Transport Throughput of Secondary Networks in Spectrum Sharing Systems

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***Abstract*—Spectrum sharing systems such as cognitive radio networks have drawn much attention recently due to their potential to resolve the conflict between increasing demand for spectrum and spectrum shortage. Such systems are typically composed of primary and secondary networks; the configuration of the latter depends on spectrum opportunity unexploited in the former. In this paper we explore the characteristics of the single hop transport throughput (STT) of the secondary network with outage constraints imposed on both networks. STT is a new metric that inherits the merits of both the traditional transport capacity and another popular metric, transmission capacity, incorporating transmission distance and outage probability into a uniform framework. We first derive the limit of STT, single hop transport capacity (STC), together with a practical upper bound for it. Then we investigate STT with secondary receivers randomly located in the field of interest. Three models regarding the selection of receivers are considered: optimally selected, randomly selected, or the nearest. Study on these models provides a comprehensive view of achievable secondary network throughput, and offers insights into the configuration of secondary networks. In addition, the broadcast transport throughput (BTT) of the secondary networks is also investigated as an extension of STT, and its similarity with STT in the nearest neighbor model is revealed.**

I. INTRODUCTION

Spectrum sharing systems have tremendous potential to alleviate the spectrum shortage problem and achieve remarkable spectrum efficiency; the inherent spectrum sharing mechanism also provides a flexible way of spectrum management. One prominent example is cognitive radio networks, where secondary (unlicensed) users are allowed to temporarily access spectrum that is not currently used by primary (licensed) users. It is generally preferable that the operation of the secondary network is transparent to the primary network, which requires that the interference incurred by secondary operations be constrained within an acceptable level.

It is of great interest to understand to what extent the secondary network can gain in transmission of its own useful data, without harming the regular operation of the primary network significantly. In literature, relevant research is dominated by the study of either the scaling law of transport capacity [1]–[3] or transmission capacity with outage consideration [4]–[7]. The former was proposed in the seminal work [8] and defined as the maximum bit·meters per second the network can achieve in aggregate; recent works in the capacity study of CR networks or ad-hoc overlaid networks [1]–[3], [9] show that there is no performance loss for the secondary network in terms of scaling law of transport capacity. Nonetheless, asymptotic analysis on the scaling law only characterizes the (rough) relationship between capacity and network size, neglecting the effect of many important system parameters. As an alternative, transmission capacity [10] quantifies the maximum spacial density under some outage probability constraint, and sparks an enormous interest recently (see [11] for its latest development). The outage probability of the secondary networks is studied in [4], [5]. In [6] it is shown that the spectrum efficiency of the whole overlaid networks can be improved if extra outage is allowed for the primary network. The *achievable* transmission capacity[1] of the secondary network is studied in [7], and maximized with respect to its transmitter density. Transmission capacity admits quantitative system analysis, but leaves out the consideration (and optimization) of transmission distance, a key parameter for wireless networks. To offer a comprehensive view of network throughput of decentralized overlaid networks, we study a new metric in this paper: single hop transport throughput (STT), which quantifies the total number of one-hop *reliable* transmissions in a unit area, weighted by corresponding transmission rates and distances. STT inherits the merits of both traditional transport capacity and transmission capacity, and incorporates transmission distance and outage probability into a uniform framework. STT deserves thorough investigation in the secondary network in that single hop transmissions may be preferred due to its inferior role in the spectrum access; it will also serve as a basis for extension to the multi-hop case. Note that our metric is similar in spirit to the *random access transport capacity* (RTC) recently proposed in [12]. Their difference is that in RTC the transmission distance is pre-determined while in STT it is dynamic across the network. On the one hand, we would like to determine the achievable network throughput when links are activated by specific network protocols (such as nearest neighbor routing). On the other hand, it is desired to explore the limiting performance of the network with transmission distance optimized.

In this work we study STT of the *secondary* network in a decentralized setting, subject to outage constraints for both the

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[1] defined as the product of the spatial density and the actual outage probability

primary and secondary network. We incorporate channel randomness and interference, two essential practical factors, into the performance analysis of CR networks, and quantitatively characterize the relationship and tradeoff among important network parameters. Our contributions are summarized below:

- We derive the limit of STT, single hop transport capacity (STC) of the secondary network, as well as a practical upper bound. The former reveals that there is no performance loss for the STC of the secondary network in terms of scaling law despite its interior role; while the latter makes the connection between transport capacity and transmission capacity.
- We further study the STT of the secondary network under the assumption that receivers are *randomly* distributed according to a Poisson Point Process (P.P.P) [13]. In such a setting achievable throughput may vary with the way how secondary transmitters select their corresponding receivers. Three important models are then considered: optimal receivers (OR), random receivers (RR) and nearest neighbors (NN). In the OR model each secondary transmitter chooses an optimal receiver so that the STT is maximized; in the RR model each secondary transmitter randomly chooses its receiver within a transmission radius; and each secondary transmitter chooses the receiver closest to it in the last model. It is shown that they share some similar properties.
- We further explore an extension of STT, broadcast transport throughput (BTT), replacing each unicast link with a local group broadcast. This metric was studied in [14] in a fading but interference-free ad hoc network. Instead, we examine a heterogeneous and interference-limited network in this work. The non-triviality of studying BTT lies in the fact that the number of connected receivers [2] and their corresponding transmission distances are all random. An interesting observation from our study is that the BTT of the secondary network admits a similar expression with its STT in the NN model. This actually makes sense since a transmitter's closest receiver is more likely connected to it than other receivers in broadcast transmissions. Additionally some properties of the average node degree of the secondary network are also revealed.

The remainder of this paper is organized as follows. The system model is given in Section II, followed by discussions on the feasible density region and the performance limits of the secondary STT in Section III. Then the STT and BTT of the secondary network are studied in Section IV and V, respectively. Finally the conclusion and future directions are provided in Section VI.

