Scheduling Gain in Spatial Diversity Systems: Asymptotic Analysis

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***Abstract*—In this paper, further in-depth asymptotic analysis on the interaction between spatial diversity and multiuser diversity in wireless networks is given, following our recent work [Zhou and Dai, ICC06]. Rigorous proofs and necessarily stronger results in terms of convergence are provided for some intuitions in this area. Equally important, explicit expressions of scheduling gain and average system capacity in various circumstances that reveal interesting inter-connections among key system parameters are given. Our results are general enough to cover many practical scenarios of interest.**

I. INTRODUCTION

Diversity has long been established as key technology that enables reliable and high-data-rate wireless communications. While diversity can be achieved in many forms, two of them attract much research interest recently. One is spatial diversity realized through employing multiple antennas at either the transmitter or receiver end, or both, the idea of which is not new but interest on which is rekindled with the introduction of multi-input multi-output (MIMO) systems [1]. In a multiuser wireless network, there is another form of diversity called multiuser diversity [2][3], which reflects the fact of independent fluctuations of different users' channels. Multiuser diversity can be exploited to increase the system throughput, through intentionally transmitting to the user(s) with good channels at each instant (opportunistic scheduling). Spatial diversity techniques typically reside in the physical layer, while multiuser diversity is obtained through user scheduling at the medium-access control layer. It is therefore interesting to understand how these two diversity techniques combine to determine overall network performance and how they interact with each other.

There exist some work on joint spatial diversity and multiuser diversity systems. In particular, the capacity analysis for Rayleigh fading channels is given in [4], and in [5] for more general Nakagami fading channels. Some have suggested that spatial diversity can actually diminish the advantages of multiuser diversity [3][6][7][8]. Intuitively, this can be explained by observing that multiuser diversity takes advantage of fading by "riding on the peak", which is unfortunately eliminated by spatial diversity. As noted in [6][5][9], however, this conclusion is valid only for open-loop but not closed-loop spatial diversity schemes, while user scheduling inherently requires feedback.

We take a different approach in our recent work [10] by conducting an asymptotic analysis on the interaction between spatial diversity and multiuser diversity, where the number of antennas (M, N) or users (K) or both are allow to go to infinity. Besides mathematical tractability, asymptotic analysis also helps reveal some fundamental relationship of key system parameters, which may be concealed in the finite case by random fluctuations and other transient properties of channel matrices. Moreover in many scenarios (especially with respect to the number of antennas), convergence to the asymptotic limit is rather fast. Our work focuses on spatial diversity systems; some related pioneer study on spatial multiplexing systems can be found in [11][12].

In [10], explicit expressions are derived for average (ergodic) capacity and scheduling gain (as defined in Section II) of joint spatial diversity and multiuser diversity systems, when K goes to infinity while M and N keep fixed. As applications, we confirm that in this scenario, there is a tradeoff between spatial diversity and multiuser diversity for an open-loop spatial diversity system, but the detrimental effect of multiple transmit antennas can be avoided with the closed-loop schemes. We also show that all closed-loop schemes perform similarly in this scenario, in the sense that their differences only occur at the second-order (i.e., $\log\log K$). This part of study assumes that the tail probability density function (PDF) of the effective link signal-to-noise ratio (SNR) takes a form of Gamma-like functions as $\alpha x^p e^{-qx}$ ($\alpha > 0$, $p \geq 0$, and $q > 0$). Further more, We show that the scheduling gain nonetheless diminishes to zero as M and N grow while K keeps fixed, no matter for open-loop or closed-loop spatial diversity systems, through asymptotic study on the mean and variance of the effective link SNR for three representative systems (further discussed in Section II and IV). In this sense, multiuser scheduling is not worthwhile in an antenna-dominant environment. On the other hand, different spatial diversity schemes make significant differences with respect to system capacity for round robin scheduling.

The main contributions of this paper are summarized below:

1). When K goes to infinity while M and N keep fixed, we generalize the results in [10], which allows us to address a larger class of problems with respect to system and channel characteristics.

2). When M and/or N grow while K keeps fixed, we give general conditions in terms of the mean and standard deviation of the normalized effective link SNR, for the convergence of both relative and absolute scheduling gain.

