

# ENERGY EFFICIENT DISTRIBUTED DETECTION VIA MULTI-HOP TRANSMISSION IN SENSOR NETWORKS

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## ABSTRACT

Most existing works on distributed detection have assumed that a bank of independent dedicated channels or a multi-access channel is used for transmitting local decisions to the fusion center. Such a transmission scheme could result in significant energy expenditure when sensors are far from the fusion center. The natural solution to energy-efficient distributed detection is to use multi-hop transmission with information fusion at intermediate nodes. In this paper, we investigate three options for multi-hop fusion schemes: Multihop Forwarding (MF), Multihop Histogram Fusion (HF) and Multihop LLR Fusion (LF). For the simplest MF scheme, the quantized observations are sent to the fusion center along the shortest path tree without further processing at relaying nodes. For HF, each sensor transmits the histogram of the observations of its descendants and itself, which achieves further energy reduction relative to MF when the number of quantization bits is small. For LF, the normalized log-likelihood ratio (LLR) values for subsets of nodes are computed and propagated along the minimum spanning tree, such that the fusion center acquires an estimate of the normalized LLR of all sensors' observations, which is used to decide the hypothesis. We show that LF exhibits the most favorable energy scaling laws with the network size among these schemes. Simulation results suggest that multihop fusion schemes significantly reduces the transmission energy compared with direct transmission, with LF requiring the least energy to achieve the same detection performance.

## 1. INTRODUCTION

Large-scale sensor networks are suitable for signal detection applications, as the detection error probability decays exponentially with the number of observations if they are perfectly available to the fusion center, or after reasonable processing at distributed sensors, such as quantization. Most existing works on distributed detection have focused on de-

signing local mapping rules and fusion rules to maximize the detection performance under a given bandwidth constraint, and relatively little work has been done to address the problem of power-scarcity in wireless sensor networks. In fact, the conventional way of treating the decision fusion independently from communication often leads to energy-inefficient designs. It is the goal of this paper to study fusion rules and transmission schemes that can be implemented with low power while yielding desirable performance.

Distributed detection with multiple sensors can be traced to Tenney and Sandell [1] and Tsitsiklis [2]. For different sensor data models and communication channel models, optimal local decision rules and fusion rules have been developed to maximize the detection performance [3–6] –see [7] for a survey on this area. Most of these works assumed a bank of independent dedicated channels [4–7] or a multi-access channel [3] is used for transmitting local decisions to the fusion center. However, direct transmission could result in significant energy expenditure, especially when sensors are far from the fusion center. Since the signal power decays extremely fast with the distance, a natural way for energy conservation is to reduce transmission distances by using serial fusion, already surveyed in [7], or more generally, multi-hop fusion, where the transmission structure along which messages are sent to the fusion center may be a general spanning tree. The idea of utilizing the spanning tree for distributed detection has been explored in [8], where each sensor transmits the real-valued log-likelihood ratio (LLR) to the next high level sensor only when the LLR is above a threshold. However, the assumption that real values can be received perfectly does not hold in practice, and makes it difficult to evaluate the true detection performance and energy expenditure. In this paper, we focus on the regime where the number of sensors is large, and investigate several options for multi-hop fusion with sensors transmitting bit sequences. The fusion rules at the sensors and at the fusion center, as well as the transmission structure of multi-hop fusion jointly determine the detection performance and the energy consumption, and both are integral parts of design. The related problem of joint treatment

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of distributed source coding and the transmission structure for data-gathering in sensor networks has been investigated by Cristescu and Vetterli [9]. Our problem can also be regarded as a particular problem of in-network computation in sensor networks [10].