II. System Model

We assume the primary and secondary transmitters are distributed in the same two-dimensional plane, and their positions are modeled as two stationary Poisson Point Processes; the former is denoted by $\Pi_o^t = \{X_o(i)\} \subset \mathcal{R}^2$ with density λ_{ot} and the latter by $\Pi_s^t = \{X_s(i)\} \subset \mathcal{R}^2$ with density λ_{st}. Primary transmitter i is paired with a receiver located at $Y_o(i)$. The locations of potential secondary receivers $Y_s(i)$ are randomly distributed according to a P.P.P Π_s^r with density λ_{sr}. According to the supposition theorem in [13] the density of secondary nodes (including both transmitters and receivers) is $\lambda_{st} + \lambda_{sr}$. The random counting measure on these point processes is denoted as $\Pi(A) = \#(\Pi \bigcap A)$, where $\Pi = \Pi_o^t, \Pi_s^t, \Pi_s^r$ and A is any Borel subset of $\mathcal{R}^2$. No cooperation between the primary and the secondary network is allowed and the common assumptions about their intra-network communications are given below:

- All the primary (secondary) transmitters use the same transmission power P_o (P_s) and their power ratio is denoted by $\theta = \frac{P_o}{P_s}$. Concurrent primary and secondary transmissions are simply treated as interference. Thermal noise is assumed negligible in this interference-limited scenario.
- For both networks large-scale path loss and small-scale Rayleigh fading are considered. Particularly the channel power gain for a communication link of length r is given by $g(r) = r^{-\alpha}u$, where $\alpha > 2$ is the path loss exponent, and u is exponentially distributed with *unit* mean. The Signal to Interference Ratio (SIR) at a primary receiver y_o, r_o-distance away from its transmitter x_o, is given by
$$SIR_o(r_o) = \frac{P_o g(r_o)}{I_o + I_{so}}, \tag{1}$$
where $I_o = \sum_{X_o(k)\in\Pi_o^t\setminus\{x_o\}} P_o g(||X_o(k) - y_o||)$ ($I_{so} = \sum_{X_s(k)\in\Pi_s^t} P_s g(||X_s(k) - y_o||)$) is the sum of interference power from concurrent primary (secondary) transmitters and $||\cdot||$ is Euclidean norm. The SIR at a secondary receiver, SIR_s, is defined similarly as
$$SIR_s(r_s) = P_s g(r_s)/(I_s + I_{os}), \tag{2}$$
where I_s (I_{os}) is the sum of interference power from concurrent secondary (primary) transmitters to a secondary receiver, r_s-distance away from its corresponding transmitter.
- The primary (secondary) transmission is successful if the SIR_o (SIR_s) is no less than a threshold T_o (T_s), assumed fixed in our study. The transmission rate is a deterministic function of this threshold $R_o = f(T_o)$ ($R_s = f(T_s)$).
- There are outage constraints imposed on primary and secondary transmission links. For the primary network, the constraint is given by:
$$\Pr(SIR_o(l_o) < T_o) \le \epsilon_o, \tag{3}$$
where l_o is a typical length of a primary link and considered constant in our analysis, whose exact value stems from specification of the primary network and is immaterial for our analysis. $\epsilon_o(< 1)$ is a predetermined

[2] A receiver is connected to a transmitter if its received SIR is larger than a pre-determined threshold.

small number. Similarly for secondary links, it is required that for any active link

$$\Pr(SIR_s < T_s) \leq \epsilon_s. \quad (4)$$

As we mentioned, the link length in the secondary network is variable, and is a key parameter in our study.

- For convenience of analysis, it is often assumed that a primary (secondary) receiver or transmitter is located at the origin, which does not change the statistics of a homogenous P.P.P. according to the Slivnyak's theory [15].

Remark 1: The settings of the primary network such as the density and outage probability are assumed fixed without considering the accommodation of the secondary network. In contrast, parameters in the secondary network can be tuned to improve its own performance, provided the primary transmission is not influenced.

In this *decentralized* framework, the metric transmission capacity has received increasing interest recently, defined as the maximum density of successful transmissions subject to an outage constraint. In our study, we will mainly consider the scenario where the transmitter density of a network is given. To avoid possible confusion, we will call relevant metrics throughputs. In particular, for a Poisson network with transmitter density λ, a typical link length r, a pre-determined SIR threshold T and transmission rate R, the transmission throughput is defined as:

$$\bar{C}(\lambda) = R\lambda(1 - \delta(\lambda, r)), \quad (5)$$

where the outage probability $\delta(\lambda, r) = \Pr(SIR(r) < T)$. The transmission capacity with outage constraint ϵ [10]–[12] is defined as $R\lambda_\epsilon(1-\epsilon)$, where λ_ϵ is the maximal density satisfying the outage constraint, i.e., $\delta(\lambda_\epsilon, r) = \epsilon$. It can be shown that when ϵ is small, the transmission capacity coincides with $\max_\lambda(\bar{C}(\lambda))$.

In the study of transmission capacity and its variants, the transmission distance is ignored. In this work, we explore a metric called single-hop transport throughput defined as follows.

Definition 1: The *single-hop transport throughput* (STT) of a Poisson network with transmitter density λ, a pre-determined SIR threshold T and transmission rate R, is defined as:

$$C(\lambda) = R\lambda E_r\left[r(1 - \delta(\lambda, r))\right] \quad (6)$$

where the outage probability $\delta(\lambda, r) = \Pr(SIR(r) < T)$. In STT the transmission distance is explicitly considered, and also allowed to change (over the space) according to some distribution. The *single-hop transport capacity* (STC)[3] with outage constraint ϵ is defined as the maximum of the single-hop transport throughput with respect to λ and the distribution of r, such that $\delta(\lambda, r) \leq \epsilon$. For a secondary network, λ and r should be chosen such that the outage constraint of the primary network is met as well (see Section III for further discussion).

[3]Some preliminary results on the STC of CR networks are provided in our recent work [16] without considering the secondary outage constraint.

In a broadcast scenario, it is natural to consider the total transmission distance for an arbitrary transmitter X as the sum distance of all successful transmissions. Denote by $\Xi = \{d_i\}$ the point process composed of all the receivers connected to X (i.e., their received SIRs are no smaller than the threshold T), where d_i is the Euclidean distance between the connected receiver i and X. Then as an extension of transport throughput above, the broadcast transport throughput of a Poisson network is defined as follows.

Definition 2: The *broadcast transport throughput* of a Poisson network with transmitter density λ, a pre-determined SIR threshold T and transmission rate R, is defined as:

$$C_b(\lambda) = R\lambda E_\Xi \sum_{i \in \Xi} d_i. \quad (7)$$

We can also define the broadcast transport capacity following the above discussion.

III. Density region and Upper bounds of STT

The purpose of this paper is to explore the single-hop transport throughput of the secondary network given the outage constraints on both the legacy network and itself. We begin by setting the boundaries for this metric. First we determine the feasible density region for the transmitter densities of the two overlaid networks. Then we go on to determine the single-hop transport capacity. Finally we provide a practical upper bound for the single-hop transport throughput, making connections with the transmission throughput.

