Joint User Selection and Feedback Bit Allocation Based on Sparsity Constraint in MIMO Virtual Cellular Networks

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Abstract—In this paper, we jointly consider the user selection and feedback design problems in a virtual cellular network (VCN), where multiple base stations (BSs) share a user set. In many practical systems, the uplink feedback channel is generally shared by multiple users. Thus, the feedback budget allocated to unselected users not only wastes the feedback resources, but harms the system throughput by decreasing the available feedback budget for the selected users. We optimize both the user selection and feedback bit allocation based on long-term average channel information of the users. We first analyze the effects of the quantization error on the average achievable rate of the VCN system. Next, we propose a user selection and feedback bit allocation protocol under each BS’s sum feedback rate constraint as well as the sparsity constraint on all users’ feedback sizes. We show that the joint optimization problem can be decoupled into several NP-hard subproblems, one for each BS. We describe the brute-force searching algorithm for the optimal solution, and propose an efficient algorithm with significantly reduced computational complexity by relaxing the sparsity constraint on the feedback sizes. As a result, only the selected users exploit the uplink feedback budget, and the system performance is improved.

Index Terms—Multiple-input multiple-output (MIMO), virtual cellular network, limited feedback, user selection

I. INTRODUCTION

In spite of the successful launch of 4G wireless communication systems (e.g., long-term evolution [1]), an increasing demand of the wireless data traffic promotes the development of new wireless communication technologies. According to the report from CISCO [2], the total network traffic in 2018 is expected to be ten times of that in 2013. As the network traffic covered by the 4G system is reaching its capacity, we need more advanced wireless technology for 5G wireless communication systems. Currently three prominent wireless technologies are considered for 5G [3]: ultra-densification, mmWave, and massive multiple-input multiple-output (MIMO).

Ultra-densification, i.e., deploying more and more transmitters in the same area, is an effective way to increase the spectral efficiency of a cellular system. In addition to simply reducing the cell size, various heterogeneous network structures have been proposed where pico-cells or femto-cells co-exist with original macro-cells for better spectral efficiency [4]–[6]. Serving mobile users becomes more challenging in the face of shrinking cell size, and more advanced user association techniques are needed. One possible solution is the virtual cellular network [6]–[9], where multiple base stations (BSs) share the user pool so that each user can be served by any BS.

To support multiple users simultaneously, each BS requires the channel state information (CSI) to effectively manage the inter-user interference. Contrary to time-division duplexing (TDD) systems, where the uplink and the downlink channels are essentially reciprocal, many practical systems adopt frequency-division duplexing (FDD), so the uplink and downlink channels are largely independent of each other. In this case, the transmitters should obtain the CSI from each user via a feedback link. Due to resource constraints, the budget for CSI feedback is usually limited and the CSI needs to be quantized. The effects of the channel quantization error are widely investigated in many wireless systems including OFDM systems [10], point-to-point MIMO systems [11], and multiuser MIMO systems [12]–[15]. Efficient channel quantization is especially important in multiuser MIMO systems as the inter-user interference due to the quantization errors harms the multiplexing gain in the high SNR region [12].

User selection under limited feedback scenarios have also been addressed in literature [16]–[19]. In conventional systems, user selection is generally placed after the channel feedback stage [16]–[18] and considered separately. The authors of [17] found the approximated sum rate of a MIMO BC as a function of the number of users and the feedback size per user. Also, the authors of [18] approximated the sum rate when every user feeds the quantized CSI, and the transmitter selects and supports the users having the semi-orthogonal channels. However, a non-zero feedback size for every user becomes unrealistic as the number of users increases.

The users in the same network generally share the uplink feedback channels. The feedback bit sharing problem is widely investigated in literature (e.g., [13], [15]), but mainly with the fixed users, and user selection was not considered. The authors of [19] proposed a user pairing method with feedback bit allocation in a distributed antenna system when the supported user set is already fixed. In many practical systems, each

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transmitter has many more users than it can support at once. Also, the feedback budget shared by the users is limited, so the average feedback size per user will significantly decrease as the number of users increases. Thus, the feedback budget allocated to the unselected users wastes the resource, and it is a natural consideration that only the selected users are allocated non-zero feedback sizes.

There have been many beamforming design algorithms for various communication systems. The authors in [20] proposed a beamforming design for weighted sum-rate maximization in a MIMO BC by solving the equivalent weighted minimum mean-square error (MMSE) minimization problem. The authors in [21] proposed iterative beamforming design algorithms for MIMO interfering broadcast channels that also can handle the user admission control. However, these method assumed perfect CSIT, thus particularly suitable for TDD systems whose uplink and downlink channels are reciprocal as shown in [22].

In this paper, we propose a user selection and feedback bit allocation protocol for a VCN system, in which the two problems are jointly considered. In our proposed scheme, each BS has the knowledge of all users’ path loss coefficients, and jointly optimizes user selection and feedback bit allocation considering the effects of the quantization errors. Since only the selected users occupy the feedback links, the feedback sizes allocated to all users are naturally sparse, and hence the joint optimization problem becomes the convex-cardinality problem, which is known as NP-hard [23, Chap. 6]. Our proposed protocol solves this problem with proper convex relaxations.

Our main contributions are summarized as follows:

- We analyze the achievable rate of the VCN system with respect to the user selection and the feedback bit allocation and find a lower bound of sum rate in an average sense.
- Next, we establish the joint optimization problem of user selection and feedback bit allocation, which is NP-hard, and show that our joint optimization problem can be solved in a distributed manner.
- To solve the optimization problem, we describe a brute-force search algorithm first with analysis of its complexity. Then, we propose an efficient algorithm with low complexity by relaxing the sparsity constraint on the feedback sizes.
- Finally, we design a user selection and feedback bit allocation protocol based on the solution of the optimization problem.

The rest of the paper is organized as follows. In Section II, we describe the system model and formulate the problem. In Section III, we analyze the effects of the quantization error on the sum rate and establish alternative optimization problems. Our user selection and feedback bit allocation protocol is proposed in Section IV and numerically evaluated in Section V. Then, Section VI concludes our paper.

Notations. For a vector $\mathbf{x}$, we denote its conjugate transpose and $l_2$-norm by $\mathbf{x}^\dagger$ and $|\mathbf{x}|$, respectively. For an integer $n$, we use $\{n\}$ to denote the set of all positive integers less than or equal to $n$, i.e., $\{n\} \triangleq \{1, \ldots, n\}$. Also, the cardinality of a set $\mathcal{S}$ is denoted by $\text{Card}(\mathcal{S})$.

II. PROBLEM FORMULATION

A. System Model

Our system model is depicted in Fig. 1. We consider a VCN system, where $N$ BSs having $N_t$ antennas each share a total of $K$ users in the network, so each user can be dynamically served by any BS. We refer to a disc of radius $r$ centered at the BS $n$ as the inner-cell $n$ and the remained part (whose distance from the BS $n$ is at least $r$) as the outer-cell $n$.