We cast the problem as a binary hypothesis testing problem with known priors. Sensors are geographically-dispersed and have different distances to the fusion center, and sensor observations are assumed to be independently and identically distributed (i.i.d.) conditioned on the hypothesis. The messages sent by sensors are bit sequences, and the energy for transmitting one bit is selected such that a certain received energy is maintained. The received messages are assumed to be error-free. The required path routes are pre-computed so we do not have to consider the overhead of any particular routing protocol. The traditional approach of direct transmission (DT) using independent parallel channels serves as a performance baseline in our work. Local observations are quantized using the optimal likelihood ratio quantizer (LRQ) [11] in the sense that the exponent for probability of error is maximized. In addition, we consider three multihop fusion schemes: Multihop Forwarding (MF), Multihop Histogram Fusion (HF) and Multihop LLR Fusion (LF). For Multihop Forwarding, the quantized observations are transmitted to the fusion center through the Shortest Path Tree (SPT) without further processing at relaying nodes. The same detection performance as direct transmission is achieved, with a significantly reduced transmission energy. For Histogram Fusion, instead of transmitting the exact quantized observations, each sensor transmits the histogram of the observations of its descendants and itself. Since the histogram is a sufficient statistic for detection with conditionally i.i.d. observations, no performance loss is incurred. Moreover, the number of bits transmitted by a node grows only logarithmically with the number of its descendants (instead of linearly in the MF scheme). We propose to use the minimum spanning tree (MST) as the transmission structure for HF, which is shown to offer further energy saving than MF when the number of quantization levels is small. For Multihop LLR Fusion, the normalized LLR values for subsets of nodes are computed and propagated along the MST, such that at the end of the algorithm, the fusion center acquires an estimate of the normalized LLR of all sensors' observations, which is used to decide the hypothesis. Unlike MF and HF, LF suffers performance loss compared with direct transmission due to information compression at intermediate nodes. Simulation results show that for detecting a deterministic signal in Gaussian noise with  $n = 100$  and SNR=-10dB, LF provides 2 to 3 folds energy reduction compared with MF to achieve the same detection performance. Finally, we show that LF exhibits the most favorable energy scaling behavior with the network size  $n$  among these schemes.

The rest of the paper is organized as follows. Section 2 introduces the system model and some notations used in the paper. In Section 3, direct transmission is considered as a performance baseline, and the optimal quantization scheme is described. In Section 4, we introduce the MF, HF and LF schemes and study their detection performance and energy expenditure. We also study the energy scaling laws associated with these schemes. Section 5 contains the numerical results and discussions, and Section 6 concludes the paper.

## 2. SYSTEM MODEL AND NOTATIONS

We assume that a bunch of sensors, indexed by  $1, \dots, n$ , are dispersed on a geographical area. Sensor  $j$  makes an observation  $Y_j$ , which is conditionally i.i.d. across sensors given the hypothesis. The cumulative distribution function under  $H_0$  and  $H_1$  are respectively given by  $F_0$  and  $F_1$ , with differentiable Lebesgue densities  $f_0$  and  $f_1$ . The fusion center, denoted as  $S$ , after receiving messages from sensors, makes a final decision about whether hypothesis  $H_0$  or  $H_1$  is true.

The sensor nodes and the fusion center constitute the vertices of a fully-connected graph  $G = (V, E)$ . The weight of an edge  $e = (i, j)$ , denoted by  $w(e)$ , is defined as the energy consumed for directly transmitting one bit between node  $i$  and  $j$ . Following the propagation model in wireless environment, we assume that  $w(e) = d(i, j)^\kappa$ , where  $d(e)$  is the Euclidean distance between  $i$  and  $j$ , and  $\kappa$  is the pathloss exponent. For wireless networks  $\kappa \in [2, 4]$ , and a typical value is 2.5. The transmission structure, along which information is propagated to the fusion center, is a spanning tree (ST) on  $G$  rooted at  $S$ . For node  $j$ ,  $ch_j$  denotes the set of children of  $j$ ,  $de_j$  denotes the set of descendants of  $j$ ,  $pa_j$  denotes the parent of  $j$ ,  $lev_j$  denotes the level of  $j$  (i.e., the number of hops from the fusion center), and  $n_j$  denotes the size of  $j$  (i.e., the number of descendants including itself).

