DOWNLINK MULTIUSER CAPACITY OF INTERFERENCE-LIMITED MIMO SYSTEMS*

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Abstract – In a companion paper [4], space-time layered architectures (BLAST) and turbo coding/processing techniques, which are effective for single-link transmission, are combined with multiuser detection (MUD) methods for combating intercell interference. It is shown that, depending on the channel configuration, linear MMSE or successive interference cancellation (SIC) can be the preferable MUD method. Based on this fact, an adaptive scheme is developed. In this paper, the downlink capacity of interference-limited multiple-input multiple-output (MIMO) cellular systems is investigated. It is found that the obtained MUD capacity is excellent in high to medium SIR scenario, but still deteriorates in strong interference environments, leaving ample room for possible improvement through other techniques. The performance of the proposed adaptive MUD scheme in a standard cellular environment, both in the non-line-of-sight (NLOS) and the line-of-sight (LOS) case, is simulated, and the advantages over the standard V-BLAST scheme are quantified.

Keywords - adaptive detection, BLAST, co-channel interference, MIMO systems, multiuser detection, turbo processing

I. INTRODUCTION

Recent information theoretic results have indicated the remarkable capacity potential of wireless communication systems with antenna arrays at both the transmitters and receivers. These so-called multiple-input multiple-output (MIMO) systems have been shown to yield tremendous capacity, which grows at least linearly with the minimum of the numbers of transmit and receive antennas [6], [10]. These large spectral efficiencies were obtained for a single link with white Gaussian noise. In a cellular environment, there will often be co-channel interference from other cells, which becomes the dominating channel impairment. In this paper, we will investigate the capacity of MIMO systems in interference-limited situations.

Motivation for our work came from a recent study by Catreux et al. [3], who showed that in an interference-limited environment, the capacity of a MIMO system is hardly larger than when using smart antennas *at the receivers only*. This seems to be related to the fact that an antenna array with N elements can eliminate N-1 interferers, so that the reuse distance (in a TDMA/FDMA system) can be chosen to be very small. The independent data streams employed by a MIMO system are all different interferers, so a receive array has no degrees of freedom with which to cancel the co-channel interferers after it separates the multiple data streams in its own cell. On the other hand, this investigation assumed a certain system structure taken from the noise-limited case, and did not try to optimize the system for interference-limited environments. To be specific, they exploited sub-optimal signal processing techniques (uncoded V-BLAST) at the receivers; no attempt was made to jointly detect desired as well as interfering signals; and no cooperation between base stations was assumed.

Any BLAST-like scheme (Bell-labs space-time layered architecture) is by its nature a multiuser detector that separates the data streams from the multiple transmit antennas of the desired base station. It thus seems logical to extend that principle also to the data streams from the interfering base stations. Space-time layered architectures and turbo coding /processing techniques have come remarkably close to the ultimate capacity limits in Gaussian ambient noise. In a recent paper [4], two of the authors investigated whether multiuser detection (MUD) can in principle increase the capacity of MIMO systems with adjacent-cell interference. This investigation assumed a fixed SNR and one or two dominant interferers. In this paper, we investigate different MUD schemes further in terms of achievable outage capacity, and examine their performance in a cellular environment with multiple, stochastically fading interferers.

This paper is organized in the following way: in Section II, we present the system model and introduce the assumptions made in the problem formulation. Section III briefly reviews the advanced detection methods we use. Next, in Section IV, we examine the downlink capacity of interference-limited MIMO systems both for simulation models and for more realistic cellular settings. A summary of the results is contained in Section V.