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II. JOINT SPATIAL DIVERSITY AND MULTIUSER DIVERSITY SYSTEM

We consider a homogeneous downlink multiuser MIMO communication scenario, which is envisioned to be of crucial importance for emerging wireless networks. It is assumed that the base station has *M* antennas and each of the *K* users has *N* antennas. Appropriate spatial diversity techniques are employed for each link. In [10] we concretize our analysis with three spatial diversity schemes. The first employs well-known space-time block coding at the transmitter and maximum ratio combining at the receiver, coined as STBC/MRC. The other two belong to closed-loop diversity schemes. One of them pursues joint maximum ratio transmission and maximum ratio combining (MRT/MRC), while the other exploits simple selection combining on both ends (SC/SC), representing the two extremes for various hybrid selection combining schemes. After diversity combining, the user with the best channel quality, in this case the highest effective link SNR, is chosen for communication in opportunistic scheduling. In contrast, the round robin scheduling simply selects the users in some deterministic order.

Assume the normalized effective *link SNR* for user k is γ_k, whose PDF and CDF are denoted by $f_\gamma(x)$ and $F_\gamma(x)$ respectively (same for all users). Let $k^* = \arg\max_k (\gamma_k)_{k=1}^K$, the resultant normalized *system SNR* seen by the base station with opportunistic scheduling is γ_{k^*}, with PDF

$$f_{\gamma_{k^*}}(x) = Kf_\gamma(x)F_\gamma^{K-1}(x). \qquad (1)$$

The corresponding average system capacity can be expressed as a function of K and M as

$$\overline{S}(K,M) = E\left(\log\left(1+\gamma_t\left(\max_{1\le k\le K}\gamma_k\right)\right)\right),$$

where γ_t is the average transmit SNR. We also define $\overline{R}(M)$, the average system capacity obtained by round-robin scheduling as a function of M as[1]

$$\overline{R}(M) = E\left(\log\left(1+\gamma_t\gamma\right)\right).$$

Finally, the scheduling gain

$$G(K,M) = \overline{S}(K,M) - \overline{R}(M)$$

measures the benefit brought by multiuser diversity. Throughout the paper, when asymptotic analysis with respect to the size of antenna array is pursued, we allow both *M* and *N* to go to infinity, with their ratio $r = N/M$ fixed. The incorporation of the large *M* and fixed *N* scenario is relatively straightforward, and will be briefly discussed as well.

In the rest of this paper, we adopt the following notations for the limiting behaviors of two functions $f(x)$ and $g(x)$ with $\lim_{\substack{x\to\infty \\ or\ x\to 0}} g(x)/f(x) = c$: $g(x) = O(f(x))$ for $0 < |c| < \infty$ and in particular $g(x) \sim f(x)$ for $c = 1$; $g(x) = o(f(x))$ for $c = 0$. When convergence of a sequence of random variables is involved, shorthand notation "*D*" stands for in distribution, "*P*" for in probability, "*r*" for in *r*th mean, and "a.s." for almost surely.

III. ASYMPTOTIC SYSTEM CAPACITY AND SCHEDULING GAIN AS *K* GOES TO INFINITY WHILE *M* KEEPS FIXED

In this section with *M* (and *N*) fixed, we will examine $\lim_{K\to\infty}\overline{S}(K,M) = \lim_{K\to\infty} G(K,M)$. Our main results are stated below, where Corollary 1 admits Theorem 1 in [10] as a special case.

Theorem 1: Let $\gamma_1,...,\gamma_K$ be independent and identically distributed (i.i.d.) positive random variables with absolutely continuous CDF $F_\gamma(x)$ and PDF $f_\gamma(x)$, whose derivative f_γ' exists for all x in (x_1,∞) for some x_1. Define the growth function

$$g_\gamma(x) = \frac{1-F_\gamma(x)}{f_\gamma(x)}. \qquad (2)$$

If $\lim_{x\to\infty} g_\gamma(x) = c \ge 0$, whose derivative $g_\gamma'(x) = O(1/x^{\delta_1})$ with $\delta_1 > 0$, and $b_K = F_\gamma^{-1}(1-1/K) = O\left((\log K)^{\delta_2}\right)$ with $0 < \delta_2 \le 1$, then

$$\lim_{K\to\infty}\left\{\overline{S}(K,M) - \log(1+\gamma_t b_K)\right\} = 0. \qquad (3)$$

Proof: See Appendix A.