## 3. DIRECT TRANSMISSION

The traditional approach, where all sensors transmit their quantized observations through the direct link to the fusion center, serves as a performance baseline in our work. It is known that under the conditional independence assumption, optimal local quantizers that minimizes the probability of error are likelihood ratio tests characterized by a finite number of thresholds [1], and for conditionally i.i.d. observations, it is asymptotically optimal (in term of the error exponent) to have each sensor use the same quantizer [2]. Tsitsiklis subsequently investigated some properties of the likelihood-ratio quantizer (LRQ) [11], and shown that LRQ is optimal when the optimization criterion is one from the important class of  $f$ -divergences, e.g., the Kullback-Leibler distance, the Chernoff Information and the Bhattacharyya distance. In this paper, we consider the regime of the num-

ber of sensors going to infinity, where the natural optimization criterion is the error exponent given by the Chernoff Information. For most detection problems, the Chernoff Information of the quantized values are difficult to evaluate, and it is convenient to replace it with the Bhattacharyya distance, which provides a lower bound for the Chernoff Information [4, 6]. Denote the log-likelihood ratio (LLR) function

$$L(y) \triangleq \log \frac{f_1(y)}{f_0(y)}, \quad (1)$$

and let  $L_j \triangleq L(y_j)$ , whose distribution functions under  $H_0$  and  $H_1$  are denoted by  $F_{L_0}$  and  $F_{L_1}$  with corresponding density functions  $f_{L_0}$  and  $f_{L_1}$ . Assume that  $k$ -bit quantization is used, then the LRQ quantizer may be characterized by the threshold test performed on the LLR's, with the threshold vector  $\mathbf{a} = [a_0, \dots, a_M]^T$ , where  $M = 2^k$  and  $-\infty = a_0 < a_1 \dots < a_{M-1} < a_M = +\infty$ . The associated  $M$  quantization intervals are  $I_i = (a_{i-1}, a_i]$ ,  $i = 1, \dots, M$ . The quantization is defined by the mapping  $U$ :

$$u_j \triangleq U(y_j) = i, \quad \text{if } L_j \in I_i. \quad (2)$$

This induces the probability mass function (p.m.f.) of the quantized observations under  $H_0$ , given by

$$p_0(i) \triangleq \Pr(u_j = i | H_0) = F_{L_0}(a_i) - F_{L_0}(a_{i-1}), \quad (3)$$

where  $1 \leq i \leq M$ , and similarly define the p.m.f.  $p_1$  under  $H_1$ . For a given  $M$ , the problem is to find the optimal  $\mathbf{a}$  that maximizes the Bhattacharyya distance:

$$B_M(\mathbf{a}) = -\log \sum_{i=1}^M \sqrt{p_0(i)p_1(i)}. \quad (4)$$

Denote the LLR for quantized values

$$L_q(i) \triangleq \log \frac{p_1(i)}{p_0(i)}, \quad i = 1, \dots, M. \quad (5)$$

The necessary condition for  $\mathbf{a}$  to be a local minimum is the gradient  $\nabla B_M(\mathbf{a}) = 0$ , which yields

$$\log \frac{f_{L_1}(a_i)}{f_{L_0}(a_i)} = \frac{L_q(i) + L_q(i+1)}{2}. \quad (6)$$

For a satisfying (6) to give a local minimum, it is sufficient that the Hessian matrix is positive definite. The Hessian matrix  $\mathbf{H}$  is a symmetric tri-diagonal matrix. Results on the definiteness of this type of matrix are given in [12] where a similar optimization problem is discussed. Newton's method may be applied to search for the minimum.

The energy consumption for DT is given by

$$E_{DT} = k \sum_{j=1}^n w((j, S)). \quad (7)$$

Since the weight of the direct path to the fusion center can be very large for far sensor nodes, the DT scheme is very energy-inefficient.