II. SYSTEM MODEL

A. MIMO System Model

We adopt the same mathematical model as in [6] and [10], which is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where **y** is the received vector, **x** is the transmitted signal, **H** is the channel matrix that captures the channel characteristics between transmit and receive antenna arrays, and **n** is the background noise. We assume an $N \times N$ MIMO system with the transmitted signal vector constrained to have overall power $E\{\mathbf{x}^H\mathbf{x}\} \le P$, and circularly symmetric Gaussian background noise with covariance matrix $\Phi_N = \sigma^2\mathbf{I}$. The entries of the complex matrix **H** are independent with uniformly distributed phase and normalized Rayleigh distributed magnitude, modeling a Rayleigh fading channel with sufficient physical separation between transmit and receive antennas. The signal-to-noise ratio (SNR) is given by $\rho = P/\sigma^2$. We list the single-cell interference-free MIMO capacity formulas below that will serve as the upper bound for the multi-cell interference-limited cases. In all cases, the channel state information (CSI) is assumed to be known at the receiver.

1. If the channel matrix **H** is known at the transmitter, then the capacity is given by

$$C_U = \sum_i \log_2(1 + \lambda_i P_i), \tag{2}$$

   where $\{\lambda_i\}$ are the eigenvalues of the matrix $\mathbf{H}^H \Phi_N{}^{-1}\mathbf{H}$, and the values of P_i, i.e., the powers assigned to the different eigenmodes, are derived from the water-filling rule.

2. If the channel matrix **H** is unknown at the transmitter, then the capacity is given by

$$C = \log_2 \det[\mathbf{I} + \frac{P}{N}\mathbf{H}^H \Phi_N{}^{-1}\mathbf{H}], \tag{3}$$

   which can be viewed as an equal power allocation to the eigenmodes due to the lack of channel knowledge.

3. If the channel matrix **H** is unknown at the transmitter, a lower bound on the capacity with $\Phi_N = \sigma^2\mathbf{I}$ is given by

$$C_L = \sum_i \log_2(1 + \frac{\rho}{N}\chi_{2i}^2), \tag{4}$$

   where χ_{2i}^2 is a χ^2 distributed random variable with $2i$ degrees of freedom and mean value i.

B. Cellular System Model

We consider a TDMA/FDMA multi-cell system, and take into account interference from the first tier of the center-excited cell configuration with reuse factor of one. We assume a frequency-flat, quasi-static fading environment, and the complex baseband channel gain between the *j*th transmit and the *i*th receive antenna is modeled by

$$h_{ij} = \sqrt{c\frac{1}{d_{ij}^{\gamma}}}\sqrt{s_{ij}}\left[\sqrt{\frac{K}{K+1}}e^{j\Phi_{ij}} + \sqrt{\frac{1}{K+1}}z_{ij}\right], \tag{5}$$

where the three terms embody the path loss, the shadow fading and multipath fading effects, respectively. For path loss, d_{ij} is the length of the link and γ is the path loss exponent; $s_{ij} = 10^{S_{ij}/10}$ is a log-normal shadow fading variable, where S_{ij} is a zero mean Gaussian random variable with standard deviation υ; for multipath fading, *K* is the so-called Ricean *K*-factor, which denotes the ratio of the direct received power (LOS component) to average scattered power (NLOS component); $\Phi_{ij} = 2\pi d_{ij}/\lambda$ is the phase shift of the line-of-sight path (λ is the wavelength); z_{ij} is modeled as a set of normalized complex Gaussian random variables, assumed to be independent for each transmit-receive link. With these assumptions, the multicell system model is given by

$$\mathbf{y} = \mathbf{H} \cdot \mathbf{x} + \sum_i \mathbf{H}_{ifi} \cdot \mathbf{x}_{ifi} + \mathbf{n}, \tag{6}$$

where the subscript "*if*" denotes interference. The channel matrices **H** and $\{\mathbf{H}_{ifi}\}$ are independent with independent and identically distributed (i.i.d.) elements given by (5). The transmitted signals from all users are assumed to be of the same format with $E\{\mathbf{x}^H\mathbf{x}\} = E\{\mathbf{x}_{ifi}{}^H\mathbf{x}_{ifi}\} \le P$, whose codebooks are known to the receivers. As above, the noise is assumed to be white and complex Gaussian with covariance matrix $\Phi_N = \sigma^2\mathbf{I}$.