We denote the set of all users in the network by $\mathcal{K} \triangleq [K]$, and each BS divides the user set $\mathcal{K}$ into the sets of inner-cell users and outer-cell users. Let $\gamma_{kn}$ be the path loss coefficient from the $n$th BS to the $k$th user. Then, the set of inner-cell users of BS $n$ denoted by $\mathcal{K}_{IN}^n$ can be defined by

$$\mathcal{K}_{IN}^n = \{k \in [K] \mid \gamma_{kn} < \gamma_{th}\},$$

where $\gamma_{th}$ is the path loss coefficient corresponding to distance $r$.\(^1\) Let $K_{IN}^n$ be the number of inner-cell users for BS $n$, i.e., $K_{IN}^n \triangleq \text{Card}(\mathcal{K}_{IN}^n)$. Then, the set of outer-cell users for the BS $n$ becomes $\mathcal{K} - \mathcal{K}_{IN}^n$ and satisfies that $\text{Card}(\mathcal{K} - \mathcal{K}_{IN}^n) = K - K_{IN}^n$.

For generality, we assume that each BS serves some of its inner-cell users and some of its outer-cell users; each BS selects $M$ users in the network, among which at most $M_{IN}$ users can be selected from the inner-cell. Denote by $\mathcal{S}_n \subset \mathcal{K}$ the set of selected users at BS $n$, which satisfies $\text{Card}(\mathcal{S}_n) = M$. Also, we have $\text{Card}(\mathcal{S}_n \cap \mathcal{K}_{IN}^n) \leq M_{IN}$, which is equivalent to

$$\text{Card}\{s \in \mathcal{S}_n \mid \gamma_{sn} < \gamma_{th}\} \leq M_{IN}. \tag{1}$$

At the same time, the sets $\mathcal{S}_1, \ldots, \mathcal{S}_N$ are assumed to be pairwise disjoint user sets, i.e., $\mathcal{S}_i \cap \mathcal{S}_j = \emptyset$ for all $i \neq j$.

Fig. 1. Illustration of a VCN system. All BSs share a user pool and serve the selected users both in inner-cell and outer-cell.
When user \( k \in [K] \) is supported by the \( n \)th BS, its received signal denoted by \( y_{kn} \) is given by
\[
y_{kn} = \sqrt{\gamma_{kn}} \mathbf{h}_{kn}^\dagger x_n + \sum_{i \in [N] \setminus \{n\}} \sqrt{\gamma_{ki}} \mathbf{h}_{ki}^\dagger x_i + n_k, \tag{2}
\]
where \( \gamma_{kn} \) is the path loss coefficient from the \( n \)th BS to the \( k \)th user, \( \mathbf{h}_{kn} \in \mathbb{C}^{N_k \times 1} \) is the vector channel from the \( n \)th BS to the \( k \)th user, and \( x_n \in \mathbb{C}^{N_k} \) is the transmitted signal from the \( n \)th BS such that \( \mathbb{E}[|x_n|^2] = P_n \) with \( P_n \) the total transmit power for the BS \( n \). Also, \( n_k \) is a complex Gaussian noise at user \( k \) with zero mean and unit variance. In (2), the term \( \sqrt{\gamma_{kn}} \mathbf{h}_{kn}^\dagger x_n \) is its received signal from the BS \( n \), which contains the inter-user interference as well as the desired signal, while the term \( \sum_{i \in [N] \setminus \{n\}} \sqrt{\gamma_{ki}} \mathbf{h}_{ki}^\dagger x_i \) is the inter-cell interference from the other BSs.

To nullify the inter-user interference, each BS adopts the zero-forcing (ZF) beamforming scheme. Thus, the BS \( n \) exploits the CSI of the selected users \( S_n \), i.e., \( \{ \mathbf{h}_{sn} : s \in S_n \} \), to find their beamforming vectors. Since perfect CSI at the BSs is impossible in practical systems, the BSs should obtain the quantized CSI from each user. In this study, as a common practice, we assume that each user feeds back the channel direction information to assist the BSs’ beamforming designs [12], [13], [15], [19]. Meanwhile, we assume that each BS already knows all path losses from all BSs to all users, as they are quasi-static large-scale propagation effects, easy to measure and share [24], [25].

We assume that user \( k \) quantizes the direction of the channel from the \( n \)th BS, i.e., \( \mathbf{h}_{kn} \triangleq \mathbf{h}_{kn}/||\mathbf{h}_{kn}|| \), to a unit vector \( \mathbf{h}_{kn} \in \mathbb{C}^{N_k \times 1} \) using a \( b_{kn} \)-bit quantizer and feeds it to the BS \( n \). In this case, the channel direction can be represented by
\[
\tilde{\mathbf{h}}_{kn} = \sqrt{1 - Z_{kn}^2} \mathbf{h}_{kn} + \sqrt{Z_{kn}} \mathbf{e}_{kn}, \tag{3}
\]
where \( Z_{kn} \in [0,1] \) is the quantization error defined as \( Z_{kn} \triangleq \sin^2(\angle(\mathbf{h}_{kn}, \mathbf{h}_{kn})) \), and \( \mathbf{e}_{kn} \in \mathbb{C}^{N_k \times 1} \) is the quantization error vector such that \( \mathbf{e}_{kn} \perp \mathbf{h}_{kn} \) and \( ||\mathbf{e}_{kn}||^2 = 1 \).

Allowing no instant CSI exchanges among the BSs, the BS \( n \) only possesses its own quantized directional vectors for all users, i.e., \( \tilde{\mathbf{h}}_{1n}, \ldots, \tilde{\mathbf{h}}_{Kn} \). When the BS \( n \) selects user \( k \), i.e., \( k \in S_n \), it supports user \( k \) with the ZF beamforming vector \( \mathbf{v}_{kn} \) randomly picked in the null spaces of the other selected users’ quantized channels, i.e., \( \{ \mathbf{h}_{sn} : s \in S_n \setminus \{k\} \} \), so it is satisfied that \( ||\mathbf{v}_{kn}||^2 = 1 \) and
\[
\mathbf{v}_{kn} \perp \{ \mathbf{h}_{sn} : s \in S_n \setminus \{k\} \}. \tag{4}
\]

As a result, the transmitted signal at the \( n \)th BS becomes \( x_n = \sum_{s \in S_n} \mathbf{v}_{sn} m_{sn} \), where \( m_{sn} \) is a data symbol from the \( n \)th BS intended for user \( s \). Without the knowledge of channel magnitudes, we assume that each BS serves the selected users with equal transmit power, i.e., \( \mathbb{E}[|m_{sn}|^2] = P_n/M \). Note that ZF beamforming with equal power allocation is optimal in the high SNR region [16]. Albeit optimal in the high SNR region, the ZF beamforming is suboptimal at low SNR. Generally, the performance of the ZF beamforming in the low SNR region can be improved by the regularized ZF beamforming (or MMSE beamforming) [12]. Otherwise, each BS can control the number of selected users according to the operating SNR. These factors can be carefully chosen offline.