## 4. MULTIHOP FUSION SCHEMES

### 4.1. Multihop Forwarding (MF)

The easiest alternative to direct transmission is multihop forwarding (MF): each node simply transmits its quantized observation via the path on the transmission structure to the fusion center. Since the intermediate nodes only forward the information without processing, the achievable performance is the same as direct transmission. The optimal spanning tree is the one that minimizes the total transmission energy

$$ST^* = \arg \min_{ST} k \sum_{j=1}^n c_j, \quad (8)$$

where  $c_j$  is the total weight of the path connecting node  $j$  and the fusion center on the spanning tree. It is well-known that the shortest path tree (SPT) is a superposition of all individual shortest paths for different nodes, hence SPT is the optimal transmission structure for the MF scheme. Let  $c_j^*$  denote the weight of the shortest path from  $j$  to  $S$ . The transmission energy of MF is given by

$$E_{MF} = k \sum_{j=1}^n c_j^*. \quad (9)$$

### 4.2. Multihop Histogram Fusion (HF)

For the MF scheme, sensor  $j$  needs to send all the data of its descendants to its parent node on the SPT. Hence for a dense network, nodes with large number of descendants generate heavy traffic and drain power fast. The overall energy efficiency can be further improved, if traffic generated by those nodes can be reduced. Note that for detection with conditionally i.i.d. observations, the histogram of sensor observations is a sufficient statistic. Each sensor can therefore only send the histogram of its descendants (including itself) quantized observations instead of the exact bits, without any loss in the detection performance. Since binary or quadratic quantization is sufficient for most detection scenarios when the number of sensors is large, transmitting histograms instead of the exact observations conserves energy and bandwidth. Histogram fusion, also known as "type-based fusion" for distributed detection has been investigated in [13] and [14], where it is assumed that all sensors transmit the histogram (or type) of their own observations simultaneously through a *multi-access channel*, and the resultant error exponent is shown to be the same as centralized detection, irrespective of the transmission power. Although theoretically appealing, the implementation of such a scheme can be difficult, and lack of MAC synchronization or channel fading can significantly degrade the detection performance. Collecting histograms of sensor data *via multi hops* has been

considered in [10] for computing the class of symmetric functions.

Here we consider histogram fusion (HF) implemented in the multihop fashion. Node  $j$  uses  $M$  time slots for transmission, and in the  $i$ th slot,  $\max\{\lceil \log_2(n_j) \rceil, 1\}$  bits are transmitted to indicate the number of descendants of  $j$  that has observation  $i$ . Hence the transmission energy for  $j$  grows only logarithmically with  $n_j$ , instead of linearly for MF, while for  $k = 1, 2$ ,  $M$  is simply twice of  $k$ . Since leaf nodes can waste energy by transmitting histograms, the original quantized bits may be transmitted instead.

The remaining problem is to find the optimal spanning tree that minimizes the total transmission energy for HF:

$$ST^* = \arg \min_{ST} 2^k \sum_{j=1}^n w((j, pa(j))) \max\{\lceil \log_2(n_j) \rceil, 1\}.$$

It is difficult to obtain an exact solution to this problem, as the edge weights and node sizes of the spanning tree are inter-dependent. Observe that  $\log_2(n_j)$  does not vary much for different  $j$ , thus we may obtain a suboptimal solution by assuming that it is a constant. In which case Under this assumption, the solution to the optimization problem is given by the minimum spanning tree (MST), which minimizes the total weight of all edges in the spanning tree. Denote the parent of  $j$  in the MST by  $pa^*(j)$ , and  $w_j^* \triangleq w((j, pa^*(j)))$ . The transmission energy for HF is therefore given by

$$E_{HF} = 2^k \sum_{j=1}^n w_j^* \max\{\lceil \log_2(n_j) \rceil, 1\}. \quad (10)$$

Note that for HF, the message structure is different for different nodes depending on their sizes. An easier alternative is to use the upper bound of  $n_j$ , which is  $n$ , at all nodes, and in that case, the optimal spanning tree is indeed given by the MST. It can be expected that in a dense network, the total cost of the MST is much lower than the sum of the weights of the shortest paths in the SPT, thus HF achieves better energy efficiency than MF when  $k$  is small.