Remark: The following result is often invoked to indicate that $\max_{1\le k\le K}\gamma_k$ "grows like" b_K in a coarse sense, and is widely used in the study of opportunistic communications involving extreme values and order statistics (e.g., [3][8]):

$$\frac{\max_{1\le k\le K}\gamma_k - b_K}{a_K} \xrightarrow{D} \Lambda(x) = \exp\left(-e^{-x}\right), \qquad (4)$$

where $a_K = (Kf_\gamma(b_K))^{-1}$. This result can actually be strengthened from existing literature [13][14]: if $c = 0$, $\max_{1\le k\le K}\gamma_k - b_K \xrightarrow{P} 0$, otherwise $\max_{1\le k\le K}\gamma_k \big/ b_K \xrightarrow{P} 1$. Nonetheless, our result (3) is a yet stronger one, which concerns the convergence of the expectation of functions of $\max_{1\le k\le K}\gamma_k$.

According to Theorem 1, the system capacity (and scheduling gain) is asymptotically equivalent to $\log(1+\gamma_t b_K)$ when given conditions are fulfilled. Note that all these conditions involve only the tail behaviors of the distributions of individual link SNR. In the following, we examine a form of special interest, which is general enough to cover common fading models and spatial diversity schemes. And we can also obtain explicit expressions for b_K.

Corollary 1: If $f_\gamma(x) \sim \alpha x^p e^{-qx^v}$ as $x\to\infty$ with $\alpha > 0$, $q > 0$, $v \ge 1$ and any p, then

$$\lim_{K\to\infty}\left\{\overline{S}(K,M) - \log(1+\gamma_t b_K)\right\} = 0,$$

where (up to the second-order approximation[2])

$$b_K = \left(\frac{1}{q}\log cK\right)^{1/v} + \frac{p+1-v}{qv^2}\frac{\log\dfrac{\log cK}{q}}{\left(\dfrac{1}{q}\log cK\right)^{(v-1)/v}}, \qquad (5)$$

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[1] The user index will be omitted from relevant notations when no ambiguity is incurred.

[2] We define the first order approximation when truncated at $\log K$, and the second order approximation when truncated at $\log\log K$.

with $c = \alpha / qv$.

Proof: See Appendix B.

Remark: This corollary includes Theorem 1 in [10], which deals with $f_\gamma(x) \sim \alpha x^p e^{-qx}$ (i.e., $v = 1$), as a special case. While previous results in [10] can well address Rayleigh and Nakagami fading, as commonly assumed in literature, this extension enables us to include Ricean and Log-normal fading as well (further discussed below). Of course when a form different from what is given in Corollary 1 appears in applications, we can still turn to Theorem 1. We also observe that, the parameter α only appears in c, which is typically not important in large K analysis. In general, a smaller v and q indicate a better system performance, as seen from the first term of (5). A larger p also helps, though only at the second-order sense.

Let us check some applications of Corollary 1. For the Log-normal distribution

$$f_\gamma(x) = \frac{1}{\sqrt{2\pi\sigma^2}\, x} e^{-(\log x - \mu)^2 / 2\sigma^2}, \tag{6}$$

the transformation $y = (\mu + \log \gamma)/\sigma$ results in a normal distribution, which dictates $\alpha = 1/\sqrt{2\pi}$, $p = 0$, $q = 1/2$, and $v = 2$. Ricean fading admits the following distribution

$$\begin{aligned} f_\gamma(x) &= \frac{1}{2\sigma^2} \exp\left(-\frac{x + s^2}{2\sigma^2} \right) I_0\left(\frac{\sqrt{x}s}{\sigma^2} \right) \\ &\sim \frac{1}{2\sigma} \frac{1}{\sqrt{2\pi}} (s^2 x)^{-1/4} \exp\left(\frac{-\left(\sqrt{x} - s\right)^2}{2\sigma^2} \right), \end{aligned} \tag{7}$$

