Distributed Detection of A Deterministic Signal in Correlated Gaussian Noise Over MAC

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***Abstract*— Distributed detection of a deterministic signal in correlated Gaussian noise in a one-dimensional sensor network is studied in this paper. In contrast to the traditional approach where a bank of dedicated parallel access channels (PAC) is used for transmitting the sensor observations to the fusion center, we explore the possibility of employing a shared multiple access channel (MAC), which significantly reduces the bandwidth requirement or detection delay. We assume that local observations are mapped according to a certain function subject to a power constraint and transmitted simultaneously to the fusion center. Using a large deviation approach, we demonstrate that with a specially-chosen mapping rule, MAC fusion achieves the same asymptotic performance as centralized detection under the average power constraint (APC), while there is always a loss in error exponents associated with PAC fusion. Under the total power constraint (TPC), MAC fusion still results in exponential decay in error exponents with the number of sensors, while PAC fusion does not. Finally, we derive an upper bound on the performance loss due to the lack of perfect synchronization over MAC, and show that the performance degradation is negligible when the phase mismatch among sensors is sufficiently small.**

I. Introduction

In this paper, we study the distributed detection of a deterministic signal in a one-dimensional (1-D) sensor network with equally-spaced sensors and correlated Gaussian observation noise. We use the measure of asymptotic error exponent as the number of sensors goes to infinity to evaluate the detection performance. The error exponent gives an estimate of the number of sensors required to reach a certain error probability, which is a useful performance index for densely-deployed sensor networks [1]–[4]. The traditional approach of studying the distributed detection problem is to assume that sensors transmit their observations (possibly quantized versions of them) through a parallel access channel (PAC), which is independent across sensors [1], [2], [5]. For large-scale sensor networks, this assumption implies a large bandwidth requirement for simultaneous transmission or a large detection delay. On the other hand, a multiple access channel (MAC) is bandwidth efficient, but with a MAC, the fusion center only receives an aggregate of sensor observations, which is typically not a sufficient statistic for detection. Nevertheless, this does not mean that distributed detection over MAC is not worthy of study. Several recent works have explored the possibility of using a MAC for distributed detection with i.i.d. discrete observations based on the method of types [3], [6]. As sensors are packed more closely together, it is reasonable to expect that their observations get more correlated [2]. Several recent works have employed large deviation theory to study the performance of centralized or distributed detection with correlated observations [2], [4]. The distributed detection of a constant signal in correlated Gaussian noise and the detection of a Gaussian-Markov process are studied in [2], where the communication channel is assumed to be a PAC. Closed-form detection error exponent for Neyman-Pearson centralized detection of a Gaussian-Markov process is obtained in [4]. In this paper, we demonstrate that a Gaussian multiple access channel can be used for detection of a deterministic signal in correlated Gaussian noise, by appropriately choosing the local mapping rule. The deterministic signal contains the constant signal as a special case. To the best of our knowledge, this has not been explored by others.

We state our assumptions as follows. Sensors transmit local decisions based on their respective local observations and a mapping rule through a MAC or a PAC. Following [3], we assume that the MAC is perfectly synchronized with identical channel gain to facilitate the analysis. If sensors experience different pathlosses, we assume that sensors have the knowledge of their own pathlosses and adjust the transmission power accordingly such that the overall channel gain is kept the same for all sensors. We assume that uncoded analog communication is used for transmission of local decisions. There is a fair body of literature including [7], [8] adopting this assumption, and our motivations are similar to theirs: firstly, it is well-known that transmitting a Gaussian source directly through a Gaussian channel leads to optimal cost-distortion tradeoff [9]; secondly, although distortionless analog transmission is difficult to realize in practice, such an idealization enables us to use the large deviation theory to obtain a good estimate of system performance.

The transmitted symbols satisfy either an average power constraint (APC) or a total power constraint (TPC). We propose a mapping rule for MAC fusion based on the observation of the optimal decision statistic, and show that 1) under APC, our proposed MAC fusion scheme yields the same error exponents as optimal centralized detection, while PAC fusion always incurs a loss in error exponents; 2) under TPC, MAC fusion still results in exponential decay of the error probability with the number of sensors, while the error probability of PAC fusion is not reduced by increasing the number of sensors.

