamma_M)) \geq P(X_M \geq (1 - \delta)) \log(1 + \gamma_i (1 - \delta) \mu_M), \quad (56)$$

which together with $X_M \xrightarrow{2} 1$ leads to

$$\liminf_{M \rightarrow \infty} \frac{E(\log(1 + \gamma_i \gamma_M))}{\log(1 + \gamma_i (1 - \delta) \mu_M)} \geq \lim_{M \rightarrow \infty} P(X_M \geq (1 - \delta)) = 1. \quad (57)$$

Now we let $\delta \rightarrow 0$ to get

$$\liminf_{M \rightarrow \infty} \frac{E(\log(1 + \gamma_t \gamma_M))}{\log(1 + \gamma_t \mu_M)} \geq 1. \quad (58)$$

On the other hand, by Jensen's Inequality $\limsup_{M \rightarrow \infty} \frac{E(\log(1 + \gamma_t \gamma_M))}{\log(1 + \gamma_t \mu_M)} \leq 1$, which leads to the first part of the weaker conclusion. For the second part, it is known [11] that $E[\gamma_{k^*}] \leq \mu_M + \frac{(K-1)\sigma_M}{\sqrt{2K-1}}$. Using Jensen's inequality, we have

$$\bar{S}(K, M) \leq \log \left(1 + \gamma_t \left(\mu_M + \frac{(K-1)\sigma_M}{\sqrt{2K-1}} \right) \right). \quad (59)$$

So

$$0 \leq \lim_{M \rightarrow \infty} \frac{G(K, M)}{\bar{R}(M)} \leq \lim_{M \rightarrow \infty} \frac{\log \left(1 + \gamma_t \left(\mu_M + \frac{(K-1)\sigma_M}{\sqrt{2K-1}} \right) \right)}{\log(1 + \gamma_t \mu_M)} - 1 = 0. \quad (60)$$

We now turn to the proof concerning the absolute mean capacity and scheduling gain. We can write

$$\log X_M = \log X_M \mathbf{I}_{(0,1)}(X_M) + \log X_M \mathbf{I}_{[1,\infty)}(X_M) = Y_M^{(1)} + Y_M^{(2)}. \quad (61)$$

First we have

$$0 \leq Y_M^{(2)} \leq (X_M - 1) \mathbf{I}_{[1,\infty)}(X_M). \quad (62)$$

Therefore $E(Y_M^{(2)}) \rightarrow 0$ since $E((X_M - 1) \mathbf{I}_{[1,\infty)}(X_M)) \leq E(|X_M - 1|) \rightarrow 0$ as $M \rightarrow \infty$. In order to show that $E(Y_M^{(1)}) = E(\log X_M \mathbf{I}_{(0,1)}(X_M)) \rightarrow 0$, we make the following claim.

Claim 1: If a random variable $X_n \xrightarrow{P} 0$ as $n \rightarrow \infty$, $X_n \mathbf{I}_E(X_n) \xrightarrow{P} 0$ for any event E .

This claim is easy to verify as $\forall \varepsilon, P(|X_n \mathbf{I}_E(X_n)| > \varepsilon) \leq P(|X_n| > \varepsilon) \rightarrow 0$ as $n \rightarrow \infty$.

Now that $X_M \xrightarrow{2} 1$ we have $\log X_M \xrightarrow{P} 0$, as X_M is positive and the logarithm function is continuous. By

claim 1 we in turn have $\log X_M \mathbf{I}_{(0,1)}(X_M) \xrightarrow{P} 0$. This together with the uniform integrability

of $\{\log X_M \mathbf{I}_{(0,1)}(X_M)\}$ results in $Y_M^{(1)} \xrightarrow{1} 0$ [21], and it follows that $\lim_{M \rightarrow \infty} \bar{R}(M) = \log(1 + \gamma_t \mu_M)$. Hence

$$\begin{aligned} 0 \leq \lim_{M \rightarrow \infty} G(K, M) &= \lim_{M \rightarrow \infty} (\bar{S}(K, M) - \bar{R}(M)) \\ &\leq \lim_{M \rightarrow \infty} \left(\log \left(1 + \gamma_t \left(\mu_M + \frac{(K-1)\sigma_M}{\sqrt{2K-1}} \right) \right) - \log(1 + \gamma_t \mu_M) \right) = 0. \end{aligned} \quad (63)$$