### 4.3. Multihop LLR Fusion (LF)

The message lengths in the HF scheme can still be large. This motivates us to explore the possibility of further constraining the number of bits each node can transmit. For the simplest case, let us consider enforcing each node in the transmission structure transmit only  $k$  bits based on its observation and the messages from children. Obviously such a scheme requires some form of information reduction performed at the intermediate nodes. In the following, we propose local fusion rules with which the same detection performance can be achieved with a smaller transmission energy compared with MF and HF.

Recall that for centralized detection in the Bayesian setting, the fusion center performs a threshold test on the normalized LLR

$$L_S \triangleq \frac{1}{n} \sum L_j \triangleq \frac{1}{n} \sum L(y_j), \quad (11)$$

and  $H_1$  is accepted (rejected) if  $L_S > 0$  (respectively,  $L_S \leq 0$ ) as  $n \rightarrow \infty$ . Denote  $T_i \triangleq \{y : L(y) \in I_i\}$ . We have

$$L_q(i) = \log \frac{p_1(i)}{p_0(i)} = \log \frac{\int_{T_i} f_1(y) dy}{\int_{T_i} f_0(y) dy} \in I_i. \quad (12)$$

This shows that for relatively fine quantization,  $L_q(i) \approx L$  for all  $L \in I_i$ . Therefore, a node having only direct children can obtain an estimate of each of its children's LLR based on their quantized values, from which the normalized LLR can be computed. Subsequently, the resultant normalized LLR is quantized again using an LRQ, and the quantized observation is sent to its parent. The parent estimates the normalized LLR's at each of its children from their quantized values, and compute the normalized LLR of all its descendants (including itself), which is quantized again. The process continues until the fusion center is reached. Thus at the end of the algorithm, the fusion center obtains an estimate of  $L_S$ , denoted as  $\hat{L}_S$ , to which the threshold test is applied. Note that the LRQ here is only an ordinary quantizer for reconstructing the LLR values with good fidelity. Since the normalized LLR at different nodes have different distribution functions depending on their positions in the transmission structure, it is impossible to design a single quantizer for all sensors such that the distortion at the fusion center is minimized. To simplify the problem, we propose to use the same LRQ discussed in Section 3 at all nodes. Obviously, for this purpose, binary or quadratic quantizers are not sufficient.

#### 4.3.1. Algorithm

Given the transmission structure, the LLR Fusion starts from highest level nodes. The level number  $l$  is set to be  $\max_j(lev_j)$  at the initial stage. During one iteration, all nodes  $j$  with  $lev_j = l$  process the data from its own measurement and from their children, following the four steps below:

1. Compute the LLR of the unquantized observation of  $j$ :  $L_j = L(y_j)$ .
2. Compute the LLR of the quantized value of every child of  $j$ :  $\hat{L}_t = L_q(u_t)$ ,  $t \in ch_j$ , where  $u_t$  is the quantized value sent by node  $t$ . This step is skipped if  $j$  is a leaf node.
3. Obtain an approximation of the normalized LLR for nodes in the subtree including  $j$  and its descendants,

denoted by  $\tilde{L}_j$ :

$$\tilde{L}_j = \frac{1}{n_j} \left( L_j + \sum_{t \in ch_j} n_t \hat{L}_t \right). \quad (13)$$

For a leaf node, the above would give  $\tilde{L}_j = L_j$ .

4. Quantize the approximate normalized LLR value using the LRQ, i.e.,

$$u_j \triangleq i, \quad \text{if } \tilde{L}_j \in I_i, \quad (14)$$

and send  $u_j$  to the parent node.

At the end of each iteration, the level number  $l$  is deducted by 1 and the process repeats until the level number becomes 0. The fusion center then computes an approximation of the normalized LLR:

$$\hat{L}_S = \frac{1}{n} \sum_{j \in ch_S} n_j \hat{L}_j, \quad (15)$$

where  $\hat{L}_j = L_q(u_j)$ . A threshold test is then performed on  $\hat{L}_S$  to decide the hypothesis.