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While perfect synchronization over MAC is a basic assumption throughout the paper, we study the impact of synchronization error on detection performance in Section IV. It is shown that there is little performance loss compared with the perfectly-synchronized case when the phase mismatch is slight.

II. System Description and Preliminaries

A. Detection Problem

We consider a 1-D sensor network, where sensors are equally-spaced over a straight line, with the location of the kth node $d_k = kd, k = 1, \cdots, n$. Consider the binary hypothesis testing problem where the observation at the kth sensor is given by

$$H_1 : x_k = s_k + v_k, \quad k = 1, 2, \cdots, n, \tag{1}$$

$$H_0 : x_k = v_k, \quad k = 1, 2, \cdots, n, \tag{2}$$

where $\{s_k\}$ is a uniformly bounded signal. The autocorrelation function of $\{s_k\}$ is recovered from the spectral distribution $G(\cdot)$ by a Riemann-Stieltjes integral

$$R(k) \doteq \lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} s_{j+k} s_j = \frac{1}{2\pi} \int_0^{2\pi} e^{ik\omega} dG(\omega), \tag{3}$$

and if $G(\cdot)$ is absolutely continuous, its derivative $G'(\omega)$ is the spectral density of $\{s_k\}$ [10]. The stationary Gaussian noise process $\{v_k\}$ has zero mean and covariance function

$$\rho(k_1, k_2) \doteq E\{v_{k1} v_{k2}\} = \sigma^2 \rho^{|k_1 - k_2|},$$

hence its covariance matrix

$$\mathbf{\Sigma} = \sigma^2 \begin{bmatrix} 1 & \rho & \cdots & \rho^{n-1} \\ \rho & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \rho \\ \rho^{n-1} & \cdots & \rho & 1 \end{bmatrix}. \tag{4}$$

B. Mapping Rule and Network Communication Channel

We assume that local observations are first mapped through a function $U(\cdot) : y_k = U(x_k)$ subject to a global power constraint, which may be an *average power constraint (APC)*, given by $\frac{1}{n}\sum_{k=1}^{n} E\{|y_k|^2\} \leq P_{av}$, or a *total power constraint (TPC)*, given by $\sum_{k=1}^{n} E\{|y_k|^2\} \leq P_{tot}$. The mapped signal is then transmitted over one of the following channels (the signal attenuation of the channel is assumed to be absorbed in $U(\cdot)$):

1) **Parallel Access Channel (PAC)**, consisting of n dedicated additive White Gaussian noise channels given by

$$r_k = y_k + z_k, \quad k = 1, 2, \cdots, n, \tag{5}$$

where $\{z_k\}$ is i.i.d. $\mathcal{N}(0, 1)$.

2) **Multiple Access Channel (MAC)**: Unless otherwise specified, we refer to MAC as a perfectly synchronized Gaussian multiple access channel given by

$$r = \sum_{k=1}^{n} y_k + z, \tag{6}$$

where $z \sim \mathcal{N}(0, 1)$.

In the following, we use the vector notation $\mathbf{s} = (s_1, \cdots, s_n)^T$, and similarly denote $\mathbf{v}$, $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ and $\mathbf{r}$.

C. Preliminaries

Absolutely Summable Toeplitz Matrix [11]: Let $\mathbf{\Sigma}^{(n)}$ be an $n \times n$ Toeplitz matrix with entries $t_k \in \mathbb{R}$ on the kth diagonal and dimension $n \to \infty$. If $\{t_k\}$ is absolutely summable, i.e., $\sum_{k=-\infty}^{\infty} |t_k| < \infty$, the spectral density function of $\mathbf{\Sigma}^{(n)}$ is given by

$$S(\omega) = \sum_{k=-\infty}^{\infty} t_k e^{-ik\omega}, \quad -\pi < \omega \leq \pi. \tag{7}$$