A. Density region

Due to the primary and secondary outage constraints, the primary and secondary *transmitter* densities are upper bounded. The density region is defined as $\{(\lambda_{ot}, \lambda_{st})\}$, a set of feasible primary and secondary transmitter densities. Since the secondary network should remain transparent to the primary network, it can be easily derived from Lemma 3.1 in [17] that

$$\lambda_{ot} \leq \lambda_{ot,m} = \frac{-\ln(1-\epsilon_o)}{K_\alpha T_o^{2/\alpha} l_o^2}, \quad (8)$$

where $K_\alpha = \frac{2\pi^2}{\alpha \sin(2\pi/\alpha)}$, regardless of the existence of the secondary network. It is the secondary network that is responsible for satisfying the primary outage constraint, as well as its own. To satisfy the primary outage constraint, the secondary transmitter density should be bounded above by:

$$\lambda_{st} \leq \lambda_{st,m} = \theta^{2/\alpha} \Delta\lambda_{ot}, \quad (9)$$

where $\theta = P_o/P_s$ and $\Delta\lambda_{ot} = (\lambda_{ot,m} - \lambda_{ot})$. The derivation is given in Appendix A.

This inequality intuitively reflects the inherent tradeoff of the overlaid networks. If the primary network is sparse, i.e., $\lambda_{ot} \ll \lambda_{ot,m}$, there is ample "white space" left to secondary users. On the contrary, if the primary network is heavily loaded (i.e., λ_{ot} is close to $\lambda_{ot,m}$), a secondary network can hardly "survive" without harming the performance of the primary network. However, an important observation is that the maximum secondary transmitter density is proportional to the

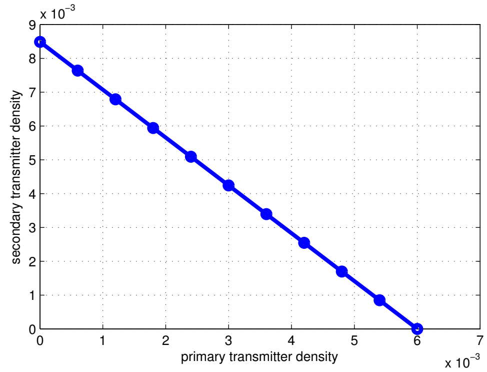

Fig. 1: the density region $(\lambda_{ot}, \lambda_{st})$: the triangle under the solid line

power ratio θ, which implies that the secondary network may still operate with low transmission power given limited "white space". In such circumstances, some low power physical layer techniques such as spread spectrum may help improve the performance of secondary networks.

As mentioned in Section II, we do not impose a common transmission range in the secondary network, and the secondary outage constraint can be satisfied by choosing appropriate transmission distance. Thus no further constraint on the secondary transmitter density is needed. One example of the density region is given in Fig. 1, where $\epsilon_o = \epsilon_s = 0.05, \alpha = 4, T_o = 3, T_s = 1, l_o = 1$.

In the following discussion, we assume all densities of interest $(\lambda_{ot}, \lambda_{st})$ are within the density region.

B. Single-Hop Transport Capacity

To explore the limiting performance of the secondary network, we assume the flexibility to choose an arbitrary distribution for the transmission distance r. It turns out that the optimal distribution is a degenerate one.

Lemma 1: Given a secondary transmitter density λ_{st}, the secondary single-hop transport throughput $C_s(\lambda_{st})$ achieves the maximum by choosing a common transmission distance r_s for each link:

$$r_s = L(\lambda_{st}) \triangleq \sqrt{\frac{\ln(1-\epsilon_s)}{-B(\lambda_{st})}} \quad \text{if} \quad \epsilon_s \leq 1 - e^{-1/2}, \tag{10}$$

and $r_s = L^*(\lambda_{st}) \triangleq 1/\sqrt{2B(\lambda_{st})}$, otherwise, where

$$B(\lambda) \triangleq K_\alpha T_s^{2/\alpha}(\lambda_{ot}\theta^{2/\alpha} + \lambda).$$

The corresponding maximum single-hop transport throughput $\hat{C}_s(\lambda_{st})$ is given by

$$\hat{C}_s(\lambda_{st}) = \frac{R_s \lambda_{st}}{\sqrt{B(\lambda_{st})}}(1-\epsilon_s)\sqrt{-\ln(1-\epsilon_s)}, \tag{11}$$

if $\epsilon_s \leq 1 - e^{-1/2}$, and $\hat{C}_s(\lambda_{st}) = \frac{R_s \lambda_{st}}{\sqrt{2B(\lambda_{st})}} e^{-1/2}$, otherwise. The sketch of proof is given in Appendix B.

$\hat{C}_s(\lambda_{st})$ obviously increases with λ_{st}. Then we have

Theorem 1: The single hop transport capacity is

$$\hat{C}_s = \hat{C}_s(\lambda_{st,m}), \tag{12}$$

with $\hat{C}_s(\lambda_{st})$ given above.

Remark 2: In practice, it is expected that ϵ_s is smaller than $1 - e^{-1/2} \approx 0.39$. Therefore in the rest of this paper, we focus on this scenario. It is also known from Appendix B that $C_s(\lambda_{st})$ monotonically increases with $r_s \in (0, L(\lambda_{st}))$ when $\epsilon_s < 1 - e^{-1/2}$, and $L(\lambda_{st})$ is the maximum allowable transmission distance satisfying the outage constraint ϵ_s.

Substituting $\lambda_{st,m}$ in Eq. (9) into Eq. (12) we get

$$\hat{C}_s = R_s(\theta\frac{T_o}{T_s})^{1/\alpha}\Delta\lambda_{ot} l_o \sqrt{\frac{\ln(1-\epsilon_s)}{2\ln(1-\epsilon_o)}}(1-\epsilon_s), \tag{13}$$

which reveals some interesting insights:

- $\hat{C}_s$ is proportional to the ratio $\theta\frac{T_o}{T_s}$. One can always improve the capacity by increasing the power ratio θ or decreasing the SIR threshold T_s.
- Our result coincides with the existing scaling law result on capacity in a dense network model where the network size is allowed to grow with the network density in a fixed area. It is shown in [8] that, in a single n-node arbitrary dense network, the sum throughput scales with[4] $\Theta(\sqrt{n})$. In our model, the secondary transmitter density $\lambda_{st,m}$ increases with power ratio θ, according to Eq. (9) (with the primary density fixed; see Remark 1). By Eq. (13) we have

  $$\frac{\hat{C}_s(\lambda_{st,m})}{\sqrt{\lambda_{st,m}}} = R_s(\frac{T_o}{T_s})^{1/\alpha} l_o \sqrt{\Delta\lambda_{ot}\frac{\ln(1-\epsilon_s)}{2\ln(1-\epsilon_o)}}(1-\epsilon_s)$$

  which is a positive constant and clearly indicates that the throughput of the secondary network is boosted as $\sqrt{\lambda_{st,m}}$ (thus there is no performance loss in transport capacity for the secondary network, in terms of scaling law). Note that this result is achieved for any finite $\lambda_{st,m}$, unlike the scaling law study which requires $\lambda_{st,m}$ be large. In addition, the critical constant term is clearly presented in the result above, which is usually unavailable in the scaling law study.