#### 4.3.2. Analysis

We have  $\hat{L}_j = L_q(u_j) \in I_{u_j}$ , as well as  $\tilde{L}_j \in I_{u_j}$ . Thus for relatively large  $k$ ,  $I_{u_j}$  is small, and  $\hat{L}_j \approx \tilde{L}_j$ .

**Proposition:** Assume that  $k \rightarrow \infty$  such that  $\hat{L}_j = \tilde{L}_j$ , then

$$\hat{L}_S = L_S. \quad (16)$$

**Proof:** It suffices to show that for  $1 \leq j \leq n$ ,  $\hat{L}_j = \frac{1}{n_j} \left( L_j + \sum_{t \in de_j} L_t \right)$ . This obviously holds for a leaf node  $j$ . Now, for an intermediate node  $j$ , assume that  $\hat{L}_t = \frac{1}{n_t} \left( L_t + \sum_{v \in de_t} L_v \right)$ ,  $\forall t \in ch_j$ . Since  $\hat{L}_t = \tilde{L}_t$ ,  $\forall t \in ch_j$ , we have

$$\begin{aligned} \tilde{L}_j &= \frac{1}{n_j} \left( L_j + \sum_{t \in ch_j} n_t \tilde{L}_t \right) \\ &= \frac{1}{n_j} \left[ L_j + \sum_{t \in ch_j} \left( L_t + \sum_{v \in de_t} L_v \right) \right] \\ &= \frac{1}{n_j} \left( L_j + \sum_{t \in de_j} L_t \right). \end{aligned}$$

□

The above shows that for fine quantization, there is little loss in performance compared with centralized detection

based on  $L_S$ . For small  $k$ , the difference between  $\hat{L}_j$  and  $\tilde{L}_j$  is non-negligible, and the performance of LF degrades. Unlike the MF and HF schemes, the detection performance of LF critically depends on the transmission structure. Intuitively, the more approximations for the LLR value of each node is incurred during fusion, the severer the distortion of the resultant normalized LLR. Hence the transmission structure for LF is relatively flexible. The best detection performance is obviously achieved when all sensor nodes are direct children of the fusion center, which corresponds to the most energy-inefficient DT scheme. On the other hand, for a given  $k$ , the best energy efficiency is achieved when the spanning tree is the MST. Simulation results show that a larger  $k$  is often required to achieve the same error probability when the spanning tree is MST compared with DT, MF and HF, but the transmission energy can still be reduced. Other transmission structures may be used, but extensive simulations suggest that MST seems to be the best choice in the sense that it requires the least energy to attain the same error probability. Therefore, in the sequel, the transmission structure for LF is implied to be MST. The total transmission energy for LF is therefore

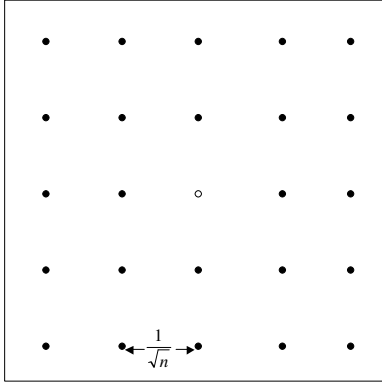
$$E_{LF} = k \sum_{e \in MST} w(e). \quad (17)$$

Furthermore, note that messages sent by nodes with a larger number of descendants carries more weight in determining the final normalized LLR. Hence we can apply LF adaptively by allowing different quantization levels at different nodes depending on their sizes—nodes with more descendants adopt finer quantization than nodes with less descendants. Simulation results suggest that some performance improvement can be realized by adaptive quantization.

#### 4.4. Energy Scaling Laws

In the above sections we have presented the formulae for computing the energy consumption of various distributed detection schemes, which depends on the number of sensors  $n$ . For large-scale sensor networks, the scaling laws for transmission energy with  $n$  provide additional insight into the comparative performance of these schemes. In this section, we establish the scaling laws of the total transmission energy with the number of nodes  $n$  (the node density grows with  $n$  with the area of the sensor field fixed).