Extended Toeplitz distribution theorem [12]: Let $\{s_k\}_{k=1}^n$ be a deterministic signal with spectral distribution $G(\omega)$. For an absolutely summable Toeplitz matrix $\mathbf{\Sigma}^{(n)}$ with spectral density $S(\omega)$, let $\{\lambda_k^{(n)}\}_{k=1}^n$ be the eigenvalues of $\mathbf{\Sigma}^{(n)}$ contained on the interval $[\delta_1, \delta_2]$, and $\{\boldsymbol{\phi}_k^{(n)}\}_{k=1}^n$ be the normalized eigenvectors of $\mathbf{\Sigma}^{(n)}$, then for any continuous function $h(\cdot)$ defined on $[\delta_1, \delta_2]$, we have

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} h(\lambda_k^{(n)}) (\mathbf{s}^T \boldsymbol{\phi}_k)^2 = \frac{1}{2\pi} \int_0^{2\pi} h(S(\omega)) dG(\omega). \tag{8}$$

Gärtner-Ellis Theorem [13]: Let $\{Z_n\} \in \mathbb{R}$ be a sequence of random variables drawn according to the probability law $\{\mu_n\}$, and define

$$\Lambda^{(n)}(\theta) = \log E[e^{\theta Z_n}]. \tag{9}$$

Assumptions: (1) For each $\theta \in \mathbb{R}$, the logarithmic moment generating function, defined as the limit $\Lambda(\theta) \doteq \lim_{n\to\infty} \frac{1}{n} \Lambda^{(n)}(n\theta)$ exists as an extended real number. (2) The interior of $D_\Lambda \doteq \{\theta \in \mathbb{R} : \Lambda(\theta) < \infty\}$, denoted by D_Λ^o, contains the origin. (3) $\Lambda(\cdot)$ is differentiable throughout D_Λ^o, and $\Lambda(\cdot)$ is steep, i.e., $\lim_{n\to\infty} \Lambda'(\theta_n) = \infty$ whenever $\{\theta_n\}$ is a sequence in D_Λ^o converging to a boundary point of D_Λ^o. Under the above assumptions, the large deviation principle (LDP) satisfied by the sequence of $\{\mu_n\}$ can be characterized by the *Fenchel-Legendre* transform of $\Lambda(\theta)$:

$$\Lambda^*(x) = \sup_{\theta\in\mathbb{R}} \{\theta x - \Lambda(\theta)\}. \tag{10}$$

That is, for any closed set $F \subset \mathbb{R}$, $\limsup_{n\to\infty} \frac{1}{n} \log \mu_n(F) \leq -\inf_{x\in F} \Lambda^*(x)$, and for any open set $G \subset \mathbb{R}$, $\liminf_{n\to\infty} \frac{1}{n} \log \mu_n(G) \geq -\inf_{x\in G} \Lambda^*(x)$.

III. Detection of Deterministic Signal In Correlated Noise

A. Optimal Centralized Detection

Optimal centralized detection, where the sensor observation vector $\mathbf{x}$ is perfectly available to the fusion center, serves as a performance baseline for distributed detection strategies. The optimal centralized detection is a threshold test on the normalized log-likelihood ratio: Choose H_1 if

$$\frac{1}{n} \log \frac{\Pr(\mathbf{x}|H_1)}{\Pr(\mathbf{x}|H_0)} = \frac{1}{n} (\mathbf{s}^T \mathbf{\Sigma}^{-1} \mathbf{x} - \frac{1}{2} \mathbf{s}^T \mathbf{\Sigma}^{-1} \mathbf{s}) > \tau, \tag{11}$$

and choose H_0 otherwise. We can rewrite the test as

$$T_n = \frac{1}{n} \mathbf{s}^T \mathbf{\Sigma}^{-1} \mathbf{x} \gtrless T. \tag{12}$$

The spectral density function for $\mathbf{\Sigma}$ is $S(\omega) = \frac{\sigma^2(1-\rho^2)}{1+\rho^2-2\rho\cos\omega}$.