Appendix E: Proof of Lemma 3

Define $\mathbf{B} = \frac{1}{M} \mathbf{H} \mathbf{H}^H$, and $\lambda_{\max}(\mathbf{B}) = \left\| \frac{1}{\sqrt{M}} \mathbf{H} \right\|^2$. Assuming $b > (1 + \sqrt{r})^2$, we have

$$E(\lambda_{\max}(\mathbf{B})) = E(\lambda_{\max}(\mathbf{B}) \mathbf{I}_{[0,b]}(\lambda_{\max}(\mathbf{B}))) + E(\lambda_{\max}(\mathbf{B}) \mathbf{I}_{(b,\infty)}(\lambda_{\max}(\mathbf{B}))). \quad (64)$$

Using dominated convergence theorem together with Theorem 3.1 of [22], we can obtain

$$\lim_{M \rightarrow \infty} E(\lambda_{\max}(\mathbf{B}) \mathbf{I}_{[0,b]}(\lambda_{\max}(\mathbf{B}))) = (1 + \sqrt{r})^2. \quad (65)$$

The remaining task is to show

$$\lim_{M \rightarrow \infty} E(\lambda_{\max}(\mathbf{B}) \mathbf{I}_{(b,\infty)}(\lambda_{\max}(\mathbf{B}))) = 0. \quad (66)$$

We define two new $N \times M$ matrices \mathbf{Y} and \mathbf{Z} based on \mathbf{H} , with each entry of \mathbf{Y} being $y_{i,j} = h_{i,j} \mathbf{I}_{[0,\delta\sqrt{M}]}(|h_{i,j}|)$ and each entry of \mathbf{Z} being $z_{i,j} = h_{i,j} \mathbf{I}_{(\delta\sqrt{M},\infty)}(|h_{i,j}|)$, where the detailed definition of $\delta = \delta_M \rightarrow 0$ can be found in [22] (see the proof of Lemma 2.2) and [23] (see the discussion below (1.8)). What we need in this proof is the following two results from [22][23]:

$$\frac{1}{\delta^2} E(|h_{1,1}|^4 \mathbf{I}_{(\delta\sqrt{M},\infty)}(|h_{1,1}|)) \rightarrow 0, \text{ as } M \rightarrow \infty, \quad (67)$$

and

$$\sum_{M=1}^{\infty} E\left(\frac{\lambda_{\max}(\mathbf{B}')} {b'}\right)^k < \infty, \quad (68)$$

where $\mathbf{B}' = \frac{1}{M} (\mathbf{Y} - E(y_{1,1}) \mathbf{1}_N \mathbf{1}_M^T) (\mathbf{Y} - E(y_{1,1}) \mathbf{1}_N \mathbf{1}_M^T)^H$ ($\mathbf{1}_s$ is an $s \times 1$ column vector with all one entries),

b' is a real number such that $(1 + \sqrt{r})^2 < b' < b$, and $k = k_M$ satisfies (4.3) and (4.4) of [22]. Further

assume $\mathbf{B}'' = \frac{1}{M} \mathbf{Z} \mathbf{Z}^H$. With these definitions,

$$\begin{aligned} E(\lambda_{\max}(\mathbf{B}) \mathbf{I}_{(b,\infty)}(\lambda_{\max}(\mathbf{B}))) &= E\left(\left\| \frac{1}{\sqrt{M}} (\mathbf{Y} - E(y_{1,1}) \mathbf{1}_N \mathbf{1}_M^T) + \frac{1}{\sqrt{M}} E(y_{1,1}) \mathbf{1}_N \mathbf{1}_M^T + \frac{1}{\sqrt{M}} \mathbf{Z} \right\|_{(b,\infty)}^2 (\lambda_{\max}(\mathbf{B}))\right) \\ &\leq 3 \left[E(\lambda_{\max}(\mathbf{B}') \mathbf{I}_{(b,\infty)}(\lambda_{\max}(\mathbf{B}))) + \frac{1}{M} |E(y_{1,1})|^2 \|\mathbf{1}_N \mathbf{1}_M^T\|^2 + E(\lambda_{\max}(\mathbf{B}'')) \right]. \end{aligned} \quad (69)$$