where $I_n(x)$ is the *n*th-order modified Bessel function of the first kind, and the tail-equivalence is due to the fact that for fixed *n*, $I_n(x) \sim e^x / \sqrt{2\pi x}$. Define $z = \sqrt{x} - s$: its PDF is acknowledged with $\alpha = 1/\sigma\sqrt{2\pi s}$, $p = 1/2$, $q = 1/2\sigma^2$, and $v = 2$. Diversity combining can be readily included in the above discussion. As an example, we can show that for i.i.d. Ricean fading, SC/SC admits

$$b_{K,Ricean}^{SC/SC} = \left(s + \sqrt{2\sigma^2 \log\left(\frac{KMN\sigma}{\sqrt{2\pi s}} \right)} - \frac{\sigma^2}{4} \frac{\log\log \frac{KMN\sigma}{\sqrt{2\pi s}}}{\sqrt{2\sigma^2 \log\left(\frac{KMN}{\sqrt{2\pi s}} \right)}} \right)^2 \tag{8}$$

with the application of (5).

IV. Asymptotic Scheduling Gain as M Goes to Infinity While K Keeps Fixed

In this section, we will examine $\lim_{M\to\infty} G(K,M) = \lim_{M\to\infty} \left(\bar{S}(K,M) - \bar{R}(M) \right)$ with K fixed. In [10] we show that $\lim_{M\to\infty} G(K,M) = 0$ for fixed K for three representative systems: STBC/MRC, SC/SC, and MRT/MRC. The following theorem unifies the case-by-case study in [10] by presenting general conditions in terms of the mean and standard deviation of the normalized effective link SNR, for the convergence of both relative and absolute scheduling gain.

Theorem 2: Let μ_M and σ_M be the mean and standard deviation of the normalized effective SNR γ_M for each individual link[3]. If $\lim_{M\to\infty} \sigma_M / \mu_M = 0$, then when K keeps fixed

$$\lim_{M\to\infty} \left\{ \bar{R}(M) / \log\left(1 + \gamma_t \mu_M\right) \right\} = 1 \tag{9}$$

and

$$\lim_{M\to\infty} G(K,M) / \bar{R}(M) = 0. \tag{10}$$

If we further have $\left\{ \log\left(\gamma_M / \mu_M \right) \mathbf{I}_{(0,1)}\left(\gamma_M / \mu_M \right) \right\}$ uniformly integrable[4], then

$$\lim_{M\to\infty} \left\{ \bar{R}(M) - \log\left(1 + \gamma_t \mu_M\right) \right\} = 0 \tag{11}$$

and

$$\lim_{M\to\infty} G(K,M) = 0. \tag{12}$$

Proof: See Appendix C.

Remark: As shown in the proof, $\lim_{M\to\infty} \sigma_M / \mu_M = 0$ leads to the conclusion that γ_M / μ_M converges to 1 in 2nd mean (mean square). It is relatively straightforward to show that mean-square convergence still holds for non-negative functions of γ_M / μ_M which grow slower than γ_M / μ_M (e.g., $\log(1 + \gamma_M / \mu_M)$). The difficulty with $\log \gamma_M / \mu_M$ occurs when the argument falls on $(0,1)$, which necessitates the condition of uniform integrability.

As applications, let us revisit three representative systems in this scenario. For simplicity we assume Rayleigh fading.

STBC/MRC

Without loss of generality, we assume that the adopted space-time block coding scheme achieves the full rate and the transmit power is equally allocated among the transmit antennas. In this case, the PDF of $\gamma_M^{STBC/MRC}$ is given by

$$f_\gamma^{STBC/MRC}(x) = \frac{M^{MN}}{(MN-1)!} x^{MN-1} e^{-Mx}, x \geq 0, \tag{13}$$

from which it's straightforward to obtain $\mu_M^{STBC/MRC} = N = rM$ and $\sigma_M^{STBC/MRC} = \sqrt{N/M}$. Clearly we have

$$\lim_{M\to\infty} \frac{\sigma_M^{STBC/MRC}}{\mu_M^{STBC/MRC}} = \lim_{M\to\infty} \frac{1}{\sqrt{rM}} = 0. \tag{14}$$