Proposition 3.1 (Centralized Detection): For the Neyman-Pearson (N-P) formulation, when $0 \le T \le \frac{1}{2\pi}\int_0^{2\pi} \frac{dG(\omega)}{S(\omega)}$, the error exponent for type I (false-alarm) and type II (miss) errors are respectively given by

$$\lim_{n\to\infty} -\frac{1}{n}\log\alpha^{(n)} = \frac{\pi T^2}{\int_0^{2\pi}\frac{dG(\omega)}{S(\omega)}}, \tag{13}$$

$$\lim_{n\to\infty} -\frac{1}{n}\log\beta^{(n)} = \frac{\pi}{\int_0^{2\pi}\frac{dG(\omega)}{S(\omega)}}\left(T - \frac{1}{2\pi}\int_0^{2\pi}\frac{dG(\omega)}{S(\omega)}\right)^2 \tag{14}$$

For the Bayesian formulation, the error exponent for the average error probability $P_e = \pi_0\alpha + \pi_1\beta$ is given by

$$\lim_{n\to\infty} -\frac{1}{n}\log P_e^{(n)} = \frac{1}{16\pi}\int_0^{2\pi}\frac{dG(\omega)}{S(\omega)}. \tag{15}$$

Proof: Let $\mathbf{\Sigma} = \mathbf{\Phi}\mathbf{\Lambda}\mathbf{\Phi}^T$, where $\mathbf{\Lambda} = \text{diag}\{\lambda_1, \cdots, \lambda_n\}$ is a diagonal matrix containing the eigenvalues of $\mathbf{\Sigma}$, and $\mathbf{\Phi}$ is a unitary matrix with eigenvectors of $\mathbf{\Sigma}$ as column vectors. Let $\mathbf{p} = \mathbf{\Phi}^T\mathbf{s}$ and $\mathbf{w} = \mathbf{\Phi}^T\mathbf{x}$. Then it is easily shown that under H_0, $\{w_k\}$ is i.i.d. $\mathcal{N}(0, \lambda_k)$, and under H_1, $\{w_k\}$ is i.i.d. $\mathcal{N}(p_k, \lambda_k)$. Thus we have

$$T_n = \frac{1}{n}\sum_{k=1}^n \frac{p_k w_k}{\lambda_k}.$$

The logarithmic moment generating functions under H_0 is

$$\Lambda_0^{(n)}(n\theta) = \log E_0\left\{e^{\theta\sum_{k=1}^n \frac{p_k w_k}{\lambda_k}}\right\} = \sum_{k=1}^n \frac{\theta^2 p_k^2}{2\lambda_k}.$$

Using the extended Toeplitz Theorem, we have

$$\Lambda_0(\theta) \;=\; \lim_{n\to\infty}\frac{1}{n}\Lambda_0^{(n)}(n\theta) = \frac{\theta^2}{4\pi}\int_0^{2\pi}\frac{dG(\omega)}{S(\omega)}.$$

It can be checked that the assumptions for Gärtner-Ellis theorem hold for T_n under H_0. Therefore,

$$\Lambda_0^*(x) = \sup_{\theta\in\mathbb{R}}\{\theta x - \Lambda_0(\theta)\} = \frac{\pi x^2}{\int_0^{2\pi}\frac{dG(\omega)}{S(\omega)}}.$$

Similarly, we obtain that for H_1,

$$\Lambda_1^*(x) = \sup_{\theta\in\mathbb{R}}\{\theta x - \Lambda_1(\theta)\} = \frac{\pi}{\int_0^{2\pi}\frac{dG(\omega)}{S(\omega)}}\left(x - \frac{\int_0^{2\pi}\frac{dG(\omega)}{S(\omega)}}{2\pi}\right)^2.$$

When $0 \le T \le \frac{1}{2\pi}\int_0^{2\pi}\frac{dG(\omega)}{S(\omega)}$, the error exponent for type I and type II errors are given by $\Lambda_0^*(T)$ and $\Lambda_1^*(T)$. The Bayesian error probability is obtained with the threshold $T = \lim_{n\to\infty}\frac{1}{2n}\mathbf{s}^T\mathbf{\Sigma}^{-1}\mathbf{s} = \frac{1}{4\pi}\int_0^{2\pi}\frac{dG(\omega)}{S(\omega)}$. ∎

B. Distributed Detection over PAC

For PAC, we assume that each sensor directly transmits an amplified version of the local observation. Denote the average power of s_k by $P_s \doteq \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^n s_k^2 = \frac{1}{2\pi}\int_0^{2\pi} dG(\omega)$, we have $\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^n E(x_k^2) = \sigma^2 + \pi_1 P_s$.