If BSs have the perfect CSI, the inter-user interference can be perfectly nullified with ZF beamforming, and hence in (2), we have \( \mathbf{h}_{kn}^\dagger x_n = \mathbf{h}_{kn}^\dagger \mathbf{v}_{sn} m_{sn} \). With quantized CSI, however, the inter-user interference still remains in (2) due to the quantization error; the equation (2) can be rewritten by
\[
y_{kn} = \sqrt{\gamma_{kn}} \mathbf{h}_{kn}^\dagger \mathbf{v}_{sn} m_{sn} + \sum_{s \in S_n \setminus \{k\}} \mathbf{h}_{kn}^\dagger \mathbf{v}_{sn} m_{sn} \tag{5}
\]
\[
\text{inter-user interference}
\]
\[
\text{inter-cell interference}
\]

For notational simplicity, we denote the desired signal power of user \( k \) served by the BS \( n \) by \( D_{kn} \), such as
\[
D_{kn} = \frac{\gamma_{kn} P_n}{M} ||\mathbf{h}_{kn}^\dagger \mathbf{v}_{sn}||^2. \tag{6}
\]

Also, at the same user, we denote the inter-user interference power by \( I_{kn}^{\text{ex}} \) and the inter-cell interference power by \( I_{kn}^{\text{ci}} \), which are respectively given by
\[
I_{kn}^{\text{ex}} = \sum_{s \in S_n \setminus \{k\}} \frac{\gamma_{kn} P_n}{M} ||\mathbf{h}_{kn}^\dagger \mathbf{v}_{sn}||^2, \tag{7}
\]
\[
I_{kn}^{\text{ci}} = \sum_{i \in [N] \setminus \{n\}} \sum_{j \in S_i} \frac{\gamma_{ki} P_i}{M} ||\mathbf{h}_{ki}^\dagger \mathbf{v}_{ji}||^2. \tag{8}
\]

Thus, the achievable rate of the \( k \)th user supported by the \( n \)th BS becomes
\[
R_{kn} = \log_2 \left( 1 + \frac{D_{kn}}{I_{kn}^{\text{ex}} + I_{kn}^{\text{ci}}} \right). \tag{9}
\]
and the BS \( n \) obtains the achievable rate \( \sum_{s \in S_n} R_{sn} \). Also, the sum rate of the VCN system becomes
\[
R_{n} = \sum_{n=1}^{N} \sum_{k \in S_n} R_{kn}. \tag{10}
\]

### B. Problem Description

As the uplink feedback channel for each BS is generally shared by the multiple users, we take into account the feedback link sharing among the users [13], [15]. The \( n \)th BS has total \( B_n \)-bit uplink feedback budget, and user \( k \) is allocated \( b_{kn} \)-bit feedback size from the BS \( n \) such that \( b_{kn} \in \{0\} \cup \mathbb{Z}_+ \). Thus, we have sum feedback rate constraint for all BSs; the feedback bit allocation from all BSs to all users, i.e., \( b_{11}, \ldots, b_{KN} \), should satisfy
\[
\sum_{k=1}^{K} b_{kn} = B_n \quad \forall n \in [N]. \tag{11}
\]
In this paper, user selection and feedback bit allocation are jointly considered for maximizing the sum achievable rate by solving the following problem:

\[(P1) \quad \text{maximize} \quad \sum_{n=1}^{N} \left[ \sum_{k \in S_n} R_{kn}(b_{1n}, \ldots, b_{Kn}) \right] \quad \text{(12)}
\]
subject to

\[S_n \subset [K], \quad \text{Card}(S_n) = M, \quad \text{Card}\{s \in S_n | \gamma_{sn} \leq \gamma_m\} \leq M^R, \quad \forall n \in [N]. \quad \text{(13)}\]

Note that in problem (P1), we omit the disjoint user set constraints \(S_i \cap S_j = \emptyset\) for all \(i \neq j\) because a practical cellular system has enough users, so the probability that the multiple BSs select the same user will be relatively small and goes to zero as the number of users increases. Even if the multiple BSs select the same user, the objective function (12) is still valid if the user decodes each stream independently treating the others as noises. Obviously, the probability that there exists a user served by multiple BSs will depend on the number of total users on the network. This probability will be evaluated in the simulation part.

Problem (P1) is hard to solve because its objective function is defined as a function of (e.g., \(v_{kn}\)), while the effect of the feedback sizes (e.g., \(b_{kn}\)) is implicit. Moreover, the cardinality constraints make the problem much harder; even an optimization problem with a convex objective function becomes NP-hard if it has a cardinality constraint [23, Chap. 6]. We analyze the effect of feedback bit allocation and establish alternative problems in the next section.

### III. ALTERNATIVE OPTIMIZATION PROBLEMS

In this section, we investigate the effects of the feedback bit allocation on the sum rate and establish alternative problems.

#### A. Effects of Feedback Bit Allocation

From (9), we find the average achievable rate of user \(k\) served by the BS \(n\) is lower bounded by

\[\mathbb{E}[R_{kn}] \geq \mathbb{E}[D_{kn}] \geq \mathbb{E}_{\mathcal{I}} \log_2 \left( 1 + \frac{\mathcal{D}_{kn}}{1 + \mathcal{I}_{kn} + \mathcal{I}_{kn}^{\text{cl}}} \right) \quad \text{(15)}\]

where the inequality (a) holds because \(i) \log(1 + 1/x)\) is a convex function of \(x\), and \(ii)\) the random variable \(\mathcal{D}_{kn}\) is independent of the random variables \(\mathcal{I}_{kn}^{\text{cl}}\) and \(\mathcal{I}_{kn}^{\text{cl}}\). Also, the inequality (b) holds because the logarithm function is a monotonically increasing function and \(\mathcal{I}_{kn}^{\text{cl}} \geq 0\).