The second term above admits

$$\begin{aligned}
& \left| E(y_{1,1}) \right|^2 \frac{1}{M} \left\| \mathbf{1}_N \mathbf{1}_M^T \right\|^2 = N \left| E \left(h_{1,1} \mathbf{I}_{[0, \delta\sqrt{M}]}(|h_{1,1}|) \right) \right|^2 = N \left| -E \left(h_{1,1} \mathbf{I}_{(\delta\sqrt{M}, \infty)}(|h_{1,1}|) \right) \right|^2 \\
& = \frac{N}{M} \frac{1}{\delta^2} \left| \delta\sqrt{M} E \left(h_{1,1} \mathbf{I}_{(\delta\sqrt{M}, \infty)}(|h_{1,1}|) \right) \right|^2 \leq \frac{N}{M} \frac{1}{\delta^2} \left| E \left(|h_{1,1}|^2 \mathbf{I}_{(\delta\sqrt{M}, \infty)}(|h_{1,1}|) \right) \right|^2 \leq \frac{N}{M} \frac{1}{\delta^2} E \left(|h_{1,1}|^4 \mathbf{I}_{(\delta\sqrt{M}, \infty)}(|h_{1,1}|) \right).
\end{aligned} \tag{70}$$

From (67), we know the above expression approaches 0 as M goes to infinity.

For the third term, we have

$$E(\lambda_{\max}(\mathbf{B}'')) \leq E\left(\frac{1}{M} \sum_{i,j} |z_{i,j}|^2\right) = NE\left(|h_{1,1}|^2 \mathbf{I}_{(\delta\sqrt{M}, \infty)}(|h_{1,1}|)\right) \leq \frac{N}{M} \frac{1}{\delta^2} E\left(|h_{1,1}|^4 \mathbf{I}_{(\delta\sqrt{M}, \infty)}(|h_{1,1}|)\right). \tag{71}$$

Therefore the above expression also approaches 0 as M goes to infinity.

Denote $a = \sqrt{b} - \sqrt{b'}$, we have

$$\mathbf{I}_{(b, \infty)}(\lambda_{\max}(\mathbf{B})) \leq \mathbf{I}_{(b', \infty)}(\lambda_{\max}(\mathbf{B}')) + \mathbf{I}_{(0, b]}(\lambda_{\max}(\mathbf{B}')) \mathbf{I}_{(a, \infty)}\left(\sqrt{\lambda_{\max}(\mathbf{B})} - \sqrt{\lambda_{\max}(\mathbf{B}')} \right). \tag{72}$$

According to Markov's inequality, we have

$$E\left(\mathbf{I}_{(a, \infty)}\left(\sqrt{\lambda_{\max}(\mathbf{B})} - \sqrt{\lambda_{\max}(\mathbf{B}')} \right)\right) = P\left(\sqrt{\lambda_{\max}(\mathbf{B})} - \sqrt{\lambda_{\max}(\mathbf{B}')} > a\right) \leq \frac{E\left(\sqrt{\lambda_{\max}(\mathbf{B})} - \sqrt{\lambda_{\max}(\mathbf{B}')} \right)}{a}. \tag{73}$$

Furthermore

$$\begin{aligned}
& \left| \sqrt{\lambda_{\max}(\mathbf{B})} - \sqrt{\lambda_{\max}(\mathbf{B}')} \right| = \left\| \frac{1}{\sqrt{M}} \mathbf{H} \right\| - \left\| \frac{1}{\sqrt{M}} (\mathbf{Y} - E(y_{1,1}) \mathbf{1}_N \mathbf{1}_M^T) \right\| \\
& \leq \frac{1}{\sqrt{M}} \left\| (\mathbf{H} - \mathbf{Y}) \right\| + \frac{1}{\sqrt{M}} \left\| E(y_{1,1}) \mathbf{1}_N \mathbf{1}_M^T \right\| = \frac{1}{\sqrt{M}} \left\| \mathbf{Z} \right\| + \left\| \frac{1}{\sqrt{M}} E(y_{1,1}) \mathbf{1}_N \mathbf{1}_M^T \right\|.
\end{aligned} \tag{74}$$