To provide a common framework that is general enough to address multiuser detection across the cell while remaining simple enough for analysis and simulation, in [4] we proposed a simplified model given as

$$\mathbf{y} = \mathbf{H} \cdot \mathbf{x} + \sum_{i=1}^{2} \mathbf{H}_{ifi} \cdot \mathbf{x}_{ifi} + \sum_{i=3}^{4} \mathbf{H}_{ifi} \cdot \mathbf{x}_{ifi} + \mathbf{n}, \tag{7}$$

where we use the same assumptions for the channel matrices and noise as (1), while assuming the channel matrices for different users are independent. We are mainly interested in two scenarios: (A) two-dominant-interferer case with $P_{if1} = P_{if2} = 4P_{if3} = 4P_{if4}$ and (B) one-dominant-interferer case with $P_{if1} = 6P_{if2} = 6P_{if3} = 6P_{if4}$.

In this paper, we will continue to investigate the downlink multiuser outage capacity in the framework of (7), and test and validate the proposed algorithms with more realistic settings (6).

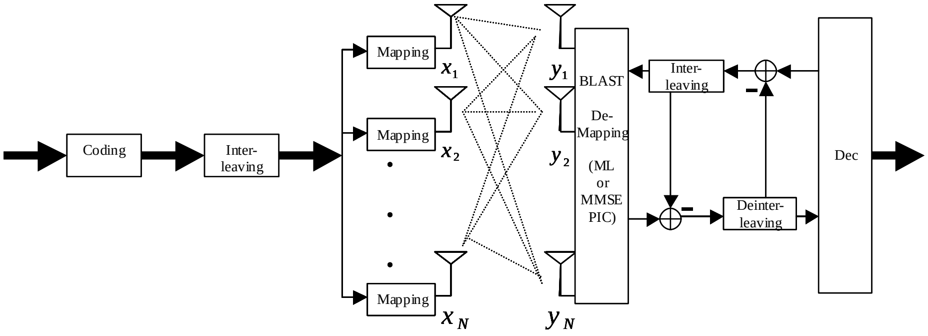

Fig. 1 Structure of Turbo-BLAST

III. MULTIUSER DETECTION WITH TURBO BLAST

In this section, we briefly introduce the key techniques that are used in detection. The reader is referred to [4] for details. The structure of Turbo-BLAST is shown in Fig. 1. The information bits are coded (not necessarily with Turbo codes) and interleaved as a whole; then the whole coded stream is demultiplexed into *N* substreams and symbol-mapped individually. At the receiver, the entire data stream is processed iteratively between a soft metric calculation stage and a decoding stage. Note that in the soft metric calculation stage, either a maximum likelihood joint detection or a MMSE multistage parallel interference cancellation (PIC) scheme can be used. Owing to the turbo processing, these two schemes achieve the same performance.

In [4], various multiuser detection methods have been studied for combating intercell interference on top of the Turbo-BLAST structure. Among them are maximum likelihood MUD, linear MMSE MUD, linear channel shortening MUD, and nonlinear group IC MUD. We have found that group IC MUD with the detection of the strongest interferer only performs the best when one interferer dominates. But when two interferers dominate that have the same power, it is no better than the simpler linear MMSE MUD scheme. It is also shown that in the two-dominant-interferer scenario, when the ratio between the two largest interferer power increases, the gap between the performance of group IC MUD and linear MMSE MUD also increases. An adaptive MUD detection scheme was proposed based on these observations, which makes a choice between the group IC MUD and linear MMSE MUD according to some fixed thresholds that are related to the structure of the interferers.

IV. SIMULATION RESULTS

A. Downlink Capacity of Interference-Limited MIMO

In this subsection, we examine the downlink capacity of interference-limited MIMO systems obtained through the techniques we have developed in the framework of (7). Figures 2 and 3 give the outage capacity for interference-limited MIMO when one and two interferers with equal power dominate, respectively.