The term \(\mathbb{E}[\mathcal{I}_{kn}^{\text{cl}}]\) in (14) can be upper bounded as follows:

\[\mathbb{E} [\mathcal{I}_{kn}^{\text{cl}}] = \mathbb{E} \left[ \sum_{i \in S_n} \frac{\gamma_{kn} P_i}{M} \| h_{kn}^i \| v_i^{\text{in}} \|^2 \right] \leq \frac{\gamma_{kn} P_n}{M} \left( \mathbb{E} [\| h_{kn} \|^2] Z_{kn} \sum_{i \in S_n} |e_i^f v_i^{\text{in}}|^2 \right) \quad \text{(a)}
\]

\[\leq \frac{\gamma_{kn} P_n}{M} \left( \mathbb{E} [\| h_{kn} \|^2] \mathbb{E} [Z_{kn}] \mathbb{E} \left[ \sum_{i \in S_n} |e_i^f v_i^{\text{in}}|^2 \right] \right) \quad \text{(b)}
\]

\[\leq \frac{\gamma_{kn} P_n}{M} \frac{N_t (M-1)}{N_t-1} 2^{-\frac{b_{kn}}{N_t}} \quad \text{(c)} \]

where the equality (a) holds by plugging (3) and from the fact that \(h_{kn} \perp \{v_i : i \in S_n \setminus \{k\}\}\). The equality (b) is due to the independency of the channel gain, the magnitude of the quantization error, and the direction of the quantization error. We have \(\mathbb{E} [\| h_{kn} \|^2] = N_t\), and \(\mathbb{E} \left[ \sum_{i \in S_n} |e_i^f v_i^{\text{in}}|^2 \right] = (M - 1)/(N_t - 1)\) because the vectors \(v_i^{\text{in}}\) and \(e_i^f\) are independent of each other and isotropic in \((N_t - 1)\)-dimensional null space of \(h_{kn}\) so that each \(|e_i^f v_i^{\text{in}}|^2\) follows the beta distribution with parameters \((1, N_t - 2)\), i.e., \(\mathbb{E} [|e_i^f v_i^{\text{in}}|^2] = 1/(N_t - 1)\) for all \(i \in S_n \setminus \{k\}\). Also, the quantization errors of all channel codebooks are statistically dominated by that of the random vector quantizer \(\text{RVQ}\) [12], so it is obvious that \(\mathbb{E} [Z_{kn}] \leq \mathbb{E} [Z_{\text{RVQ}}]\), where \(Z_{\text{RVQ}}\) is the quantized error using \(b_{kn}\)-bit RVQ. Also, it is well known that [26]

\[\mathbb{E} [Z_{\text{RVQ}}] = 2^{b_{kn}} \beta \left( \frac{b_{kn}}{N_t}; \frac{N_t}{N_t-1} \right) \leq 2^{-\frac{b_{kn}}{N_t}} \quad \text{(16)}\]

where \(\beta(x, y)\) is beta function defined by \(\beta(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} \text{d}t\). Thus, we obtain the inequality (c).

On the other hand, from the definition in (8), we find \(\mathbb{E} [\mathcal{I}_{kn}^{\text{cl}}]\) in (14) as follows:

\[\mathbb{E} [\mathcal{I}_{kn}^{\text{cl}}] = \mathbb{E} \left[ \sum_{j \in [N] \setminus \{n\}} \sum_{i \in S_j} \frac{\gamma_{kj} P_j}{M} \| h_{kj}^i \| v_i^{\text{lj}} \|^2 \right] \leq \sum_{j \in [N] \setminus \{n\}} \sum_{i \in S_j} \frac{\gamma_{kj} P_j}{M} \mathbb{E} \left[ \| h_{kj}^i \|^2 \right] \mathbb{E} \left[ \| h_{kj}^i \| v_i^{\text{lj}} \|^2 \right] \quad \text{(a)}
\]

\[= \sum_{j \in [N] \setminus \{n\}} \frac{\gamma_{kj} P_j}{M} N_t \mathbb{E} \left[ \| h_{kj} \|^2 \right] \mathbb{E} \left[ \| h_{kj} \| v_i^{\text{lj}} \|^2 \right] \quad \text{(b)}
\]

\[= \frac{N_t}{N_t-1} \sum_{j \in [N] \setminus \{n\}} \gamma_{kj} P_j \quad \text{(17)}\]

\(\mathcal{D}_{kn}\) given in (6), the vector \(v_{kn}\) is determined by the other users’ channels, so it is independent of \(h_{kn}\). Thus, \(h_{kn}v_{kn}\) becomes the inner product of two vectors independent of each other, and the distribution of \(\mathcal{D}_{kn}\) is not affected by the values of \(\mathcal{I}_{kn}^{\text{cl}}\) and \(\mathcal{I}_{kn}^{\text{cl}}\).

\(\text{3}\)The random vector codebook consists of isotropic random unit vectors independent of each other.
where the equality (a) comes from the decomposition of the channel vector into the magnitude and the direction, which are independent of each other. Also, the equality (b) is due to the fact that $E[|\mathbf{h}_{kj}|^2] = N_t$ and $E[|\bar{h}_{kj}^l v_{lj}|^2] = N_t/(N_t - 1)$, where $\bar{h}_{kj}$ and $v_{lj}$ are independent and isotropic unit vectors in $\mathbb{C}^{N_t}$ so that $|\bar{h}_{kj}^l v_{lj}|^2$ follows the beta distribution with parameters $(N_t - 1, 1)$.

Thus, by substituting (15) and (16) into (14), we obtain a further lower bound of $\bar{R}_kn$ denoted by $\bar{R}_kn$, i.e.,

$$E[\bar{R}_kn] \geq \bar{R}_kn,$$

where

$$\bar{R}_kn = E_h \left[ \log_2 \left( 1 + D_{kn} + \frac{N_t^2}{N_t - 1} \sum_{j \in [N]\setminus\{n\}} \gamma_{kj} P_j \right) \right] - \log_2 \left( 1 + \frac{N_t(M-1)}{N_t-1} \frac{\gamma_{kn} P_n}{M} 2^{-\frac{b_{kn}}{\kappa \beta \delta}} + \frac{N_t^2}{N_t - 1} \sum_{j \in [N]\setminus\{n\}} \gamma_{kj} P_j \right).$$

(18) is a concave function of $b_{kn}$; the concavity of (18) with respect to $b_{kn}$ can be easily checked from the convexity of $\log_2(\kappa + \beta \cdot 2^{-\delta x})$ with respect to $x$ when $\beta, \delta, \kappa > 0$. The function $\log_2(\kappa + \beta \cdot 2^{-\delta x})$ has a second derivative given by

$$\frac{\partial^2}{\partial x^2} \left[ \log_2(\kappa + \beta \cdot 2^{-\delta x}) \right] = \frac{(\kappa \beta \delta^2) \cdot \ln 2 \cdot 2^{\delta x}}{(\kappa \beta \delta^2 + 1)^2},$$

so when $\beta, \delta, \kappa > 0$, it becomes a convex function whose second derivative is positive. The tightness of the lower bound (18) will be shown in the simulation part.

In (18), we can observe that $\bar{R}_kn$ is only affected by user $k$’s path loss coefficients and its feedback size for the BS $n$, i.e., $\{\gamma_{k1}, \ldots, \gamma_{kN}\}$ and $b_{kn}$. In ZF MIMO BC, a similar phenomenon, i.e., the achievable rate at each user only depends on its own path loss and feedback size, is mathematically proven in [13].