1) Average Power Constraint: Under APC, the mapping rule can be written as $y_k = a x_k$, where $a = \sqrt{\frac{P_{av}}{\sigma^2+\pi_1 P_s}}$ is a constant independent of n. Let

$$r'_k = \frac{r_k}{a} = x_k + \frac{z_k}{a}. \tag{16}$$

The optimal test becomes

$$T'_n = \frac{1}{n}\mathbf{s}^T\mathbf{\Sigma}'^{-1}\mathbf{r}' \gtrless T. \tag{17}$$

where $\mathbf{\Sigma}' = \mathbf{\Sigma} + \frac{1}{a^2}\mathbf{I}$ is the covariance matrix of $\mathbf{r}'$. Following similar analysis as in Section III. A, the expressions of error exponents are the same as for centralized detection, except that $S(\omega)$ is replaced with $S'(\omega) = S(\omega) + \frac{1}{a^2}$. Consequently under the average power constraint, detection over PAC suffers from a loss in asymptotic performance.

2) Total Power Constraint: Under TPC, the mapping rule becomes $y_k = \frac{a}{\sqrt{n}}x_k$, where $a = \sqrt{\frac{P_{tot}}{\sigma^2+\pi_1 P_s}}$. Then $r''_k = \frac{\sqrt{n}r_k}{a} = x_k + \frac{\sqrt{n}z_k}{a}$ has covariance matrix $\mathbf{\Sigma}'' = \mathbf{\Sigma} + \frac{n}{a^2}\mathbf{I}$. Note that $\mathbf{\Sigma}''$ is not absolutely summable, hence the theorems in Section II can not be applied to analyze this case. In fact, simulation shows that under TPC, PAC fusion no longer results in exponential decay in the average error probability (see Section III. D).

C. Distributed Detection over MAC

With a MAC, the fusion center no longer have access to individual sensor observations. Therefore, the mapping rule should be carefully chosen so that the received signal yields a useful decision statistic for detection. Observe that if we let $\boldsymbol{\gamma} = \mathbf{\Sigma}^{-1}\mathbf{s}$, the optimal decision statistic for centralized detection can be written as

$$T_n = \frac{1}{n}\mathbf{s}^T\mathbf{\Sigma}^{-1}\mathbf{x} = \frac{1}{n}\boldsymbol{\gamma}^T\mathbf{x} = \frac{1}{n}\sum_{k=1}^n \gamma_k x_k. \tag{18}$$

Assuming sensor k is informed of γ_k, if sensor k transmits a scaled version of $\gamma_k x_k$, the noise-free output of MAC readily yields the decision statistic. It can be shown that

$$\gamma_k = \frac{1}{\sigma^2(1-\rho^2)}\cdot\begin{cases} s_1 - \rho s_2, & k=1,\\ s_n - \rho s_{n-1}, & k=n,\\ (1+\rho^2)s_k - \rho(s_{k-1}+s_{k+1}), & \text{else.}\end{cases}$$

Therefore, if sensor k knows ρ, σ^2, s_{k-1}, s_k and s_{k+1}, it can compute γ_k.

1) Average Power Constraint: Under the average power constraint, the mapping rule is given by $y_k = a\gamma_k x_k$, where $a = \sqrt{\frac{P_{av}}{\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^n E(\gamma_k^2 x_k^2)}}$. Note that $\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^n \mathbb{E}(\gamma_k^2 x_k^2)$ exists because $\{s_k\}$ is uniformly bounded. The exact value of a can be computed for specific applications–see Section III.D.

Theorem 3.1 (Asymptotic Optimality of MAC Fusion Under APC): For the mapping rule $y_k = a\gamma_k x_k$, where a is a constant independent of n, the threshold test on

$$T_n^{APC} = \frac{1}{na}r = \frac{1}{n}\sum_{k=1}^n \gamma_k x_k + \frac{z}{na} \tag{19}$$

achieves the same error exponent as optimal centralized detection, and the error exponents do not depend on a.