Therefore

$$\begin{aligned}
& E\left(\lambda_{\max}(\mathbf{B}') \mathbf{I}_{(0, b]}(\lambda_{\max}(\mathbf{B}')) \mathbf{I}_{(a, \infty)}\left(\sqrt{\lambda_{\max}(\mathbf{B})} - \sqrt{\lambda_{\max}(\mathbf{B}')} \right)\right) \leq b' E\left(\mathbf{I}_{(a, \infty)}\left(\sqrt{\lambda_{\max}(\mathbf{B})} - \sqrt{\lambda_{\max}(\mathbf{B}')} \right)\right) \\
& \leq \frac{b'}{a} \left(\frac{1}{\sqrt{M}} \left\| \mathbf{Z} \right\| + \left\| \frac{1}{\sqrt{M}} E(y_{1,1}) \mathbf{1}_N \mathbf{1}_M^T \right\| \right) \leq \frac{2b'}{a} \left(\frac{N}{M} \right)^{1/2} \frac{1}{\delta} E^{1/2}\left(|h_{1,1}|^4 \mathbf{I}_{(\delta\sqrt{M}, \infty)}\right).
\end{aligned} \tag{75}$$

Again by (67), the above expression also approaches 0 as M goes to infinity. Finally we have

$$\begin{aligned}
& E\left(\lambda_{\max}(\mathbf{B}') \mathbf{I}_{(b', \infty)}(\lambda_{\max}(\mathbf{B}'))\right) = b' E\left(\frac{\lambda_{\max}(\mathbf{B}')}{b'} \mathbf{I}_{(b', \infty)}(\lambda_{\max}(\mathbf{B}'))\right) \\
& \leq b' E\left(\left(\frac{\lambda_{\max}(\mathbf{B}')}{b'}\right)^k \mathbf{I}_{(b', \infty)}(\lambda_{\max}(\mathbf{B}'))\right) \leq b' E\left(\frac{\lambda_{\max}(\mathbf{B}')}{b'}\right)^k,
\end{aligned} \tag{76}$$

By (68), we have

$$\lim_{M \rightarrow \infty} E \left(\frac{\lambda_{\max}(\mathbf{B}')}{b'} \right)^k = 0. \quad (77)$$

Therefore from (72)-(77), we can obtain

$$\lim_{M \rightarrow \infty} E \left(\lambda_{\max}(\mathbf{B}') \mathbf{I}_{(b, \infty)}(\lambda_{\max}(\mathbf{B})) \right) = 0. \quad (78)$$

From (69), (70), (71) and (78), we can get $\lim_{M \rightarrow \infty} E \left(\lambda_{\max}(\mathbf{B}) \mathbf{I}_{(b, \infty)}(\lambda_{\max}(\mathbf{B})) \right) = 0$. Thus

$$\lim_{M \rightarrow \infty} E \left(\lambda_{\max} \left(\frac{1}{M} \mathbf{H} \mathbf{H}^H \right) \right) = (1 + \sqrt{r})^2. \quad (79)$$

Along a similar line discussed above, we can show $\lim_{M \rightarrow \infty} E \left(\lambda_{\max}^2 \left(\frac{1}{M} \mathbf{H} \mathbf{H}^H \right) \right) = (1 + \sqrt{r})^4$, therefore

$$\lim_{M \rightarrow \infty} \sigma^2 \left(\lambda_{\max} \left(\frac{1}{M} \mathbf{H} \mathbf{H}^H \right) \right) = \lim_{M \rightarrow \infty} \left(E \left(\lambda_{\max}^2 \left(\frac{1}{M} \mathbf{H} \mathbf{H}^H \right) \right) - E^2 \left(\lambda_{\max} \left(\frac{1}{M} \mathbf{H} \mathbf{H}^H \right) \right) \right) = 0. \quad (80)$$