### B. Alternative Optimization Problems

For an alternative optimization problem of problem (P1), we use $\bar{R}_kn$, given in (18) instead of $R_kn$. Since $\bar{R}_kn$ is a function of $b_{kn}$, we denote it by $\bar{R}_kn(b_{kn})$ and establish an alternative optimization problem (P2) as follows:

$$\begin{align*}
\text{(P2)} & \quad \text{maximize} & & \sum_{n=1}^{N} \left[ \sum_{k \in S_n} \bar{R}_kn(b_{kn}) \right] \\
& \text{subject to} & & S_n \subset [K], \quad \text{Card}(S_n) = M, \quad \text{Card}(\{s \in S_n | \gamma_{sn} \leq \gamma_n\}) \leq M^n, \\
& & & \sum_{k=1}^{K} b_{kn} = B_n \quad \forall n \in [N].
\end{align*}$$

Remark 1. In problem (P2), each BS’s objective function, i.e., (18), is only affected by its own user selection and the feedback bit allocation results, i.e., the value $\sum_{k \in S_n} \bar{R}_kn(b_{kn})$ is only affected by $S_n$ and $\{b_{1n}, \ldots, b_{Kn}\}$.

Thus, we can split problem (P2) into equivalent $N$ sub-problems, each of which is solved at each BS in a distributed manner. In this case, the BS $n$ obtains the selected user group and the feedback bit allocations, i.e., $S_n$ and $\{b_{1n}, \ldots, b_{Kn}\}$, by solving the following sub-problem:

$$\begin{align*}
\text{(P3)} & \quad \text{maximize} & & \sum_{k \in S_n} \bar{R}_kn(b_{kn}) \\
& \text{subject to} & & S_n \subset [K], \quad \text{Card}(S_n) = M, \\
& & & \text{Card}(\{s \in S_n | \gamma_{sn} \leq \gamma_n\}) \leq M^n, \quad \sum_{k=1}^{K} b_{kn} = B_n.
\end{align*}$$

We have the following remarks.

Remark 2. The objective function given in (21) is only related with the path loss coefficients of all BS and user pairs, i.e., $\gamma_{11}, \ldots, \gamma_{KN}$. Thus, each BS can solve the problem (P3) only with the path loss coefficients.

Remark 3. To maximize the objective function given in (19), all feedback budget should be allocated to the selected users, i.e., the selected users in $S_n$ should fully share total feedback budget $B_n$.

In our previous work [13], we already found the feedback bit sharing strategy in MIMO BC. However, the MIMO BC only has a single transmitter so that each user is not interfered by other transmitters. Furthermore, the user selection was not considered in [13]. In this paper, on the other hand, there are multiple transmitters (BSs) and much more users in the network, so we should consider not only the feedback bit allocation itself, but also user selection and the effects of the inter-cell interference.

### IV. PROPOSED LIMITED FEEDBACK PROTOCOL FOR VCN SYSTEM

In this section, we first solve problem (P3) and then propose limited feedback protocols based on the solutions of problem (P3) for VCN systems.

#### A. Solution of the Problem (P3)

The problem (P3) is a non-convex problem because the feedback sizes are non-negative integers and there are cardinality constraints (e.g., $\text{Card}(S_n) = M$). Thus, it can be referred to as a mixed-integer problem and a convex-cardinality problem, both of which are generally known as NP-hard problems [23, Chap. 6]. We first describe the brute-force algorithm, and then propose an efficient solution for the relaxed convex problem. Without loss of generality, we consider the problem solving at the BS $n$.

1) Optimal Solution (Brute-Force Search Algorithm): In the brute-force search algorithm, each BS considers all possible user selections and the feedback bits allocations. Let $b_n \in \{(0) \cup \mathbb{Z}^+\}^K$ be the feedback bit allocation vector of the BS $n$ defined by $b_n \triangleq [b_{1n}, \ldots, b_{Kn}]$. Then, with a slight abuse of the notation, we denote the objective function (21) by $\bar{R}_n(S_n, b_n)$ such as

$$\bar{R}_n(S_n, b_n) \triangleq \sum_{k \in S_n} \bar{R}_kn([b_n]_k),$$

(23)
Algorithm 1 Brute-Force User Selection and Feedback Bit Allocation Algorithm at the BS $n$

1: **Initialization:** Randomly choose the user group $S_n$ and feedback bit allocation vector $b_n$

2: **for all** $S \in S_n$ **do**

3: **for all** $b \in B(S_n)$ **do**

4: if $\mathcal{R}_n(S, b) \geq \mathcal{R}_n(S_n, b_n)$ then

5: $S_n = S; \quad b_n = b$

6: **end if**

7: **end for**

8: **end for**

9: **Output:** the optimal user group $S_n$ and feedback bit allocation $b_n$

where $[.]_k$ denotes the $k$th element of a vector.

We define a class $S_n$ as a collection of all possible user sets at the BS $n$ such as

$$S_n = \{S \subseteq K \mid \text{Card}(S) = M, \quad \text{and} \quad \text{Card}(\{s \in S \mid \gamma_{sn} \leq \gamma_{mn}\}) \leq M^R\}. \quad (24)$$

Also, for an arbitrary selected user set $S$ for the BS $n$, we define the set of all possible feedback bit allocation vectors denoted by $B(S)$, given by

$$B(S) = \left\{ b \in (\{0\} \cup \mathbb{Z}^+)^K : \sum_{s \in S} |b_s| = B_n, \quad |b_k| = 0 \text{ for all } k \notin S \right\}. \quad (25)$$

Then, the BS $n$ can choose the optimal user selection and feedback allocation (maximizing (23)) among $S_n$ and $B(S)$ as described in Algorithm 1.