Proof: For T_n^{APC}, we have for H_0,

$$\Lambda_0^{(n)}(n\theta) = \log E_0 \left\{ e^{\theta(\sum_{k=1}^{n} \frac{p_k w_k}{\lambda_k} + \frac{z}{a})} \right\} = \sum_{k=1}^{n} \frac{\theta^2 p_k^2}{2\lambda_k} + \frac{\theta^2}{2a^2},$$

Since $\Lambda_0(\theta) = \lim_{n\to\infty} \frac{1}{n}\Lambda_0^{(n)}(n\theta)$, the second term $\frac{\theta^2}{2na^2}$ vanishes asymptotically, and $\Lambda_0(\theta)$ is the same as for optimal centralized detection, and similarly for H_1. Therefore the error exponents are the same as for optimal centralized detection. ■

Theorem 3.1 suggests that, with a properly chosen mapping rule, detection over MAC can be asymptotically optimal, provided that the total transmit power scales with the number of sensors n. In contrast to PAC, the effect of channel noise on MAC is washed out asymptotically under APC.

2) Total Power Constraint: Under the total power constraint, the mapping rule is given by $y_k = \frac{a}{\sqrt{n}}\gamma_k x_k$, where $a = \sqrt{\frac{P_{tot}}{\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} E(\gamma_k^2 x_k^2)}}$.

Theorem 3.2 (MAC Fusion Under TPC): For the mapping rule $y_k = \frac{a}{\sqrt{n}}\gamma_k x_k$, where a is a constant independent of n, the threshold test on

$$T_n^{TPC} = \frac{1}{\sqrt{n}a} r = \frac{1}{n}\sum_{k=1}^{n} \gamma_k x_k + \frac{z}{\sqrt{n}a} \tag{20}$$

yields suboptimal asymptotic performance: when the threshold $0 \le T \le \frac{1}{2\pi}\int_0^{2\pi} \frac{dG(\omega)}{S(\omega)}$, the error exponent for type I and type II errors are respectively given by

$$\lim_{n\to\infty} -\frac{1}{n}\log \alpha^{(n)} = \frac{T^2}{2\left(\frac{1}{2\pi}\int_0^{2\pi}\frac{dG(\omega)}{S(\omega)} + \frac{1}{a^2}\right)}, \tag{21}$$

$$\lim_{n\to\infty} -\frac{1}{n}\log \beta^{(n)} = \frac{\left(T - \frac{1}{2\pi}\int_0^{2\pi}\frac{dG(\omega)}{S(\omega)}\right)^2}{2\left(\frac{1}{2\pi}\int_0^{2\pi}\frac{dG(\omega)}{S(\omega)} + \frac{1}{a^2}\right)}. \tag{22}$$

The Bayesian error exponent is achieved with $T = \frac{1}{4\pi}\int_0^{2\pi}\frac{dG(\omega)}{S(\omega)}$ in the above expressions.

The proof is omitted due to the space constraint. Note that the asymptotic optimality of MAC fusion is lost if each sensor is forced to use diminishing power as the number of sensors increases. Nevertheless, MAC fusion is still preferred under TPC as it results in exponential decay in error probability.

D. Numerical Example: Detection of A Sinusoid Signal

Consider a sinusoid signal over a straight line:

$$s_k = \sqrt{2}m\cos(\omega_0 k), \quad k = 1, \cdots, n. \tag{23}$$

The spectral density of this signal is $G'(\omega) = \pi m^2[\delta(\omega-\omega_0) + \delta(\omega - (2\pi - \omega_0))]$. Fig. 1 plots the simulated Bayesian error exponents for centralized detection and various distributed detection schemes, where $\pi_0 = \pi_1 = 1/2$, $\rho = 0.5$, $m = \sigma = 1$, $\omega_0 = \pi/4$, $P_{av} = 2$, and $P_{tot} = 10$. Note that in this example, we have $\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n} E(\gamma_k^2 x_k^2) = \frac{1}{4}\sum_{k=1}^{4}\gamma_k^2(\frac{s_k^2}{2} + \sigma^2)$, from which the scaling factor for MAC fusion can be obtained. The scaling factors as well as the theoretical Bayesian exponents are given in Table I. It can be seen that the simulated

TABLE I

SCALING FACTORS AND BAYESIAN ERROR EXPONENTS, $\rho = 0.5$, $m = \sigma = 1$, $\omega_0 = \pi/4$, $P_{av} = 2$, AND $P_{tot} = 10$