C. A practical upper bound

With additional information in practical scenarios we can obtain a tighter upper bound. For a secondary network with transmitter density λ_{st} and average length of all allowable links $E(r) = l_s$, its STT is upper bounded by

$$\begin{aligned} C_s(\lambda_{st}, l_s) &= R\lambda_{st} E_r(r(1-\delta(\lambda_{st}, r))) \\ &\leq R\lambda_{st} l_s(1-\delta(\lambda_{st}, l_s)) \triangleq \hat{C}_s^p(\lambda_{st}, l_s), \end{aligned}$$

where the inequality is due to Jensen's inequality [18] and the fact that function $y(r) = r(1-\delta(\lambda_{st}, r))$ is concave when $\epsilon_s < 1 - e^{-1/2}$ (c.f. Appendix B).

Proposition 1: Assume $E(r)$ is fixed for a network configuration, a practical upper bound of STT is

$$\hat{C}_s^p = \max_{\lambda_{st}} \hat{C}_s^p(\lambda_{st}) = \hat{C}_s^p(\lambda_{st}^*),$$

where $\lambda_{st}^* = \min(\frac{1}{K_\alpha T_s^{2/\alpha} l_s^2}, \lambda_{st,m})$.

[4] Note that the lower bound in [8] is also obtained in a one-hop setting.

Proof: The conclusion follows the concavity of $\hat{C}_s^p(\lambda_{st})$, when $\lambda_{st} < \frac{2}{K_\alpha T_s^{2/\alpha} l_s^2}$. ■

Note that $\hat{C}_s^p \leq \hat{C}_s$ and the equality holds iff $l_s = L(\lambda_{st})$.

Remark 3: $\hat{C}_s^p(\lambda_{st}, l_s)$ above coincides with transmission throughput defined in Eq. (5) if l_s is considered a constant and omitted in the expression. The intuition behind this point is that we can reasonably interpret the typical link length, assumed in the definition of transmission throughput, as the average length of links in practical scenarios. Therefore, the transmission throughput, scaled by the average link length, could serve as an upper bound for STT.

IV. Transport throughput of secondary network

In this section we investigate the single hop transport throughput of the secondary network defined in Section II, with the secondary receivers modeled as the P.P.P. Π_s^r with density λ_{sr}. The two overlaid networks operate under the primary and secondary outage constraints, i.e., their transmitter densities are within the density region.

Communication links can be formed in different manners, depending on how transmitters select their corresponding receivers, which plays an essential role in determining the STT. To present a comprehensive view of throughput we are interested in the following settings:

1) Optimal receivers (OR): the secondary transmitters choose their corresponding receivers in order to achieve the best performance.
2) Random receivers (RR): the secondary transmitters randomly choose their corresponding receivers within a maximal allowable transmission radius.
3) Nearest neighbors (NN): namely, each secondary transmitter communicates with its nearest neighbor, which is a conservative but easy-to-implement approach.

STC of the secondary network discussed in the last section serves as a benchmark for our study. We'll first derive the throughput in these settings and then discuss these results in the last subsection, together with some numerical results.

A. Optimal Receivers

The optimal receiver case demonstrates the maximal throughput achieved in this setting. According to Lemma 1, given the secondary transmitter density λ_{st}, the throughput is increased over transmission distance $r_s \in (0, L]$, where L is [5]the longest allowable transmission distance given in Eq. (10). Therefore, the throughput is maximized if a secondary transmitter communicates with the furthest receiver within the range of L. In particular, for a secondary transmitter X_s, we order the potential secondary receivers $\{Y_s(i)\}$ according to their Euclidean distance $r_i \triangleq ||Y_s(i) - X_s||$ to X_s such that $r_i \leq r_j,\ \forall i < j$. The receiver chosen by transmitter X_s is given by

$$k^* = \begin{cases} \max\{i : L - r_i \geq 0\} & r_1 \leq L, \\ NULL & otherwise. \end{cases} \tag{14}$$

[5]For ease of notation, we drop the argument λ_{st} in the expression.

Theorem 2: Given a secondary transmitter density λ_{st} and a secondary receiver density λ_{sr}, the largest achievable single-hop transport throughput is given by:

$$C_s^o(\lambda_{st}, \lambda_{sr}) = \begin{cases} A_1\left[\frac{Le^{aL^2}}{2a} - \frac{\sqrt{\pi}erfi(\sqrt{a}L)}{4a^{3/2}}\right] & a > 0 \\ A_1 L^3/3 & a = 0 \\ A_1\left[\frac{Le^{aL^2}}{2a} + \frac{\sqrt{\pi}erf(\sqrt{-a}L)}{4(-a)^{3/2}}\right] & a < 0 \end{cases}$$

where $A_1 = 2\lambda_{st}R_s\lambda_{sr}\pi e^{-\lambda_{sr}\pi L^2}$, $a = \lambda_{sr}\pi - B(\lambda_{st})$, $B(\lambda_{st})$ is given in Lemma 1, $erfi(x) = 2/\sqrt{\pi}\int_0^x e^{t^2}dt$ is the imaginary error function and $erf(x) = 2/\sqrt{\pi}\int_0^x e^{-t^2}dt$ is the error function.

Proof: Given that there is at least one receiver within the circle with radius L, the event $\{r_{k^*} > l\}$, where $l \leq L$, is equivalent to the event {there is at least one receiver in the annulus $C(l, L)$}. Therefore,

$$\Pr(r_{k^*} > l | r_1 \leq L) = \frac{1 - e^{-\lambda_{sr}\pi(L^2 - l^2)}}{1 - e^{-\lambda_{sr}\pi L^2}},$$

which leads to the conditional pdf of r_k,

$$f_{r_{k^*}|r_1 \leq L}(l) = \frac{e^{-\lambda_{sr}\pi L^2} 2\lambda_{sr}\pi l}{1 - e^{-\lambda_{sr}\pi L^2}} e^{\lambda_{sr}\pi l^2}.$$

It follows that

$$\begin{aligned} C_s^o &= \lambda_{st}R_s E_{r_{k^*}}(r_{k^*}(1 - \delta_s(r_{k^*}))) \qquad (15) \\ &= \lambda_{st}R_s \Pr(r_1 \leq L) E_{r_{k^*}|r_1 \leq L}(r_{k^*}(1 - \delta_s(r_{k^*}))) \\ &= \lambda_{st}R_s(1 - e^{-\lambda_{sr}\pi L^2}) \int_0^L l e^{-B(\lambda_{st})l^2} f_{r_{k^*}|r_1 \leq L}(l) dl, \end{aligned}$$

where the outage δ_s is given in Eq. (31). Eq. (15) is then derived after some calculation. ■

B. Random Receivers

The RR model demonstrates the average STT, where each secondary transmitter randomly choose a receiver in a range at most L distance away.