The placement of sensors maybe deterministic or random. For large-scale sensor networks, random placement of nodes is considered more practical for its lower cost and latency in the deployment of the network. It is easier but instructive to obtain the scaling laws for the deterministic case when we can control the placement. As an example, we consider the energy consumption for the grid network structure frequently-adopted in the literature, i.e.,  $n$  sensor nodes and the fusion center (located at the center) form



**Fig. 1.** Grid deployment over the square region of unit area, fusion center at the center

a grid with internode spacing  $\frac{1}{\sqrt{n}}$ , as depicted in Fig. 1. For direct transmission, since the transmissions are independent across sensors, the total transmission energy obviously scales linearly with  $n$ . The shortest path between any sensor node and the fusion center consists of  $\Theta(\sqrt{n})$  hops, with the weight for each hop on the order of  $\left(\frac{1}{\sqrt{n}}\right)^\kappa$ . Hence the sum of the shortest path lengths for all nodes in the SPT is on the order of  $n \cdot \sqrt{n} \cdot \left(\frac{1}{\sqrt{n}}\right)^\kappa = n^{\frac{3-\kappa}{2}}$ . This suggests that the energy cost for Multihop Forwarding is  $\Theta\left(n^{\frac{3-\kappa}{2}}\right)$ . The minimum spanning tree connecting  $n+1$  nodes has  $n$  edges with the weight of each edge on the order of  $\left(\frac{1}{\sqrt{n}}\right)^\kappa$ . Thus the cost for LLR fusion, which equals the cost of the MST, is  $\Theta\left(n^{1-\frac{\kappa}{2}}\right)$ . For Histogram Fusion, using  $\log_2(n)$  as an upper bound of  $\log(n_j)$  at all nodes gives an upper bound on the energy consumption, which scales like  $\Theta\left(n^{1-\frac{\kappa}{2}} \log n\right)$ . The growth rate using the original HF scheme is between  $\Theta\left(n^{1-\frac{\kappa}{2}}\right)$  and  $\Theta\left(n^{1-\frac{\kappa}{2}} \log n\right)$ . The above results provide interesting insights: LF exhibits favorable energy scaling behavior, and the energy consumption decreases with  $n$  as long as  $\kappa > 2$ . For the typical value of  $\kappa = 2.5$ , the energy expenditure for DT, MF, HF and LF respectively scale like  $\Theta(n)$ ,  $\Theta\left(n^{\frac{1}{4}}\right)$ ,  $\Theta\left(n^{-\frac{1}{4}} \log n\right)$  and  $\Theta\left(n^{-\frac{1}{4}}\right)$ .

Now let us consider random placement, with  $n$  sensor nodes independently and uniformly located over the square region of unit area, and the fusion center at the center. For this case, the growth rate of  $\Theta\left(n^{1-\frac{\kappa}{2}}\right)$  for the path length of minimum spanning trees has been well-established for both  $\kappa < 2$  and  $\kappa \geq 2$  [15]. To the authors' knowledge, rigorous result pertaining to the total path lengths for SPT in a random graph with Euclidean-powered edge weights is still lacking. We conjecture that the above law  $\Theta\left(n^{\frac{3-\kappa}{2}}\right)$  also holds for uniform deployment, which is supported by simulation results. The scaling laws of the transmission energy for various schemes are summarized in Table 1.

**Table 1.** Scaling Laws of the Total Transmission Energy

Direct Transmission	$\Theta(n)$
Multihop Forwarding	$\Theta\left(n^{\frac{3-\kappa}{2}}\right)$
Histogram Fusion	$\Theta\left(n^{1-\frac{\kappa}{2}} \log n\right)$
LLR Fusion	$\Theta\left(n^{1-\frac{\kappa}{2}}\right)$

**Table 2.** Best Bhattacharyya distances and error probabilities for detecting deterministic signal in Gaussian noise,  $n = 100$ , SNR=-10dB

$k$	$\infty$	1	2	3	4	5
$B$	0.05	0.0318	0.0441	0.0483	0.0495	0.0498
$P_e$	7.83e-4	6e-3	1.5e-3	1.0e-3	9e-4	8e-4