The PDF of $X_M^{STBC/MRC} = \gamma_M^{STBC/MRC} / \mu_M^{STBC/MRC}$ is given by (for some positive constant *C*)

$$f_M^{STBC/MRC}(x) = \frac{MN^{MN}}{(MN-1)!} x^{MN-1} e^{-MNx} \leq Cx, \text{ when } x < 1, \tag{15}$$

where we can bound the coefficient due to Stiring's formula. Clearly, $\left\{ \log X_M^{STBC/MRC} \mathbf{I}_{(0,1)}\left(X_M^{STBC/MRC} \right) \right\}$ is uniformly integrable. So

$$\lim_{M\to\infty} \left\{ \bar{R}^{STBC/MRC}(M) - \log\left(1 + \gamma_t r M\right) \right\} = 0, \tag{16}$$

$$\lim_{M\to\infty} G^{STBC/MRC}(K,M) = 0. \tag{17}$$

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[3] We use subscript *M* to explicitly denote the dependence of corresponding quantities on *M*.

[4] $\mathbf{I}_A(\bullet)$ is the indicator function on the set *A*.

SC/SC

In this case, the PDF of $\gamma_M^{SC/SC}$ is given by

$$f_\gamma^{SC/SC}(x) = MNe^{-x}(1-e^{-x})^{MN-1}, x \geq 0. \tag{18}$$

It is easy to obtain $\mu_M^{SC/SC} = \sum_{i=1}^{MN} 1/i \to \log\left(MN\right) + C_0$ and $\sigma_M^{SC/SC} = \sqrt{\sum_{i=1}^{MN} 1/i^2} \to \sqrt{\pi^2/6}$ as $M \to \infty$, where C_0 is the Euler's constant. Clearly we have

$$\lim_{M\to\infty} \frac{\sigma_M^{SC/SC}}{\mu_M^{SC/C}} = \lim_{M\to\infty} \frac{1}{\log(MN)} = 0\,. \tag{19}$$

When $M \to \infty$, the single-link SC/SC is equivalent to the corresponding multiuser scheduling scenario when $K \to \infty$. So instead of checking the uniform integrability, we can invoke Corollary 1 with (18) to obtain

$$\lim_{M\to\infty} \left\{ \bar{R}^{SC/SC}\left(M\right) - \log\left(1+\gamma_t \log(MN)\right) \right\} = 0\,, \tag{20}$$

$$\lim_{M\to\infty} G^{SC/SC}(K,M) = 0\,. \tag{21}$$

MRT/MRC

In this diversity scheme the principal right and left singular vector of the channel matrix are applied at the transmit and receive side, respectively, to obtain a SISO link with equivalent channel gain $\sigma_{\max} = \sqrt{\gamma_M^{MRT/MRC}}$, the largest singular value. The closed-form expressions for the $\mu_M^{MRT/MRC}$ and $\sigma_M^{MRT/MRC}$ are unknown. In the asymptotic scenario, it is known that $\gamma_M^{MRT/MRC}/M \xrightarrow{a.s.} (1+\sqrt{r})^2$ [17]. But surprisingly, $\lim_{M\to\infty} E\left(\gamma_M^{MRT/MRC}/M\right)$ remains an open problem in literature. This problem is solved through the following lemma, whose proof is omitted due to space limitations[5].

Lemma 1: Let $\mathbf{H}$ be an $N \times M$ matrix with i.i.d. complex entries with $E(h_{ij}) = 0$, $E(|h_{ij}|^2) = 1$, and $E(|h_{ij}|^4) < \infty$. Define $\lambda_{\max}(\cdot)$ the largest eigenvalue of a square matrix, $\sigma(\cdot)$ the standard deviation of a random variable. Then

$$\lim_{M\to\infty} E\left(\lambda_{\max}\left(\tfrac{1}{M}\mathbf{H}\mathbf{H}^*\right)\right) = \left(1+\sqrt{r}\right)^2,\ r = \lim_{M,N\to\infty} N/M\,, \tag{22}$$

$$\lim_{M\to\infty} \sigma\left(\lambda_{\max}\left(\tfrac{1}{M}\mathbf{H}\mathbf{H}^*\right)\right) = 0\,. \tag{23}$$