Theorem 3: Given a secondary transmitter density λ_{st} and a secondary receiver density λ_{sr}, the single hop transport throughput in the RR model is

$$C_s^r(\lambda_{st}, \lambda_{sr}) = A_2 \frac{R_s\lambda_{st}}{\sqrt{B(\lambda_{st})}} (1 - (1 - \epsilon_s)^{\lambda_{sr}\pi/B(\lambda_{st})}) \tag{16}$$

where $A_2 = \frac{\sqrt{\pi}erf\left(\sqrt{\ln\frac{1}{1-\epsilon_s}}\right) - 2(1-\epsilon_s)\sqrt{\ln\frac{1}{1-\epsilon_s}}}{-2\ln(1-\epsilon_s)}$ and $B(\lambda_{st})$ is given in Lemma 1.

Proof: Denote by E_k the event that there are k receivers in the circle centered at a secondary transmitter with radius L. Given the number of receivers in the circle, all the receivers are uniformly distributed with probability density function (pdf) $f(r) = \frac{2r}{L^2}$ according to the Poisson property. Then we have

$$\begin{aligned} & E\left[r(1 - \delta_s(r))\right] = E[E(r(1 - \delta_s(r))|E_k)] \\ = & \sum_{k=1}^{\infty} \frac{(\lambda_{sr}\pi L^2)^k e^{-\lambda_{sr}\pi L^2}}{k!} \int_0^L r\exp(-B(\lambda_{st})r^2)\frac{2r}{L^2}dr \\ = & (1 - \frac{1}{e^{\lambda_{sr}\pi L^2}})\left[\frac{\sqrt{\pi}erf\left(L\sqrt{B(\lambda_{st})}\right)}{2L^2 B(\lambda_{st})^{3/2}} - \frac{e^{-B(\lambda_{st})L^2}}{B(\lambda_{st})L}\right]. \end{aligned}$$

The theorem follows after substituting L in Eq. (10) to the equation above.

■

C. Nearest neighbors

Nearest neighbors model is one of the popular models in analysis of wireless networks, especially in the study of network connectivity ([19] and reference there). In this subsection we consider secondary transmitters communicate with their nearest receivers satisfying the outage constraints. For the ordered receivers introduced in Sec. IV-A, the receiver chosen by X_s in this model is $Y_s(1)$ if $r_1 \leq L$; otherwise no receiver is chosen since all the receivers are outside of the allowable transmission range.

Theorem 4: Given a secondary transmitter density λ_{st} and a secondary receiver density λ_{sr}, the STT in the NN model is given by:

$$C_s^n(\lambda_{st}, \lambda_{sr}) = A_1 \left(-\frac{Le^{-B(\lambda_{st})L^2}}{2b} + \frac{\sqrt{\pi} erf(\sqrt{b}L)}{4e^{-\lambda_{sr}\pi L^2} b^{3/2}} \right), \quad (17)$$

where $b = B(\lambda_{st}) + \lambda_{sr}\pi$, $B(\lambda_{st})$ is given in Lemma 1, and A_1 is given in Theorem 2.

Proof: Given that there is at least one receiver within the circle with radius L, the event $\{r_1 > l\}$ with $l \leq L$ is equivalent to the event {there is no receiver in the circle with radius l centered at X_s}. Thus,

$$\Pr(r_1 > l) = \frac{e^{-\lambda_{sr}\pi l^2}}{1 - e^{-\lambda_{sr}\pi L^2}},$$

which leads to the conditional pdf of r_1,

$$f_{r_1}(l) = \frac{2\lambda_{sr}\pi l e^{-\lambda_{sr}\pi l^2}}{1 - e^{-\lambda_{sr}\pi L^2}}.$$

Therefore,

$$\begin{aligned} C_s^n &= \lambda_{st} R_s E_{r_1}(r_1(1 - \delta_s(r_1))) \quad (18) \\ &= \lambda_{st} R_s (1 - e^{-\lambda_{sr}\pi L^2}) \int_0^L l e^{-B(\lambda_{st})l^2} f_{r_1}(l) dl. \end{aligned}$$

The theorem then follows after some calculation. ■

We are also interested in the case without the secondary constraint, which will allow us to draw connection with the broadcast transport throughput discussed in Section V.

Corollary 1: Without the secondary outage constraint, the throughput in the NN model is given by:

$$\tilde{C}_s^n(\lambda_{st}, \lambda_{sr}) = \frac{\lambda_{st} R_s \lambda_{sr} \pi^{3/2}}{2(B(\lambda_{st}) + \lambda_{sr}\pi)^{3/2}}, \quad (19)$$

where $B(\lambda_{st})$ is given in Lemma 1.

D. Discussions and Numerical results

Define[6] $k = \frac{\lambda_{sr}\pi}{B(\lambda_{st})}$ and substitute it into Eq. (15) ($a > 0$ case), (16) and (17), we get

$$\begin{aligned} C_s^o(\lambda_{st}, k) &= Q_o(k) \frac{R_s \lambda_{st}}{\sqrt{B(\lambda_{st})}} (k > 1), \quad (20) \\ C_s^r(\lambda_{st}, k) &= Q_r(k) \frac{R_s \lambda_{st}}{\sqrt{B(\lambda_{st})}}, \quad (21) \\ C_s^n(\lambda_{st}, k) &= Q_n(k) \frac{R_s \lambda_{st}}{\sqrt{B(\lambda_{st})}}, \quad (22) \end{aligned}$$

where
$Q_o(k) = \frac{k\sqrt{-\ln(1-\epsilon_s)}(1-\epsilon_s)}{k-1} - \frac{k\sqrt{\pi} erfi(\sqrt{(1-k)\ln(1-\epsilon_s)})}{2(k-1)^{3/2}(1-\epsilon_s)^{-k}}$,
$Q_r(k) = A_2(1 - (1-\epsilon)^k)$, and
$Q_n(k) = \frac{k\sqrt{-\ln(1-\epsilon_s)}(1-\epsilon_s)^{1+k}}{-k-1} + \frac{k\sqrt{\pi} erf(\sqrt{-(1+k)\ln(1-\epsilon_s)})}{2(k+1)^{3/2}}$.