When there are enough users on the network, the BS $n$ chooses $M^{R_n}$ users out of $K^{n}_n$ inner-cell users and choose $M - M^{R_n}$ users out of $K - K^{n}_n$ inner-cell users, so there are total

$$\binom{K^{n}_n}{M^{R_n}} \binom{K - K^{n}_n}{M - M^{R_n}}$$

cases for user selection. Also, for each user selection case, allocating $B_n$ bits to the selected $M$ users has $\binom{B_n + M - 1}{M - 1}$ possible ways. Therefore, the BS $n$ should compare total $\binom{K^{n}_n}{M^{R_n}} \binom{K - K^{n}_n}{M - M^{R_n}} \binom{B_n + M - 1}{M - 1}$ cases. Using the relationship

$$\binom{k}{n} \leq \binom{k}{n} \leq \binom{k}{n}_L,$$

we can conclude that the brute-force algorithm has the complexity of $O\left(\binom{K^{n}_n}{M^{R_n}} \cdot \binom{K - K^{n}_n}{M - M^{R_n}} \cdot \binom{B_n + M - 1}{M - 1}\right)$ at the BS $n$. As a sub-optimal search, each BS can reduce the computational complexity by considering the nearest users in the inner-cell and the outer-cell. If the BS $n$ only makes selections among $K^{n}_n \geq M^{R_n}$ nearest users in inner-cell $n$ and $K^{n}_n \geq K^{n}_n$ nearest users in the cell edge, the computational complexity is reduced to

$$O\left(\binom{K^{n}_n}{M^{R_n}} \cdot \binom{K^{n}_n}{M - M^{R_n}} \cdot \binom{B_n + M - 1}{M - 1}\right). \quad (26)$$

2) Convex Relaxation: Although each BS can obtain the user selection and feedback allocation results from the brute-force algorithm, it becomes prohibitive when the number of users or total feedback budget increases. Thus, we need an efficient algorithm to solve problem (P3).

First, we relax the integer feedback size to non-negative real numbers over which (18) becomes a continuous function. However, problem (P3) is still difficult because it has non-convex cardinality constraints i.e., $\text{Card}(S_n) = M$ and $\text{Card}(\{s \in S \mid \gamma_{sn} \leq \gamma_{mn}\}) \leq M^{R_n}$. To relax the cardinality constraints, we invoke Remark 3, which indicates that the number of users having non-zero feedback bits should be at most $M$. Thus, as an alternative to the cardinality constraint, we can use the sparsity constraint on the feedback allocation vector $b_n$ given by $\|b_n\|_0 \leq M$, where $\| \cdot \|_0$ denotes $l_0$-norm of a vector, i.e., the number of nonzero elements. Meanwhile, the set of feedback sizes allocated for the inner-cell users becomes $\{k \in [K] \mid \gamma_{kn} \leq \gamma_{mn}, \quad b_n[k] > 0\}$, so the constraint (1) can be replaced by the sparsity constraint on the inner-cell user selection given by

$$\text{Card}(\{k \in [K] \mid \gamma_{kn} \leq \gamma_{mn}, \quad b_n[k] > 0\}) \leq M^{R_n}. \quad (27)$$

To handle the non-convex $l_0$-norm (sparsity) constraint, $l_1$-norm relaxation is widely used in literature [23, Chap. 6] [27]. In our case, $l_0$-norm constraint given by $\|b_n\|_0 \leq M$ is relaxed to $l_1$-norm constraint $\|b_n\|_1 \leq \eta$ (or $\sum_{k=1}^{K} b_{nk} \leq \eta$, where $\eta$ is a regularization term to obtain a desired sparsity. Thus, we adjust $\eta$ to make an $M$-sparse feedback bit allocation vector maintaining $M^{R_n}$-sparse feedback bit allocation for inner-cell users, in which only $M$ users have been assigned the positive feedback bits. However, we also have another sum feedback bit constraint $\sum_{k=1}^{K} b_{nk} = B_n$, so we should reallocate the feedback bits after the user selection.

Thus, we solve the relaxed problem with the following two steps:

- **Step 1 (User selection).** Relax the feedback sizes to non-negative real numbers and find the $M$-sparse feedback bit allocation vector with $M^{R_n}$-sparse feedback sizes for the inner-cell users by adjusting $\eta$ in the $l_1$-norm constraint $\|b_n\|_1 \leq \eta$. Then, obtain the selected user group by choosing the users who are allocated non-zero feedback bits.

- **Step 2 (Feedback bit re-allocation).** Reallocate the feedback bits to the selected users in Step 1 under total feedback rate constraint $\sum_{k=1}^{K} b_{nk} = B_n$.

**Step 1.** The BS $n$ solves the convex problem

$$(P4) \quad \text{maximize } b_n = \{b_{n1}, \ldots, b_{nK}\} \sum_{k=1}^{K} \tilde{R}_{kn}(b_{kn}) \quad \text{subject to } \sum_{k=1}^{K} b_{kn} \leq \eta, \quad (28)$$

where the feedback sizes are non-negative real numbers. Since (18) is a convex function of $b_{kn}$, so is (28). We construct a Lagrangian $\mathcal{L}$ given by

$$\mathcal{L} \triangleq \sum_{k=1}^{K} \tilde{R}_{kn}(b_{kn}) - \lambda \left( \sum_{k=1}^{K} b_{kn} - \eta \right), \quad (30)$$

*The number of all possible feedback allocations is equivalent to the number of all possible sequences consisting of $B_n$ ’s and $M - 1$ ’0’s.

*For two functions $f(x)$ and $g(x)$, we say $f(x) = O(g(x))$ if and only if there exist constants $c > 0$ such that $|f(x)| \leq c|g(x)|$ when $x$ is sufficiently large.
where \( \lambda \) is the Lagrangian multiplier. Then, the optimal solution denoted by \( \hat{b}_n = [\hat{b}_{1n}, \ldots, \hat{b}_{Kn}] \) is obtained at
\[
\frac{\partial \mathcal{L}(b_{kn})}{\partial b_{kn}} = 0
\]
for all \( k \in [K] \), given by
\[
\hat{b}_{kn} = \left[ \frac{\eta}{K} + (N_t - 1) \left( A_{kn} - \frac{1}{K} \sum_{i=1}^{K} A_{in} \right) \right]^+, \quad (32)
\]
where \( [\cdot]^+ \) is max(0, \cdot), and
\[
A_{in} \triangleq \log_2 \left( \frac{\sum_{i \in |N| \backslash \{n\}} \gamma_{ij} P_j}{1 + \frac{N_t}{N_t - 1} \sum_{j \in |N| \backslash \{n\}} \gamma_{ij} P_j} \right) \quad \forall i \in [K]. \quad (33)
\]

In (32), we can observe that the sparsity of \( \hat{b}_n (\triangleq [\hat{b}_{1n}, \ldots, \hat{b}_{Kn}]) \) decreases as \( \eta \) increases. Interestingly, we can choose \( M \) users without finding the exact value of \( \eta \) because the selected inner-cell (or outer-cell) users will have the larger values of (33) than the unselected inner-cell (or outer-cell) users. Thus, the BS \( n \) simply sorts \( \{A_{1n}, \ldots, A_{Kn}\} \) in the descending order and select up to \( M_n \) users in \( \mathcal{K}_n \) of the largest \( \mathcal{R}_n \) values, and then select \( M - M_n \) users in \( \mathcal{K} - \mathcal{K}_n \) of \( M - M_n \) largest values. As a result, the selected user group \( \mathcal{S}_n \) satisfies that \( \text{Card}(\mathcal{S}_n \cap \mathcal{K}_n) = M_n, \text{Card}(\mathcal{S}_n) = M \), and
\[
\min \left\{ A_{sn} | s \in \mathcal{S}_n \cap \mathcal{K}_n \right\} \geq \max \left\{ A_{sn} | s \notin \mathcal{S}_n \cap \mathcal{K}_n \right\},
\]
\[
\min \left\{ A_{sn} | s \in \mathcal{S}_n \cap (\mathcal{K} - \mathcal{K}_n) \right\} \geq \max \left\{ A_{sn} | s \notin \mathcal{S}_n \cap (\mathcal{K} - \mathcal{K}_n) \right\}.
\]