Remark: A more important conclusion from Lemma 1 is

$$\lim_{M\to\infty} \frac{\sigma_M^{MRT/MRC}}{\mu_M^{MRT/MRC}} = 0\,. \tag{24}$$

Uniform integrability of $\left\{\log X_M^{MRT/MRC} \mathbf{I}_{(0,1)}\left(X_M^{MRT/MRC}\right)\right\}$ is already verified in Proposition 4.2 of [18]. Therefore,

$$\lim_{M\to\infty} \left\{ \bar{R}^{MRT/MRC}\left(M\right) - \log\left(1+\gamma_t (1+\sqrt{r})^2 M\right) \right\} = 0\,, \tag{25}$$

$$\lim_{M\to\infty} G^{MRT/MRC}(K,M) = 0\,. \tag{26}$$

Theorem 2 and the above examples indicate that, given the number of users, the scheduling gain will diminish when the number of antennas goes to infinity, if the mean of the link SNR grows at a higher-order rate than its variance. This is reminiscent of the multiple-antenna channel hardening effect studied in [11]. It is also interesting to see the difference in $\lim_{M\to\infty} \bar{R}\left(M\right)$ for different diversity techniques. For STBC/MRC, it achieves a constant unless the number of receive antennas N also grows with M[6]. For SC/SC, it grows like $\log\log M$, but less impressive than that achieved by MRT/MRC, $\log M$.

V. Conclusion and Future Work

This paper presents further in-depth asymptotic analysis on the interaction between spatial diversity and multiuser diversity in wireless networks. Our results are general enough to cover many practical scenarios of interest.

Our future work includes a more rigorous analysis for the scenario when both M and K go to infinity (see [10] for some preliminary results), and extension to the situations when users' channels are heterogeneous, together with the associated fairness issues.

Appendix A: Sketch of Proof of Theorem 1

First based on a result in [15] we have for some $\kappa > 0$

$$\begin{aligned} &\mathrm{P}\left\{-\kappa \log\log K \leq \left(\max_{1\leq k\leq K} \gamma_k\right) - b_K \leq \kappa \log\log K\right\} \\ &\geq 1 - O\left(\frac{1}{\log K}\right). \end{aligned} \tag{27}$$

Then we can obtain a tight lower bound of $\bar{S}\left(K,M\right)$ through an extension of Markov's inequality

$$\begin{aligned} &\bar{S}\left(K,M\right) = E\left(\log\left(1+\gamma_t\left(\max_{1\leq k\leq K}\gamma_k\right)\right)\right) \\ &\geq P\left(\max_{1\leq k\leq K}\gamma_k \geq b_K - \kappa\log\log K\right) \times \log\left(1+\gamma_t\left(b_K - \kappa\log\log K\right)\right) \\ &\geq \left(1 - O\left(\frac{1}{\log K}\right)\right) \times \log\left(1+\gamma_t\left(b_K - \kappa\log\log K\right)\right) \\ &\geq \log\left(1+\gamma_t b_K\right) - o(1). \end{aligned}$$

As for a tight upper bound, we split $\bar{S}\left(K,M\right)$ in two parts as follows

$$\begin{aligned} &\bar{S}\left(K,M\right) = \int_0^\infty P(S\left(K,M\right) > x)dx \\ &= \int_0^{\log(1+\gamma_t b_K)} P(S\left(K,M\right) > x)dx + \int_{\log(1+\gamma_t b_K)}^{+\infty} P(S\left(K,M\right) > x)dx \\ &\leq \log\left(1+\gamma_t b_K\right) + \int_{\log(1+\gamma_t b_K)}^{+\infty} P(S\left(K,M\right) > x)dx, \end{aligned} \tag{28}$$

and show that the second term above diminishes as $K \to \infty$.

Appendix B: Sketch of Proof of Corollary 1

First we check that

$$\lim_{x\to\infty}\left(qvx^{v-1}\frac{1-F_\gamma(x)}{f_\gamma(x)}\right) = 1 \tag{29}$$

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[5] The authors thank Professor J. W. Silverstein and Professor Z. D. Bai for helping with the proof.