All the above results, as well as the maximal throughput derived in Eq (11), take a similar form: given k, they increase with the transmitter density λ_{st} and scale as $\theta(\sqrt{\lambda_{st}})$. It should be noted that, to achieve this scaling the receiver density λ_{sr} should also increase with the transmitter density since we keep k fixed. In the following, we continue to explore how the transport throughput scales with λ_{st} or λ_{sr} when the other is fixed.

1) Throughput versus secondary receiver density : We fix λ_{st} but adjust k, or equivalently, the secondary receiver density, λ_{st}. It's easy to check that $C_s^o(\lambda_{st}, k)$ and $C_s^r(\lambda_{st}, k)$ increase with k, and

$$\lim_{k \to \infty} C_s^o(\lambda_{st}, k) = \hat{C}_s(\lambda_{st}), \quad (23)$$

$$\lim_{k \to \infty} C_s^r(\lambda_{st}, k) = A_2 \frac{R_s \lambda_{st}}{\sqrt{B(\lambda_{st})}}, \quad (24)$$

where $\hat{C}_s(\lambda_{st})$ is given in Eq. (11). These facts show that 1) the maximum throughput is achieved when the receiver density in the OR model goes to infinity, which is intuitively right since in this case we can always find a secondary receiver at the optimal distance; 2) the RR model suffers a performance loss by a constant factor. The NN model behaves differently from the OR and RR model. Since $\lim_{k\to\infty} C_s^n(\lambda_{st}, k) = 0$ there exists an optimal k such that the throughput is maximized. From its first derivative with respective to k

$$\begin{aligned} \frac{dC_s^n(\lambda_{st}, k)}{dk} &\approx \sqrt{-\ln(1-\epsilon_s)} \frac{(1-(1-\epsilon_s)^{1+k})k}{1+k} \\ &\approx \sqrt{-\ln(1-\epsilon_s)} \frac{\epsilon_s^2}{2}(-k^2 + \frac{2}{\epsilon_s}k), \end{aligned}$$

where the approximation is made due to the fact that $erf(\sqrt{-(1+k)\ln(1-\epsilon_s)}) \approx 2/\sqrt{\pi}\sqrt{-(1+k)\ln(1-\epsilon_s)}$ and $(1-\epsilon_s)^{1+k} \approx 1 - \epsilon_s(1+k) + \epsilon_s^2 k(1+k)/2$ for small ϵ_s, the optimal $k \approx \frac{2}{\epsilon_s}$.

These throughput are numerically depicted in Fig. 2 and Fig. 3, where $T_o = 3, T_s = 2, l_o = 1, \alpha = 4, \theta = 2, \lambda_{st} = \lambda_{st,m}, \epsilon_o = 0.05$. We can see that in the low receiver density

[6] k is actually the average node degree if broadcast transmission is applied, as further discussed in the next section.

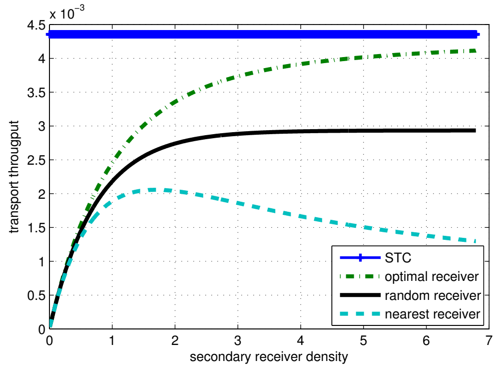

Fig. 2: secondary STT VS secondary receiver density $\epsilon_s = 0.05$

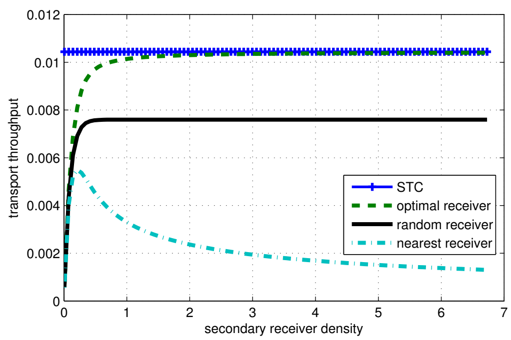

Fig. 3: secondary STT VS secondary receiver density $\epsilon_s = 0.35$

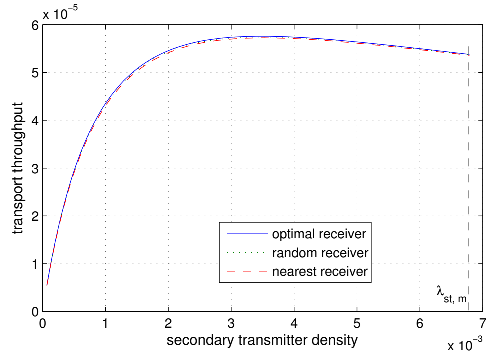

Fig. 4: secondary STT VS secondary transmitter density: low receiver density $\lambda_{sr} = 10\lambda_{st,m}$

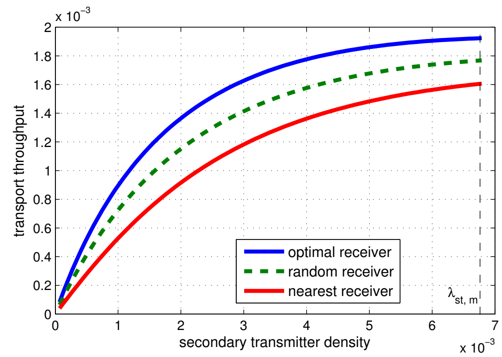

Fig. 5: secondary STT VS secondary transmitter density: high receiver density $\lambda_{sr} = 100\lambda_{st,m}$

region, three models perform almost the same, but with the receiver density increased, the NN model performs the worst among the three models. The throughput in both OR and RR model increases with λ_{sr}; while there exists an optimal receiver density such that the throughput in the NN model is maximized. These observations coincide with our discussion above and admit an intuitive interpretation: with a low receiver density the receivers chosen by a transmitter in these models often coincide so that their performance is indiscernible; with more receivers the transmitters have more chance to select the receivers at the optimal positions in the OR and RR model; however, the distance between a transmitter and its closest receiver becomes smaller so that the throughput is decreased in the NN model. Another interesting observation is that with low outage probability imposed on the secondary network (Fig. 2), the throughput in the OR model approaches the upper bound rather slowly with the increase of the secondary receiver density; its converging speed is substantially faster with a high outage probability (Fig. 3). The reason behind this phenomenon is that with a high outage probability the optimal transmission distance is much larger (c.f. (10)), which can be easily satisfied even when the receiver density is low. It is also found that the practical upper bounds (Section III-C) for these models are quite tight, barely distinguishable from the STT curves and thus omitted in the figures.