Appendix F: Scheduling Gain When both M and K Go to Infinity – Fixed N

It is also interesting to extend the analysis in Section V to the case of fixed N and large M . For STBC/MRC, the same norming constants can be used but with different interpretation; note that in this scenario, the shrinking of the link standard deviation with respect to the link mean occurs at a slower rate ($1/\sqrt{M}$ rather than $1/M$). As a consequence, we have

$$G_N^{STBC/MRC}(K, M) \rightarrow \begin{cases} 0, & \text{when } \log K = o(M) \\ c, & \text{when } \log K = O(M) \\ +\infty, & \text{when } \log K = \omega(M). \end{cases} \quad (81)$$

But this is achieved at the price of a saturated link capacity even when $M \rightarrow \infty$ (see Section IV). The analysis (30) for SC/SC remains unchanged for fixed N , i.e.,

$$G_N^{SC/SC}(K, M) \sim G^{SC/SC}(K, M). \quad (82)$$

For MRT/MRC, the fixed N analysis deviates from above in that w_k formed as in (33) does not follow the Tracy-Widom law. Alternatively, by law of large numbers, it can be shown that $\max \{ \gamma(k, M) \}_{k=1}^K$ approaches the largest element among KN i.i.d. random variables of the form $\sum_{i=1}^M |h_{i,1}|^2$. Following the

same approach, we can obtain the asymptotic scheduling gain as $\log\left(1 + \sqrt{\frac{2\log(KN)}{M}}\right)$, which still leads

to

$$G_N^{MRT/MRC}(K, M) \sim G^{MRT/MRC}(K, M). \quad (83)$$

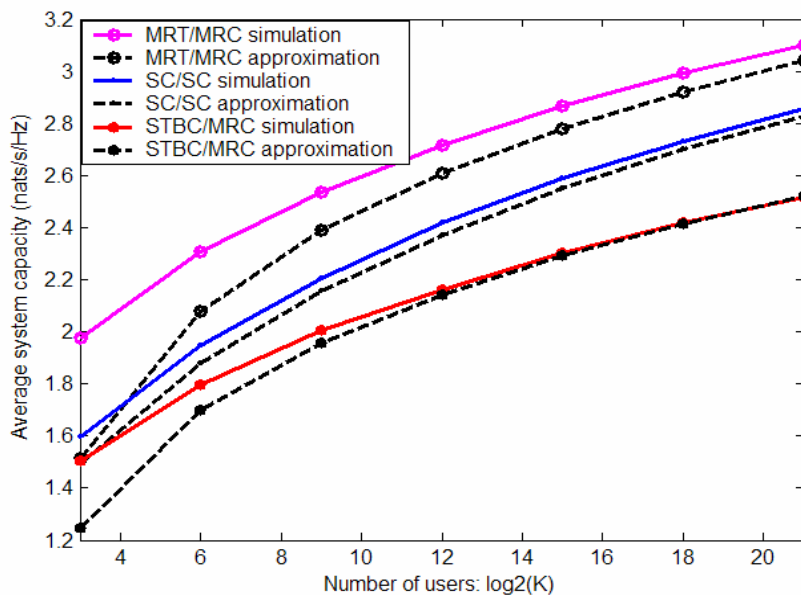


Fig.1. Average system capacity of opportunistic scheduling ($\gamma_t = 0$ dB, $M = N = 2$)