	Centralized	PAC-APC	PAC-TPC	MAC-APC	MAC-TPC
a	N/A	1.15	2.58	1.47	3.30
Exp	0.0905	0.0586	N/A	0.0905	0.0803

error exponents for various detection schemes approach the predicted values as the number of sensors becomes larger. The MAC fusion scheme achieves similar performance as centralized detection under APC. Under both APC and TPC, the MAC fusion scheme significantly outperforms PAC fusion. In particular, under TPC, the error probability for MAC fusion still decays exponentially with the number of sensors, while the error probability for PAC fusion does not decrease with the number of sensors.

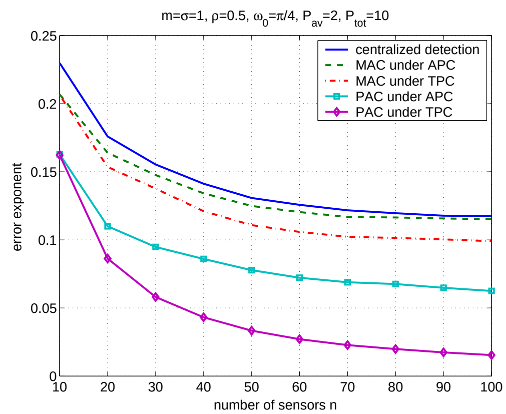

Fig. 1. Error exponents for detection of a sinusoid signal, $m = 1$, $\sigma = 1$, $\rho = 0.5$, $\omega_0 = \pi/4$, $P_{av} = 2$, $P_{tot} = 10$

IV. IMPACT OF PHASE ERROR ON DISTRIBUTED DETECTION OVER MAC

An important assumption for the proposed MAC fusion scheme to achieve the predicted asymptotic performance is the perfect synchronization among sensors. The sensor synchronization required in our application can be achieved with essentially the same strategy as described in [14], but instead of taking a star topology, we assume that the synchronization head is a node located at coordinate 0 on the line, thus every sensor needs to transmit a delayed signal with a phase shift according to their respective distance to the synchronization head. The synchronization error is manifested by the phase mismatch among sensors. In this section we study its impact on detection performance under the average power constraint.

Denote the phase error for the kth sensor as φ_k. We assume that $\{\varphi_k\}$ is i.i.d. and independent of $\{x_k\}$. The received baseband signal at the fusion center is $r = \mathrm{Re}\left\{a\sum_{k=1}^{n}\gamma_k x_k e^{j\varphi_k} + z\right\}$, where $z \sim \mathcal{CN}(0,1)$, and the decision statistic is given by

$$\tilde{T}_n = \frac{1}{na} r = \frac{1}{n}\sum_{k=1}^{n}\gamma_k x_k \cos\varphi_k + \frac{\mathrm{Re}(z)}{na}. \tag{24}$$

The asymptotic logarithmic generating function is then given by (note that the noise term vanishes under APC):

$$\tilde{\Lambda}_i(\theta) = \lim_{n\to\infty}\frac{1}{n}\log E_i\left\{e^{\theta\sum_{k=1}^{n}\gamma_k x_k\cos\varphi_k}\right\}, \tag{25}$$

where $i\in\{0,1\}$. Note that since $\tilde{T}_n$ is a random variable generated by $\mathbf{x}$ and $\boldsymbol{\varphi}$, the expectation should be taken with respect to both variables. It can be shown that, unless φ_k is the same for all k, i.e., sensors are perfectly synchronized, there is always a loss in the Bayesian error exponent.

In the following, we assume that $\varphi_k\sim\mathcal{N}(0,\sigma_\varphi^2)$, where σ_φ is small, and we seek an upper bound on the performance loss due to phase mismatch. Taking expectation with respect to $\boldsymbol{\varphi}$ and using the approximation $\sin\varphi_k\approx\varphi_k$ for small φ_k,

$$E_{\boldsymbol{\varphi}}\left\{e^{\theta\sum_{k=1}^{n}\gamma_k x_k\cos\varphi_k}\right\}\approx E_{\boldsymbol{\varphi}}\left\{e^{\theta\sum_{k=1}^{n}\gamma_k x_k\ \ 1-\frac{\varphi_k^2}{2}}\right\}$$
$$=\prod_{k=1}^{n}\frac{e^{\theta\gamma_k x_k}}{\sqrt{1+\theta\gamma_k x_k\sigma_\varphi^2}}. \tag{26}$$