[6] The results in (16), (20) and (25) hold for large M and fixed N as well. In this case, rM in (16) should be replaced with N, and r in (25) taken as 0.

when $f_\gamma(x) \sim \alpha x^p e^{-qx^v}$. This leads to the conclusion that $\lim_{x\to\infty} g(x) = c \geq 0$ ($c = 0$ when $v > 1$), and $g'(x) = O\left(1/x^v\right)$, therefore by Theorem 1, we are left to verify (5).

It can also be referred from (29) that

$$\lim_{x\to\infty}\left(1 - F(x)\right)/\varphi(x) = 1,$$

where $\varphi(x) = \alpha x^{p+1-v} e^{-qx^v}/qv$. Therefore, we only need to solve $\varphi(b_K) = 1/K$, i.e.,

$$b_K^v = \frac{1}{q}\log\frac{K\alpha}{qv} + \frac{p+1-v}{q}\log b_K. \tag{30}$$

The first order approximation for b_K is readily given by $b_K^{(1)} = \left(\log cK/q\right)^{1/v}$. To obtain the second order approximation, we just replace b_K on the right hand side of (30) with $b_K^{(1)}$.

APPENDIX C: SKETCH OF PROOF OF THEOREM 2

Let $X_M = \gamma_M/\mu_M$, $\lim_{M\to\infty} \sigma_M/\mu_M = 0$ indicates that $X_M \xrightarrow{2} 1$ as $M \to \infty$.

First we show the weaker conclusion that the relative scheduling gain diminishes as $M \to \infty$. Fix $\delta > 0$. By Markov's inequality, we have

$$E\left(\log(1+\gamma_t\gamma_M)\right) \geq P\left(X_M \geq (1-\delta)\right)\log\left(1+\gamma_t(1-\delta)\mu_M\right), \tag{31}$$

which together with $X_M \xrightarrow{2} 1$ leads to

$$\liminf_{M\to\infty} \frac{E\left(\log(1+\gamma_t\gamma_M)\right)}{\log\left(1+\gamma_t(1-\delta)\mu_M\right)} \geq \lim_{M\to\infty} P\left(X_M \geq (1-\delta)\right) = 1. \tag{32}$$

Now let $\delta \to 0$ to get

$$\liminf_{M\to\infty} \frac{E\left(\log(1+\gamma_t\gamma_M)\right)}{\log\left(1+\gamma_t\mu_M\right)} \geq 1. \tag{33}$$

On the other hand, by Jensen's Inequality

$$\limsup_{M\to\infty} \frac{E\left(\log(1+\gamma_t\gamma_M)\right)}{\log\left(1+\gamma_t\mu_M\right)} \leq 1, \tag{34}$$

which completes the proof of (9).

As for the convergence of the absolute scheduling gain, we can write

$$\begin{aligned}\log X_M &= \log X_M \mathbf{I}_{(0,1)}\left(X_M\right) + \log X_M \mathbf{I}_{[1,\infty)}\left(X_M\right)\\ &= Y_M^{(1)} + Y_M^{(2)}.\end{aligned} \tag{35}$$

First we have $0 \leq Y_M^{(2)} \leq (X_M - 1)\mathbf{I}_{[1,\infty)}\left(X_M\right)$. Therefore $E\left(Y_M^{(2)}\right) \to 0$ as $E\left((X_M-1)\mathbf{I}_{[1,\infty)}\left(X_M\right)\right) \leq E\left(\left|X_M - 1\right|\right) \to 0$ when $M \to \infty$. In order to show that $E\left(Y_M^{(1)}\right) \to 0$, we make the following claim.

Claim 1: If a random variable $X_n \xrightarrow{P} 0$ as $n \to \infty$, $X_n I_E(X_n) \xrightarrow{P} 0$ for any event E.

This claim is easy to verify as $\forall\varepsilon$,

$$P\left(\left|X_n I_E(X_n)\right| > \varepsilon\right) \leq P\left(\left|X_n\right| > \varepsilon\right) \to 0 \text{ as } n \to \infty.$$

Now that $X_M \xrightarrow{2} 1$ we have $\log X_M \xrightarrow{P} 0$. By claim 1 we in turn have $\log X_M \mathbf{I}_{(0,1)}\left(X_M\right) \xrightarrow{P} 0$. This together with the uniform integrability of $\left\{\log X_M \mathbf{I}_{(0,1)}\left(X_M\right)\right\}$ results in $Y_M^{(1)} \xrightarrow{1} 0$ [16], and (11) follows.