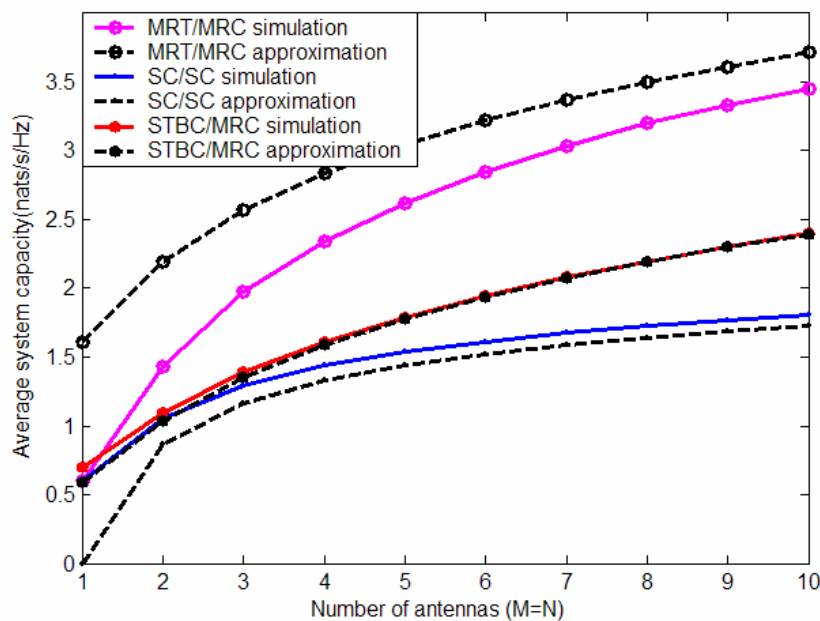


Fig.2. Average system capacity of round robin scheduling ($\gamma_t = 0$ dB)

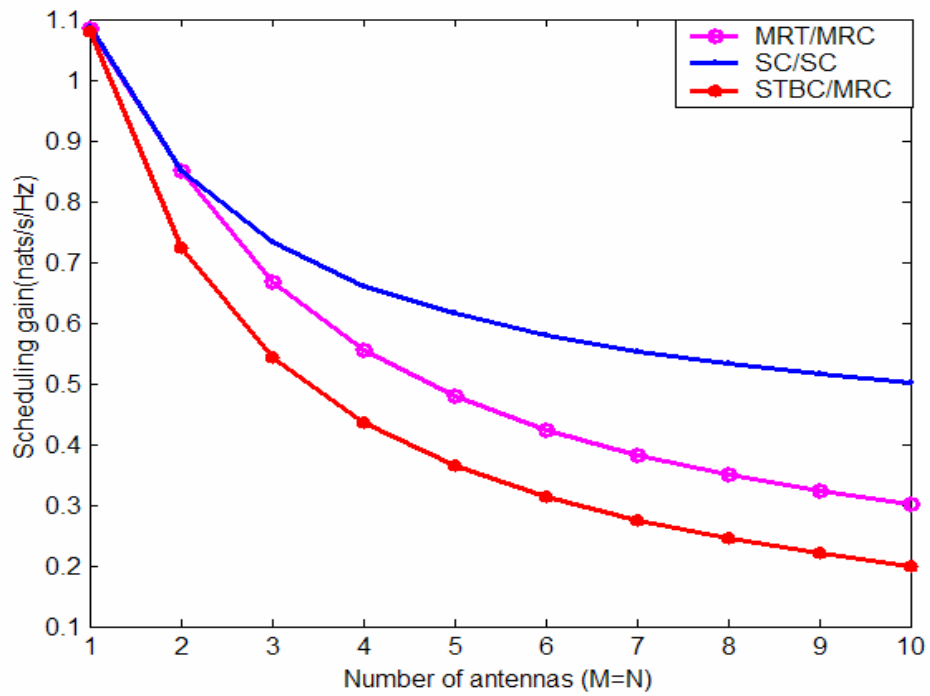


Fig.3. Scheduling gain as the number of antennas grows ($\gamma_t = 0$ dB, $K = 50$)