## 5. NUMERICAL RESULTS

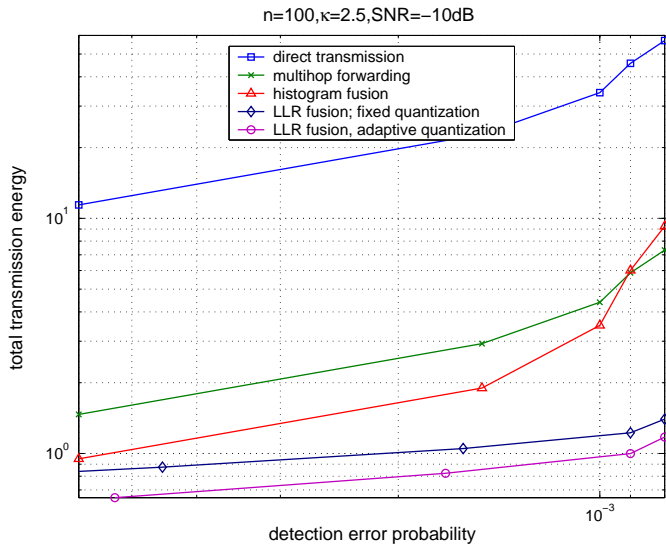
Large-scale sensor network is useful for detection of weak signals. As an example, let us consider detection of a deterministic signal in Gaussian noise

$$H_0 : y = -m + z,$$

$$H_1 : y = m + z,$$

where  $m$  is the signal mean, and  $z$  is a zero-mean Gaussian noise with variance  $\sigma^2$ . The Signal-to-Noise Ratio (SNR) is  $\frac{m^2}{\sigma^2}$ . The Bhattacharyya distance as well as the error probability for centralized detection DT, MF and HF using the optimal quantizer with  $n = 100$  and SNR= -10dB (representing an adverse SNR) are given in Table. 2. The error exponent and error probability for centralized detection case ( $k = \infty$ ) are computed and included for comparison. We observe that the detection performance of these distributed detection schemes converges to that of the centralized detection fast with  $k$ , and the performance improvement beyond  $k = 3$  is only marginal.

We assume the random uniform sensor deployment with  $n = 100$  and the pathloss exponent  $\kappa = 2.5$ . For LLR Fusion, two quantization options are considered: 1) Fixed quantization: all nodes use the same quantization level  $k$ . 2) Adaptive quantization: for a given  $k$ ,  $k-1$ ,  $k+1$  bit quantization are respectively used by nodes with  $n_j < 5$ ,  $n_j < 20$ , and  $n_j \geq 20$ . Fig. 2 illustrates the total power consumption v.s. detection error probabilities in logarithmic scale for various distributed detection schemes. It can be seen that multi-hop transmission strategies greatly reduce the transmission energy as compared with direct transmission. There is a cross-over of the two curves corresponding to MF and HF, with HF being more energy efficient for an error probability no greater than  $9e-4$ , a value already very close to the error probability for centralized detection. Simulation results show that for the parameters used, LF with fixed quantization requires roughly 4 more bits for quan-



**Fig. 2.** Total transmission energy for direct transmission and various multihop fusion schemes,  $n = 100$ ,  $\kappa = 2.5$ .

tization to achieve the same error probability as DT, MF or HF. Nevertheless, it is interesting to note that for the same detection performance, LF provides 2 to 3 folds reduction in transmission energy compared with MF. Moreover, adaptive quantization offers some further improvement than fixed quantization. For example, adaptive quantization with  $k = 5$  achieves similar error probability as fixed quantization with  $k = 6$ , but the energy consumption is about the same as fixed quantization with  $k = 5$ .

## 6. CONCLUSION

Several options for energy-efficient distributed detection with multihop transmission and fusion at intermediate nodes are investigated. The main novelty of this paper lies in the LLR fusion scheme, which is shown to outperform multihop forwarding and histogram fusion in terms of the required energy to achieve a certain probability of error, as well as the energy scaling law with the network size.

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