**Step 2.** The BS \( n \) reallocates the feedback bits to the selected users in \( \mathcal{S}_n \) found in Step 1. Following a similar procedure used in Step 1, the reallocated feedback size to the selected user \( s \in \mathcal{S}_n \) denoted by \( b_{sn}^* \) is given by
\[
b_{sn}^* \triangleq \left[ \frac{\eta'}{M} + (N_t - 1) \left( A_{sn} - \frac{1}{M} \sum_{i \in \mathcal{S}_n} A_{in} \right) \right]^+, \quad (34)
\]
where \( A_{in} \) is defined in (33), and \( \eta' \) is numerically found to satisfy \( \sum_{i \in \mathcal{S}_n} b_{sn}^* = B_n \).

In our propose scheme, the BS \( n \) calculates \( A_{in} \) for all \( i \in [K] \) and selects \( M \) users after sorting them. The calculation of the \( K \) values and sorting them take the complexities of \( O(K) \) and \( O(K \log K) \), respectively [28, Chap. 2]. Thus, the computational complexity of our proposed scheme is \( O(K \log K) \) at each BS, which is much lower than that of brute force algorithm given in (26).

**B. User Selection and Feedback Bit Allocation Procedures**

Our proposed VCN protocol is based on problem (P3). As described in Remark 2, each BS can solve problem (P3) only with the path loss coefficients. Since the path losses are mainly determined by the distances between the BSs and the users, each BS can directly measure the path loss coefficients from the uplink channels. Moreover, they can be easily shared among the BSs because they are long-term average information. Thus, we assume that each BS already knows the path loss coefficients of all users.

Our proposed protocol for the VCN system consists of the following procedures:

1. Each BS solves problem (P3) and determines the selected user set and corresponding feedback bit allocation.
2. Each BS informs the selected users and their feedback sizes.
3. The selected users quantize their channels and feed the quantized information back to the corresponding BS.
4. Each BS supports the selected users with ZF beamforming.

**C. Extension for Fairness**

Although we mainly considered sum rate maximization in our manuscript, fairness among the users is an important issue in many practical systems. Extension of our proposed scheme for fairness is worth further investigation, but in this subsection, we briefly explain how the fairness can be considered in our proposed scheme.

We consider the \( \alpha \)-fair resource allocation [29]–[31]; for \( T \) time slots, the \( \alpha \)-fair resource allocation maximizes the objective function given by
\[
\sum_{k \in K} \left( \frac{\bar{R}_k^{(\alpha)} - \bar{R}_k^{(\alpha - 1)}}{1 - \alpha} \right),
\]
where \( \bar{R}_k^{(\alpha)} \) is an accumulated rate at user \( k \) until time slot \( T \), and \( \alpha \in [0, \infty) \) is a factor that controls the trade-off between achievable rate and fairness. In this case, \( \alpha = 0 \) corresponds to sum rate maximization, whereas \( \alpha = 1 \) and \( \alpha = \infty \) correspond to proportional fair and max-min fair problems, respectively [29]–[31].

If user \( k \) is served by BS \( n \) at time slot \( T \), the accumulated rate of user \( k \) after time slot \( T \) can be represented by
\[
\bar{R}_{kn} + \bar{R}_k^{(\alpha - 1)}. \quad (35)
\]

Thus, for the transmission at time slot \( T \), the BSs should solve the following problem:
\[
\begin{aligned}
&\text{maximize} & & \sum_{n=1}^{N} \sum_{k \in \mathcal{S}_n} \left[ R_{kn}(b_{1n}, \ldots, b_{Kn}) + \bar{R}_k^{(\alpha - 1)} \right]^{(1-\alpha)} \\
&\text{subject to} & & A_{in} \sum_{i \in \mathcal{S}_n} b_{sn}^* = B_n \\
& & & \text{with the same constraint of problem (P1).}
\end{aligned}
\]

Note that this becomes problem (P1) when \( \alpha = 0 \). Since problem (P1') is not easy to solve, we establish an alternative problem; we define an alternative of (35) given by
\[
f_{kn}(b_{kn}) \triangleq \bar{R}_{kn}(b_{kn}) + \bar{R}_k^{(\alpha - 1)} + \xi, \quad (36)
\]
where \( \bar{R}_{kn}(b_{kn}) \) is the lower bound of \( \mathbb{E}[\bar{R}_{kn}] \) defined in (18) used in the objective function of problem (P2), and \( \xi \in \mathbb{R}_+ \cup \{0\} \) is a non-negative value that makes \( f_{kn}(b_{kn}) \) a non-negative function for any feedback bit allocation \( b_{kn} \in [0, B_n] \), i.e., \( f_{kn}(b_{kn}) \geq 0 \) for all \( b_{kn} \in [0, B_n] \). For example, we can use \( \xi \) given by
\[
\xi = \max_{k \in K} \left\{ -\bar{R}_{kn}(0) - \bar{R}_k^{(\alpha - 1)} \right\}.
\]
average achievable rate per selected user (bps/Hz)

$-200$ $-150$ $-100$ $-50$ $0$ $50$ $100$ $150$ $200$

\[ x = \sum_{n=1}^{N} \sum_{k \in S_n} \left(1 - \alpha \right) \frac{\left[f_{kn}(b_{kn})\right]}{(1 - \alpha)} \]

\[ \alpha \] is fixed to $4$, so BS $n$ only needs to solve the following problem:

\[ \text{(P3') maximize } \sum_{k \in S_n} \left[1 - \alpha \right] \frac{\left[f_{kn}(b_{kn})\right]}{(1 - \alpha)} \]. \[ (37) \]

As explained in our paper, the term $\hat{R}_{kn}(b_{kn})$ is a concave function of $b_{kn}$, so is $f_{kn}(b_{kn})$. Also, it is not difficult to show that if a function $f(x)$ is a non-negative concave function, then $[f(x)]^{1-\alpha}$ is also a concave function when $\alpha \in [0,1]$. Thus, the term $\left[f_{kn}(b_{kn})\right]^{1-\alpha}$ in (37) becomes a concave function of $b_{kn}$, and we can conclude that the problem (P3') for $\alpha$-fairness such that $\alpha \in [0,1]$ can be solved with in same way we used to solve problem (P3).