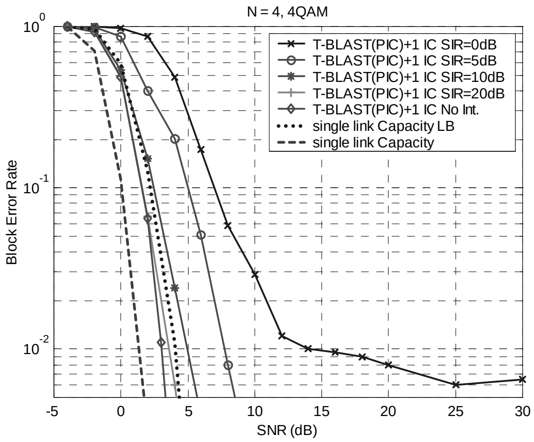

Fig. 2 Downlink capacity of interference-limited MIMO when one interferer dominates

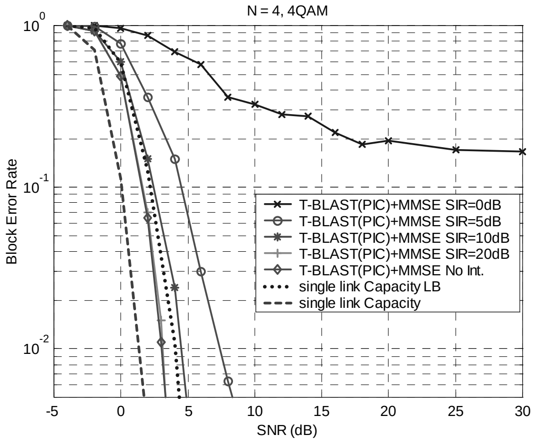

Fig. 3 Downlink capacity of interference-limited MIMO when two interferers dominate

An upper bound (corresponding to the no-interference situation) is derived from (3), where the block error rate is defined as the probability that the specified spectral efficiency (8/3 bits/s/Hz for 1/3 coded 4QAM-modulated 4×4 MIMO system) is not supported by the randomly generated channels. The Foschini approximation (single link capacity lower bound) is similarly derived from (4). For the one-dominant-interferer case, the Turbo-BLAST with a parallel interference cancellation demodulation stage, with one group IC MUD (T-BLAST (PIC)+1 IC) is employed, while for the two-equal-power-dominant-interferer case, the Turbo-BLAST with a parallel interference cancellation demodulation stage and with linear MMSE MUD pre-processing (T-BLAST (PIC)+MMSE) is used, as they achieve the best performance in each respective case [4].

The results are given for five situations: interference-free, SIR = 20, 10, 5 and 0dB. We see that in the noise-dominating scenario (interference-free, SIR = 20 dB), the MUD capacity is excellent, even better than the Foschini approximation (Turbo-BLAST usually yields better performance than D-BLAST). Even when the SIR = 10 dB, the MUD capacity is quite close to the Foschini approximation, which is only 2-3 dB away from the exact interference-free capacity. However, when the interference gets stronger, the MUD capacity gets worse, and eventually saturates, which indicates the limitations of this method in strong interference environment and leaves ample room for possible improvement through other techniques.

B. Large-Scale Simulation Results

In the last subsection, the performance evaluations have been done in the framework of (7), where we deliberately set the SNR, SIR, and power distributions among the interferers to fixed values that represent some typical cases. In this subsection, we test the performance in the more complete model of (6). The receivers of interest are 1) Coded V-BLAST treating intercell interference as noise (V-BLAST), which serves as a baseline reference; 2) Turbo-BLAST with a parallel interference cancellation demodulation stage, with linear MMSE MUD (T-BLAST (PIC)+MMSE); 3) Turbo-BLAST with a parallel interference cancellation demodulation stage, with adaptive MUD detection (T-BLAST (PIC)+ADPT); 4) Turbo-BLAST with a parallel interference cancellation demodulation stage, with the better between linear MMSE and Group IC MUD detection (T-BLAST (PIC)+IDEAL). The figure of merit is the CDF of the BLER performance for these four receivers. We collect 1000 points for this CDF profile.