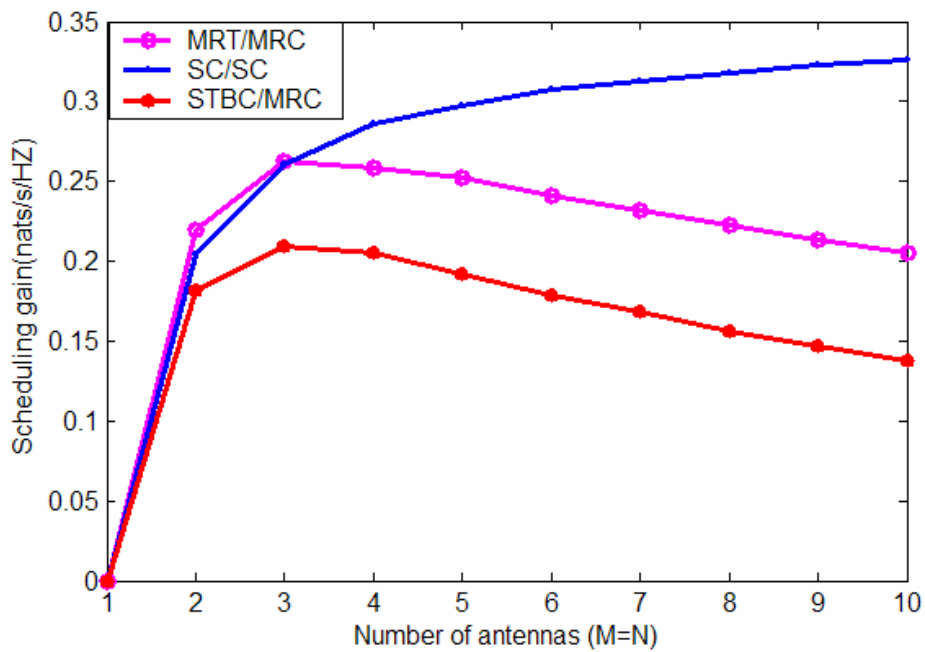


Fig.4. Scheduling gain as the number of antennas and users grow ($\gamma_t = 0$ dB, $K = M$)

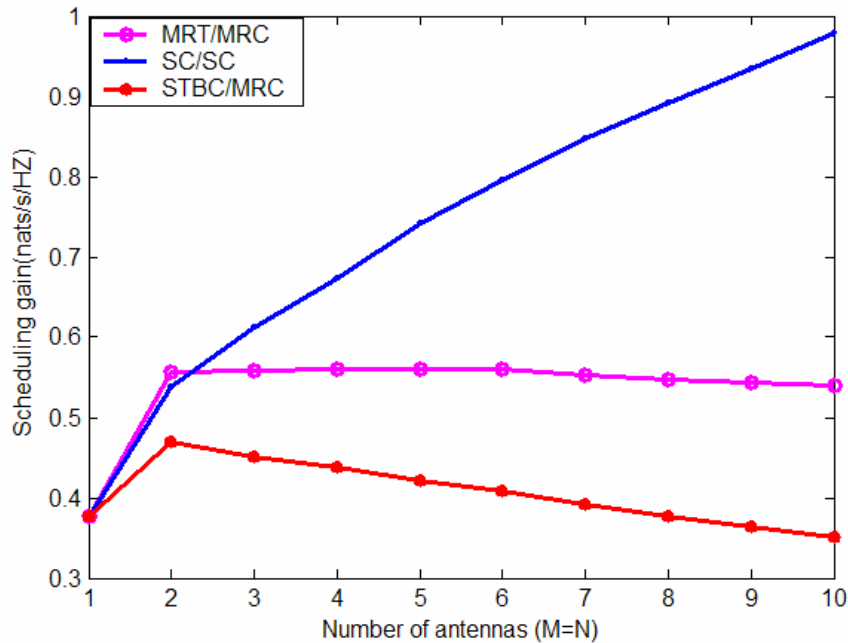


Fig.5. Scheduling gain as the number of antennas and users grow ($\gamma_t = 0$ dB, $K = e^M$)

References

- [1] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels", *IEEE J. Select. Areas Commun.*, vol. 21, pp. 684-702, June 2003.
- [2] D. Gesbert et al., "From theory to practice: An overview of MIMO space-time coded wireless systems", *IEEE J. Select. Areas Commun.*, vol. 21, pp. 281-302, Apr. 2003.
- [3] R. Knopp and P. Humblet, "Information capacity and power control in single user cell multiuser communications," in *Proc. International Conference on Communications*, vol. 1, Seattle, WA, June 1995, pp. 331-335.
- [4] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1277-1294, June 2002.
- [5] C. Mun, et al., "Exact capacity analysis of multiuser diversity combined with transmit diversity," *Electronic Letters*, vol. 40, no. 22, pp. 1423-1424, Oct. 2004.
- [6] C.-J. Chen and L.-C. Wang, "A unified capacity analysis for wireless systems with joint antenna and multiuser diversity in Nakagami fading channels," in *Proc. International Conference on Communications*, vol.6, Paris, June 2004, pp.3523-3527.
- [7] V. K. N. Lau, Y. Liu and T. A. Chen, "The role of transmit Diversity on wireless communications – Reverse link analysis with partial feedback," *IEEE Transactions on Communications*, vol. 50, no. 12, pp. 2082-2090, Dec. 2002.
- [8] R. Gozali, R. M. Buehrer and B. D. Woerner, "The impact of multiuser diversity on space-time block coding," *IEEE Communications Letters*, vol. 7, no. 5, pp. 213-215, May 2003.
- [9] J. Jiang, R. M. Buehrer and W. H. Tranter, "Antenna diversity in multiuser data networks," *IEEE Transactions on Communications*, vol. 52, no. 3, pp. 490-497, Mar. 2004.
- [10] E. G. Larsson, "On the combination of spatial diversity and multiuser diversity," *IEEE Communications Letters*, vol.8, no. 8, pp. 517-519, Aug. 2004.