V. NUMERICAL RESULTS

This section evaluates our proposed protocol. For evaluation, we consider four BSs whose positions are fixed to $(X,Y)$'s such that $X \in \{-100,100\}$ and $Y \in \{-100,100\}$. Each BS has four transmit antennas, and the path loss exponent is fixed to $-4$. An exemplary geometry of a VCN system is illustrated in Fig. 2. In the following simulation, the following parameters are assumed unless otherwise noticed: the inner-cell radius is set as 50 (i.e., $r = 50$), there are total 40 users (i.e., $K = 40$), and each BS supports four users simultaneously, where the number of selected inner-cell users is at most one, i.e., $M = 4$ and $M^N = 1$. The transmit SNR of each BS is 100dB, and the uplink feedback budget for each BS is 16 bits, i.e., $B_n = \sum_{k \in S_n} b_{kn} = 16$ for all $n \in [N]$. The channel quantization error is modeled using a random vector codebook, whereas each user can perfectly estimate the channels.

As reference schemes, we consider two user selection and feedback allocation schemes as follows:

- Reference scheme 1 (MAX-SNR user selection with equal feedback allocation): Each BS selects four users having the largest SNRs (i.e., users in the closest distances), and the selected users are served by the equal feedback size.

- Reference scheme 2 (MAX-SINR user selection with equal feedback allocation): Each BS selects four users having the largest transmit SINRs (for user $k$'s, the BS $n$ uses $\gamma_{kn} P_n / (1 + \sum_{j \in [N] \setminus \{n\}} \gamma_{kj} P_j)$), and the selected users are allocated by the equal feedback size.

Since our proposed protocol is based on problem (P2) that uses the lower bound $\hat{R}_{kn}$, we first show in Fig. 3 the gap between an arbitrary selected user's real average achievable rate $E[R_{kn}]$ and the lower bound $\hat{R}_{kn}$ defined in (18) with respect to the feedback sizes. As shown in Fig. 3, we can observe that the lower bound given in (18) well reflects the real achievable rate.

In Fig. 4(a) and Fig. 4(b), we compare the achievable rates of various schemes varying the transmit SNR when there are a total of 20 users and 40 users, respectively. In the equal feedback schemes, each selected user feeds 4-bit quantized CSI to the belonging BS. In the proposed scheme, however, each user’s feedback size is obtained from (34) taking into account the path loss coefficients (all users’ positions). In Fig. 4, we can observe that our proposed scheme outperforms the conventional user selection schemes in both cases. Also, we can observe that the proposed scheme based on the relaxed convex optimization problem achieves a similar performance to that based on the brute-force algorithm.

Fig. 2. An exemplary of geometry of a VCN system. Four BSs and 40 users are considered.

Fig. 3. Gap between the real achievable rate and the approximated achievable rate with respect to the feedback size.

Fig. 4. Comparison of achievable rates of various schemes varying the transmit SNR when there are a total of 20 users and 40 users, respectively. In the equal feedback schemes, each selected user feeds 4-bit quantized CSI to the belonging BS. In the proposed scheme, however, each user’s feedback size is obtained from (34) taking into account the path loss coefficients (all users’ positions). In Fig. 4, we can observe that our proposed scheme outperforms the conventional user selection schemes in both cases. Also, we can observe that the proposed scheme based on the relaxed convex optimization problem achieves a similar performance to that based on the brute-force algorithm.

\[ f(x) = \sqrt{(t-1) \cdot f(x)^{t-2} \cdot [f(x)]^2 + t \cdot f(x)^{t-1}} \]

\[ f''(x) \leq 0 \] when $t \in [0,1]$.\[ 6 \]

\[ f''(x) \leq 0 \] when $t \in [0,1]$.\[ 6 \]
Fig. 4. Achievable rates of various schemes when there are in total 20 users and 40 users (i.e., $K = 20$ and $K = 40$) in the network, respectively.

In Fig. 5, we compare the achievable rates of our proposed scheme and the reference schemes with respect to the number of total users. We can observe that our proposed scheme also outperforms the reference schemes for any number of total users, and the solution based on the relaxed optimization problem shows similar performance with the solution obtained from the brute-force algorithm. Also, in Fig. 6, we compare the achievable rates of our proposed scheme and the reference schemes by varying the uplink feedback budget. As we can see in Fig. 6, our proposed scheme outperforms the conventional schemes for all feedback budget.

In the same environment, the probabilities that there exists a user selected by multiple BSs are evaluated and shown in Fig. 7. As we can see, this probability is 2.8% (i.e., 0.028) for our proposed scheme when there are 32 users and tends to zero quickly when the number of users further increases.

In Fig. 8, we compare the achievable rates of our proposed scheme and the reference schemes by varying the inner-cell size. When the radius of each inner-cell is small, the selected users will be in close distance from their serving BSs, so the inter-cell interference is relatively small compared to the inter-user interference caused by imprecise ZF beamforming due to
we can observe in Fig. 9, the achievable rates decrease as delay is assumed for information sharing among the BSs. As time, each user moves to a random direction, and one second variance in the movement of the users and show how much the de-

six streams (M=6) decreases, while the gap between the MAX-SINR scheme and the MAX-SNR scheme increases.

Delayed/inexact path-loss information affects the perfor-
mance of our proposed scheme. Since the path losses are mainly determined by the positions of users, we consider the movement of the users and show how much the delayed/inexact path-loss information affects the performance. In Fig. 9, we have plotted the achievable rates with the various schemes with respect to the user speed. At the same time, each user moves to a random direction, and one second delay is assumed for information sharing among the BSs. As we can observe in Fig. 9, the achievable rates decrease as the user speed increases because the path loss information becomes more inaccurate. However, we can observe that for all user speeds, our proposed scheme efficiently increases the achievable rates.

In Fig. 10, we plot the achievable rates of various schemes according to the number of selected users, i.e., M, varying the transmit SNR in the same environment as Fig. 4(b). In such an interference-limited system, it is obvious that reducing the number of the selected users (data streams) can increase the achievable rate, so this metric should be carefully chosen offline considering many factors. However, in any case, our proposed schemes efficiently increase the achievable rates.

VI. CONCLUSIONS

In this paper, we proposed an efficient user selection and the feedback bit allocation protocol for MIMO VCN system. We firstly analyzed the effects of the quantization error on the system performance, and then proposed the user selection and feedback bit allocation protocol. In our proposed protocol, each BS jointly optimizes the user selection and feedback bit allocation exploiting all users’ path loss coefficients. Our proposed protocol significantly increases the system performance by allocating the optimized feedback sizes to the selected users and by exploiting total feedback budget only for the selected users.

REFERENCES


The user speeds (1, 10, 30) m/s correspond to (3.6, 36, 108) km/h and (2.24, 22.4, 67.1) mi/h.