1) NLOS Scenario

We assume a center-excited pico-cell structure with radius $d = 200$ m. The transmit antenna array sends out signals simultaneously from all elements with a total power of 1W at the 2.45GHz band, which undergo free-space path loss up to a distance of 10m, and then suffer path loss according to a power law with exponent $\eta = 3.7$. The lognormal shadow fading standard deviation υ = 8dB and Ricean *K*-factor = 0. The multipath fading is assumed to be zero-mean complex Gaussian with variance 0.5 per dimension. Each user is randomly located, according to a uniform distribution over the cell.

The simulation results are shown in Fig. 4, from which we can see that 1) advanced signal processing and coding techniques substantially improve the performance over the well-known V-BLAST technique with coding (roughly 30% more at 1% outage for the linear MMSE); 2) the adaptive scheme affords further gain over linear MMSE (roughly 9% more at 1% outage for the ideal case); 3) the adaptive detection scheme illustrated in IV-A approaches the ideal performance at the low BER area, which is of practical interest. The threshold values of the adaptive detection scheme could be refined to get better performance in practice.

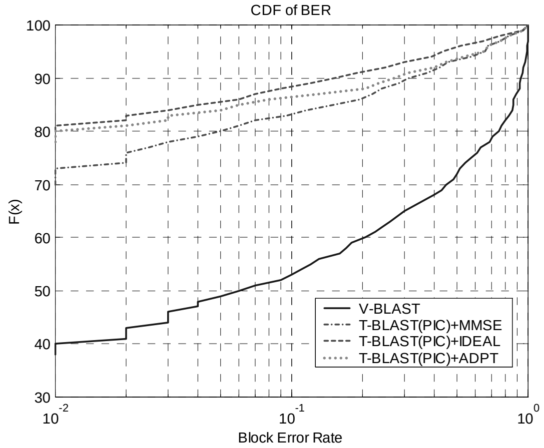

Fig. 4 CDF of block error rate for different receivers experienced by a mobile in Rayleigh fading

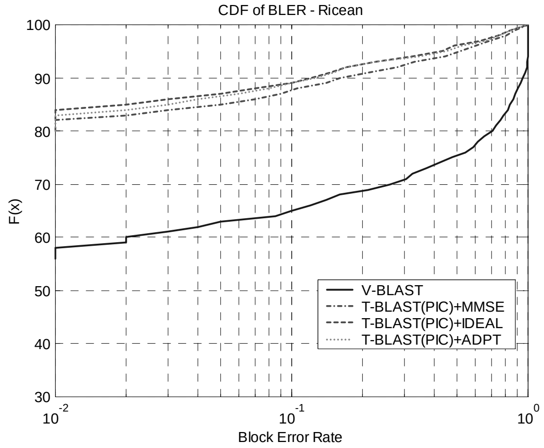

Fig. 5 CDF of block error rate for different receivers experienced by a mobile in Ricean fading

2) LOS Scenario

A user is randomly located as before, and the probability for LOS decreases linearly with its distance to a base station, until a "cutoff point", which is set at 300m [9]. If the signal from some base station is NLOS, the same parameters as IV. B. 1) is used. Otherwise, we set the Ricean factor to

$$K = 13 - 0.03d \text{ dB}, \tag{8}$$

where d is the distance to some base station, and the path-loss exponent to 2. Slightly different from model (5), we assume no shadowing for the LOS component. Furthermore, we assume that the transmitter and receiver are positioned far apart from each other compared with the antenna spacing, so we get a rank 1 system matrix for the LOS component with energy equally distributed between real and imaginary parts [5].