2) Throughput versus secondary transmitter density : We then fix λ_{sr} and change λ_{st} in its allowable range. It is easy to check from Eq. (15, 16, 17) that there exists a λ_{st}^* for each model such that its throughput is maximized. The optimal secondary transmitter density is depicted in Fig. 4 and 5, where we adopt the same set of parameters as in Fig. 2 and Fig. 3, except $\epsilon_o = \epsilon_s = 0.05$. In the low receiver density region (Fig. 4) the optimal secondary transmitter density is within the range $(0, \lambda_{st,m}]$, while in the high receiver density region (Fig. 5) it is $\lambda_{st,m}$ since $\lambda_{st}^* > \lambda_{st,m}$ in this case.

3) Throughput versus primary transmitter density: Fig. 6 shows the influence of the primary transmitter density on these three throughputs, where we adopt the same set of parameters as above and $\lambda_{st} = \lambda_{st,m}$, given λ_{ot}. Note that the maximal secondary transmitter density is *inversely* proportional to the primary transmitter density (c.f. Eq. (9)). Therefore, the secondary throughput decreases with λ_{ot}, which is clearly presented in Fig. 6.

To sum up, these three models share some common properties. In particular, when the secondary receiver density is fixed, there exists an optimal secondary transmitter density for each model such that the corresponding throughput is maximized; and all three throughputs decrease with the primary transmitter density. The NN model performs slightly differently from the other two: when the secondary transmitter density is fixed, the throughput in the NN model eventually decreases with the secondary receiver density while the other two always grow

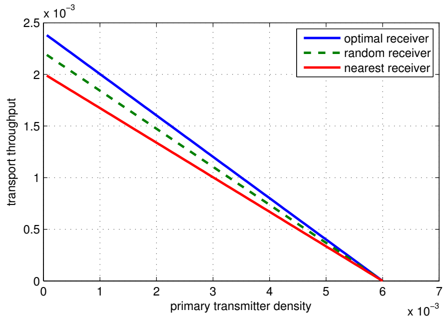

Fig. 6: secondary STT versus primary transmitter density

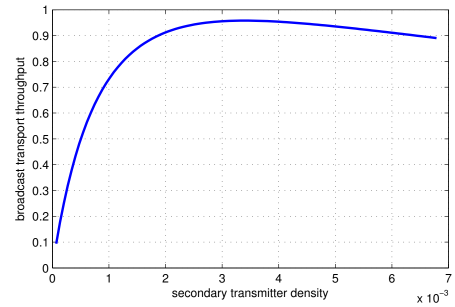

Fig. 7: Broadcast transport throughput VS secondary transmitter density

with the secondary receiver density.

V. Broadcast transport throughput

We continue our study by investigating BTT defined in Eq. (7), an extension of STT, with the assumption that the interference components at receivers are independent[7]. Suppose an arbitrary secondary transmitter at the origin, the point process Ξ (of the ordered receiver distances) has the following property:

Proposition 2: Ξ is a one dimensional inhomogeneous Poisson process with density $\delta(x) = 2\lambda_{sr}\pi x \exp(-B(\lambda_{st})x^2)$.

Proof: By definition $\Pi_s^r \triangleq \{Y_s(i)\}$ is Poisson so is the process $\Phi = \{r_i\}$ by the Mapping Theorem [13], where $r_i = |Y_s(i) - o|$, the Euclidean distance between node $Y_s(i)$ and origin. Then each point r in Φ is associated with an SIR, random variable $s_r = \frac{r^{-\alpha}uP_s}{I_s+I_{os}}$, independent for different points in Φ. Therefore, according to the Marking Theorem in [13] the set $\Phi^* = \{(r, s_r); r \in \Phi, s_r \in \mathcal{R}\}$ is Poisson on the product space $\mathcal{R}^2$. Therefore $\Xi = \{(r, s_r); r \in \Phi, s_r \in (T_s, \infty)\}$ is Poisson.

The density is derived as follows. We first consider a circle $c(o, a)$ of radius a centered at the origin and condition on finite number of nodes in this disk. And then we allow the radius $a \to \infty$. In particular, denote by $\mathcal{N}_a$ the number of receivers in $c(o, a)$ connected to the origin, then we have

$$\begin{aligned} E(\mathcal{N}_a) &= E\left[\sum_{i=1}^{N_a} I_i\right] = E\left[E\left(\sum_{i=1}^{k} I_i | N_a = k\right)\right] \quad (25) \\ &= \sum_{k=1}^{\infty} \frac{(\lambda_{sr}\pi a^2)^k e^{-\lambda_{sr}\pi a^2}}{k!} E\left(\sum_{i=1}^{k} I_i | N_a = k\right) \end{aligned}$$

where $N_a = \Pi_s^r(c(o, a))$ is the number of receivers in $c(o, a)$ and I_i is the indicator function such that $I_i = 1$ if receiver i is connected to the origin.

The probability density function that a secondary receiver is at a distance $r \le a$ from the origin is $f_r(r) = \frac{2r}{a^2}$. Therefore,

$$E\left(\sum_{i=1}^{k} I_i | N_a = k\right) = kE(I_1) = k\int_0^a (1 - \delta_s) f_r(r) dr. \quad (26)$$

where δ_s is given in Eq. (31).

[7]This assumption is partly justified by [20], and greatly simplifies the analysis.

Taking Eq. (26) into Eq. (25) and noticing the fact that $\sum_{k=1}^{\infty} \frac{(\lambda_{sr}\pi a^2)^{k-1} e^{-\lambda_{sr}\pi a^2}}{(k-1)!} = 1$, we get

$$\begin{aligned} E(\mathcal{N}_a) &= \lambda_{sr}\pi a^2 \int_0^a (1 - \delta_s) f_r(r) dr \quad (27) \\ &= \lambda_{sr}\pi \frac{1 - \exp(-B(\lambda_{st})a^2)}{B(\lambda_{st})}. \end{aligned}$$

Due to the Mapping Theorem and Poisson property of Ξ, the density of Ξ is given by $\delta(a) = \frac{dE(\mathcal{N}_a)}{da}$ after some calculation. ■

Now we are ready to calculate the broadcast transport throughput of the secondary network C_b.