- [11] H. A. David and H. N. Nagaraja, *Order Statistics*, 3rd edition, New York: Wiley, 2003.
- [12] J. Galambos, *the Asymptotic Theory of Extreme Order Statistics*, 2nd edition, New York: Wiley, 1978.
- [13] M. Sharif and B. Hassibi, "On the capacity of MIMO broadcast channels with partial side information," *IEEE Transactions on Information Theory*, vol. 51, no. 2, pp. 506-522, Feb. 2005.
- [14] N. T. Uzgoren, "The asymptotic development of the distribution of the extreme values of a sample," in *Studies in Mathematics and Mechanics Presented to Richard von Mises*, New York: Academic, 1954, pp. 346-353.
- [15] A. F. Molisch, "MIMO systems with antenna selection-an overview", *IEEE Microwave Magazine*, vol. 5, no. 1, pp 46-56, Mar. 2004.
- [16] T. K. Y. Lo, "Maximum ratio transmission," *IEEE. Trans. Commun.*, vol. 47, no. 10, pp. 1458-1461, Oct. 1999.
- [17] M. Z. Win and J. H. Winters, "Analysis of hybrid selection/maximum ratio combining in Rayleigh fading," *IEEE Transactions on Communications*, vol. 47, no. 12, pp. 1773-1776, Dec. 1999.
- [18] P. A. Dighe, R. K. Mallik, and S. S. Jamuar, "Analysis of transmit-receive diversity in Rayleigh fading," *IEEE Transactions on Communications*, vol. 51, no. 4, pp. 694 -703, April 2003.
- [19] C. G. Khatri, "Distribution of the largest or the smallest characteristic root under null hypothesis concerning complex multivariate normal populations," *Ann. Math. Stat.*, vol. 35, pp. 1807-1810, Dec. 1964.
- [20] M. Kang, and M-Slim Alouini, "Largest eigenvalue of complex wishart matrices and performance analysis of MIMO MRC systems," *IEEE Journal of Selected Areas in Communications*, vol. 21, no. 3, pp. 418-426, April, 2003.
- [21] G. Grimmett and D. Stirzaker, *Probability and Random Processes*, 3rd edition, Oxford University, 2001.
- [22] Y. Q. Yin, Z. D. Bai, and P.R. Krishnaiah, "On the limit of the largest eigenvalue of the large dimensional sample covariance matrix," *Probability Theory and Related Fields*, vol. 78, pp. 509-521, 1988.
- [23] Z. D. Bai, and Jack W. Silverstein, "CLT for linear spectral statistics of large-dimensional sample covariance matrices," *The Annals of Probability*, vol. 32, no. 1A, pp.553-605, 2004.
- [24] A. Edelman, "Eigenvalues and condition numbers of random matrices," *Siam J. Matrix Anal. Appl.*, vol.9, no. 4, pp. 543-560, Oct. 1988.
- [25] B. M. Hochwald, T. L. Marzetta, and V. Tarokh, "Multi-antenna channel-hardening and its implications for rate feedback and scheduling," *IEEE Trans. Inform. Theory*, vol. 50, pp. 1893-1909, Sept. 2004.
- [26] I. M. Johnstone, "On the distribution of the largest eigenvalue in principal components analysis," *The Annals of Statistics*, vol. 29. no. 2, pp. 295-327, 2001.