The simulation results are shown in Fig. 5. Compared with Fig. 4, we see that the performance of the V-BLAST technique with coding significantly increases due to less signal fading [7]. MUD techniques with the Turbo-BLAST structure still greatly improve the system performance over the V-BLAST. But the advantage of the adaptive scheme over the linear MMSE is negligible.

V. CONCLUSIONS

This paper has explored the downlink capacity of interference-limited MIMO cellular fading systems. In contrast to the single-cell MIMO system considered in previous studies, where the intercell interference, when accounted for, is added to ambient Gaussian noise, we take the approach of modeling the whole downlink cellular system as a broadcast/interference channel, the capacity of which has long been an open question. As block fading is assumed, we are interested in outage capacity instead of the Shannon ergodic capacity. Upper bounds for this capacity are obtained from the interference-free single-link theoretical formulas. We have primarily addressed the issue of how closely we can approach those bounds without any base station cooperation by implementation and simulation of advanced techniques.

A much better downlink capacity over the well-known V-BLAST techniques with coding in this interference-limited cellular environment has been obtained. Nonetheless, numerical results indicate that, due to complexity constraints and adverse environments, there is a significant performance gap between MUD capacity and interference-free capacity, especially in environments with strong interference (SIR of 5 dB or less). This indicates a need to exploit more complex schemes, such as base station cooperation (macrodiversity) for interference reduction, to enhance the system throughput.

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REFERENCES

[1] S. L. Ariyavisitakul, "Turbo space-time processing to improve wireless channel capacity," *IEEE Trans. Commun.*, vol. 48, no. 8, pp. 1347-1359, Aug. 2000.

[2] E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: information-theoretic and communications aspect," *IEEE Trans. Inform. Theory*, vol. 44, no. 6, pp. 2619-2692, Oct. 1998.

[3] S. Catreux, P. F. Driessen and L. J. Greenstein, “Simulation results for an interference-limited multiple-input multiple-output cellular system,” *IEEE Comm. Lett.*, vol. 4, no. 11, pp. 334-336, Nov. 2000

[4] H. Dai and A. F. Molisch, “Multiuser Detection for Interference-Limited MIMO Systems,” *Proc. IEEE VTC 2002 Spring*, Birmingham, AL, May 2002.

[5] D. F. Driessen and G. J. Foschini, “On the capacity formula for multiple input-multiple output wireless channels: a geometric interpretation,” *IEEE Trans. Commun.*, vol. 47, no. 2, pp. 173-176, Feb. 1999

[6] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas, " *Wireless Personal Communications*, vol. 6, no. 3, pp. 311-335, Mar. 1998.

[7] M. A. Khalighi et al., “On capacity of Ricean MIMO channels,” *Proc. IEEE PIMRC 2001*, San Diego, CA, Sep 30-Oct 3. 2002.

[8] S. Shamai and A. Wyner, "Information theoretic considerations for symmetric, cellular multiple-access fading channels – part I and part II," *IEEE Trans. Inform. Theory*, vol. 43, no. 6, pp. 1877-1911, Nov. 1997.

[9] M. Steinbauer and A. F. Molisch (eds.), “Directional channel modeling,” in L. Correia (ed.), *Flexible Personalized Wireless Communications*, Wiley, 2001.

[10] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecomm.*, vol. 10, no. 6, pp. 585-595, Nov.-Dec. 1999.

[11] S. Verdu, *Multiuser Detection*, Cambridge, U.K.: Cambridge Univ. Press, 1998.

[12] X. Wang and H. V. Poor, “ Iterative (turbo) soft interference cancellation and decoding for coded CDMA”, *IEEE Trans. Commun.*, vol. 47, no. 7, pp. 1046-1061, July 1999.

[13] A. Wyner, "Shannon-theoretic approach to a Gaussian cellular multiple-access channel," *IEEE Trans. Inform. Theory*, vol. 40, no. 6, pp.1713-1727, Nov. 1994.

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