(10) and (12) follow readily from (9), (11) and the fact [13]

$$E\left[\max_{1\leq k\leq K} \gamma_k\right] \leq \mu_M + \frac{(K-1)\sigma_M}{\sqrt{2K-1}}. \tag{36}$$

REFERENCES

[1] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, “Capacity limits of MIMO channels”, *IEEE J. Select. Areas Commun.*, vol. 21, pp. 684-702, June 2003.

[2] R. Knopp and P. Humblet, “Information capacity and power control in single cell multiuser communications,” in *Proc. Int. Conf. Commun.*, vol. 1, Seattle, WA, June 1995, pp. 331-335.

[3] P. Viswanath, D. N. C. Tse, and R. Laroia, “Opportunistic beamforming using dumb antennas,” *IEEE Trans. Inf. Theory*, vol. 48, no. 6 , pp. 1277 -1294, June 2002.

[4] C. Mun et al, “Exact capacity analysis of multiuser diversity combined with transmit diversity,” *Electron. Lett.*, vol. 40, no. 22, pp. 1423-1424, Oct. 2004.

[5] C.-J. Chen and L.-C. Wang, “A unified capacity analysis for wireless systems with joint antenna and multiuser diversity in Nakagami fading channels,” *Proc. Int. Conf. Commun.*, vol. 6, Paris, June 2004, pp. 3523-3527.

[6] V. K. N. Lau, Y. Liu and T. A. Chen, “The role of transmit diversity on wireless communications – Reverse link analysis with partial feedback,” *IEEE Trans. Commun.*, vol. 50, no. 12, pp. 2082-2090, Dec. 2002.

[7] R. Gozali, R. M. Buehrer and B. D. Woerner, “The impact of multiuser diversity on space-time block coding,” *IEEE Commun. Lett.*, vol.7, no.5, pp213-215, May 2003.

[8] J. Jiang, R. M. Buehrer and W. H. Tranter, “Antenna diversity in multiuser data networks,” *IEEE Trans. Commun.*, vol. 52, no. 3, pp. 490-497, Mar. 2004.

[9] E. G. Larsson, “On the combination of spatial diversity and multiuser diversity,” *IEEE Commun. Lett.*, vol. 8, no.8, pp.517-519, Aug. 2004.

[10] Q. Zhou and H. Dai, “Asymptotic analysis on spatial diversity versus multiuser diversity in wireless networks,” *Proc. Int. Conf. Commun.*, Istanbul, Turkey, June 2006.

[11] B. M. Hochwald, T. L. Marzetta, and V. Tarokh, “Multiple-antenna channel hardening and its implications for rate feedback and scheduling,” *IEEE Trans. Inf. Theory*, vol.50, no.9, pp.1893-1909, Sep. 2004.

[12] M. Sharif and B. Hassibi, “On the capacity of MIMO broadcast channels with partial side information,” *IEEE Trans. Inf. Theory*, vol.51, no.2, pp.506-522, Feb. 2005.

[13] H. A. David and H. N. Nagaraja, *Order Statistics*, 3rd edition, New York: Wiley, 2003.

[14] J. Galambos, *the Asymptotic Theory of Extreme Order Statistics*, 2nd edition, New York: Wiley, 1978.

[15] N. T. Uzgoren, “The asymptotic development of the distribution of the extreme values of a sample,” *Studies in Mathematics and Mechanics Presented to Richard von Mises*, New York: Academic, 1954, pp. 346-353.

[16] G. Grimmett and D. Stirzaker, *Probability and Random Processes*, 3rd edition, Oxford University, 2001.

[17] Y. Q. Yin, Z. D. Bai, and P. R. Krishnaiah, “On the limit of the largest eigenvalue of the large dimensional sample covariance matrix,” *Probability Theory and Related Fields*, vol. 78, pp. 509-521, 1988.

[18] A. Edelman, “Eigenvalues and condition numbers of random matrices,” *Siam J. Matrix Anal. Appl.*, vol.9, no. 4, pp. 543-560, Oct. 1988.