Theorem 5:

$$C_b = \frac{R_s \pi^{3/2} \lambda_{st} \lambda_{sr}}{2B(\lambda_{st})^{3/2}}, \quad (28)$$

where $B(\lambda_{st})$ is given in Lemma 1.

Proof: According to Campbell's theorem in [13],

$$C_b = \lambda_{st} R_s E\left[\sum_{d_i \in \Xi} d_i\right] = \lambda_{st} R_s \int_0^{\infty} x\delta(x) dx$$

The conclusion follows after some calculation. ■

Some interesting observations are listed as follows:

- The expression of Eq. (28) above is very similar to Eq. (19) in the NN case. This actually makes sense due to the fact that the nearest receivers are more likely connected to their corresponding transmitters when broadcast communications is considered. It is not difficult to show that there exists an optimal secondary transmitter density $\min(\lambda_{st,m}, 2\lambda_{ot}\theta^{2/\alpha})$ such that C_b is maximized (see Fig. 7). However C_b always increases with receiver density λ_{sr}, while the throughput in NN model eventually decreases with it.
- The average node degree can be evaluated as $E(\mathcal{N}) = \lim_{a\to\infty} E(\mathcal{N}_a) = \frac{\lambda_{sr}\pi}{B(\lambda_{st})}$, which is a vital property of network connectivity. In particular if the density ratio $\beta = \frac{\lambda_{sr}}{\lambda_{st}}$ is fixed,

$$E(\mathcal{N}) = \frac{\beta\pi}{K_\alpha T_s^{2/\alpha}(\frac{\lambda_{ot}\theta^{2/\alpha}}{\lambda_{st}} + 1)} \quad (29)$$

increases with λ_{st} and $E(\mathcal{N}) \leq \frac{\beta\pi}{K_\alpha T_s^{2/\alpha}(\frac{\lambda_{ot}\theta^{2/\alpha}}{\lambda_{st,m}}+1)} = \frac{\pi\beta\Delta\lambda_{ot}}{K_\alpha T_s^{2/\alpha}\lambda_{ot,m}}$ (c.f. (9)), which only depends on β.

VI. Conclusions and Future Work

We have made some quantitative study on the throughput of the secondary network in spectrum sharing systems subject to the outage constraints for both the legacy network and the secondary network, aiming at revealing the relationship and tradeoff among key system parameters and providing insights into system design and optimization. As part of our future work, we plan to explore multi-hop transport throughput in secondary networks; some pioneer work in this direction can be found in [12], [17], [21].

Appendix A
Derivation of Outage Probability

Overlaid with the secondary network, the probability of a successful primary transmission, $\Pr(SNR_o(l_o) \geq T_o)$, is given by:

$$\begin{aligned}\Pr\left(\left(\frac{P_o l_o^{-\alpha}u}{I_o+I_{so}}\right) \geq T_o\right) &= \Pr\left(u \geq \frac{T_o(I_o+I_{so})}{P_o l_o^{-\alpha}}\right) \\ &= \int_0^\infty \exp\left(-\frac{T_o}{P_o l_o^{-\alpha}}i\right) f_I(i)di \\ &= \psi_I\left(\frac{T_o l_o^\alpha}{P_o}\right),\end{aligned}$$

where $I = I_o + I_{so}$, with pdf f_I, and $\psi_I(i)$ is the Laplace transform of f_I. Due to the independence of I_o and I_{so},

$$\begin{aligned}\psi_I\left(\frac{T_o l_o^\alpha}{P_o}\right) &= \psi_{I_o}(\frac{T_o l_o^\alpha}{P_o})\psi_{I_{so}}(\frac{T_o l_o^\alpha}{P_o}) \qquad (30)\\ &= \exp\left[-K_\alpha T_o^{2/\alpha} l_o^2\left(\lambda_{st}\frac{1}{\theta^{2/\alpha}}+\lambda_{ot}\right)\right],\end{aligned}$$

where $\psi_{I_o}(i) = \exp\left[-K_\alpha\lambda_{ot}(P_o i)^{2/\alpha}\right]$, $\psi_{I_{so}}(i) = \exp\left[-K_\alpha\lambda_{st}(P_s i)^{2/\alpha}\right]$ (K_α is defined after (8)). Eq. (9) is obtained by letting $\Pr(SNR_o(l_o) \geq T_o) \geq 1-\epsilon_o$.

Following the same line above, the outage probability of the secondary network can be calculated as follows:

$$\begin{aligned}\delta_s(r,\lambda_{st}) &= \Pr\left(\frac{P_s u r^{-\alpha}}{I_s+I_{os}} \leq T_s\right) \qquad (31)\\ &= 1-\exp\left[-K_\alpha T_s^{2/\alpha} r^2\left(\lambda_{ot}\theta^{2/\alpha}+\lambda_{st}\right)\right],\end{aligned}$$

where r is the transmission distance between a secondary transmitter and its corresponding receiver.

Appendix B
Sketch of proof of Lemma 1

From the definition of transport throughput in Eq. (6),

$$C_s(\lambda_{st}) \leq R_s\lambda_{st}\max_{r_s} y(r_s) \triangleq \hat{C}_s(\lambda_{st}), \qquad (32)$$

subject to the outage constraint $\delta_s(r_s,\lambda_{st}) \leq \epsilon_s$, where $y(r) = r(1-\delta_s(r,\lambda_{st}))$ and $\delta_s(r_s,\lambda_{st}) = 1-e^{-B(\lambda_{st})r_s^2}$ is given in (31).

Regardless of the outage constraint, it's easy to check that $y(r_s)$ is concave when $r_s \leq \frac{\sqrt{3}}{\sqrt{2B(\lambda_{st})}}$ and achieves the maximum at $r_s = L^*(\lambda_{st}) = 1/\sqrt{2B(\lambda_{st})}$, where the corresponding outage probability is $1-e^{-1/2}$, and the throughput (and outage) monotonically increases over transmission range $r_s \in (0, L^*(\lambda_{st})]$. Thus if $\epsilon_s > 1-e^{-1/2}$ the throughput is maximized at the transmission range $L^*(\lambda_{st})$; otherwise at $L(\lambda_{st})(\leq L^*(\lambda_{st}))$, which is obtained by solving the equation $\delta_s(L(\lambda_{st}),\lambda_{st}) = \epsilon_s$. Their corresponding maximal throughput is $\lambda_{st}L^*(\lambda_{st})e^{-1/2}$ and $\lambda_{st}L(\lambda_{st})(1-\epsilon_s)$, respectively.

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