Since x_k is Gaussian, there exists a constant ϱ such that the probability of $|\theta\gamma_k x_k\sigma_\varphi^2|\geq\varrho$ is negligible. Then we can obtain a constant $M_\varrho\geq 1$ with which $\left(1+\theta\gamma_k x_k\sigma_\varphi^2\right)^{-\frac{1}{2}}\leq M_\varrho e^{-\frac{1}{2}\theta\gamma_k x_k\sigma_\varphi^2}$ holds. Therefore we have

$$\tilde{\Lambda}_i(\theta)\leq\lim_{n\to\infty}\frac{1}{n}\log E_i\left\{e^{\theta\left(1-\frac{1}{2}\sigma_\varphi^2\right)\sum_{k=1}^{n}\gamma_k x_k}\right\}+\log M_\varrho. \tag{27}$$

It can be shown that the Fenchel-Legendre transform of the first term on the right-hand-side of (27), is $\Lambda_i^*\left(\frac{x}{1-\frac{1}{2}\sigma_\varphi^2}\right)$, with $\Lambda_i^*(x)$ being the rate function governing the LDP associated with T_n. It follows from the definition of the Fenchel-Legendre transform in (10) that

$$\tilde{\Lambda}_i^*(x)\geq\Lambda_i^*\left(\frac{x}{1-\frac{1}{2}\sigma_\varphi^2}\right)-\log M_\varrho. \tag{28}$$

Note that the Bayesian error exponent is achieved at the intersection of both rate functions $\Lambda_0^*(T)=\Lambda_1^*(T)$. Denote the Bayesian exponent under perfect synchronization by B and that under phase error by $\tilde{B}$, we have $\tilde{B}\geq B-\log M_\varrho$, and the relative loss in Bayesian error exponent is bounded by

$$\epsilon\leq\frac{\log M_\varrho}{B}. \tag{29}$$

Generally, little performance loss can be ensured with a relatively small σ_φ. To illustrate this we take the detection of a constant signal $s_k=m$ as an example. Ignoring the probability of $|x_k|>|m|+3\sigma$, we have $\varrho=0.5\sigma_\varphi^2\left[\left(\frac{|m|}{\sigma}\right)^2+3\frac{|m|}{\sigma}\right]\frac{1-\rho}{1+\rho}$. Thus, when $\frac{|m|}{\sigma}=2$, $\rho=0.5$, $\sigma_\varphi=0.1\pi$, we obtain $\varrho=0.1648$, $M_\varrho=1.0077$, $B=0.1667$, and the relative loss $\epsilon\leq 4.6\%$. When $\sigma_\varphi=0.1$ with other parameters kept the same, the relative loss is only $\epsilon\leq 0.06\%$. Fig. 2 depicts the simulated error exponents with $\pi_0=\pi_1=1/2$, $m=1$, $\sigma=0.5$, $\rho=0.5$ and $P_{av}=1$. We observe that when $\sigma_\varphi=0.1\pi$, there is only a slight loss in the performance of MAC fusion compared with the perfectly-synchronized case. When $\sigma_\varphi=0.2\pi$, however, the performance degrades significantly, but MAC fusion still largely outperforms PAC fusion.

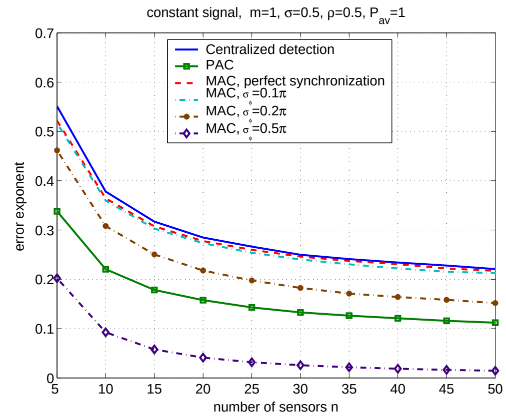

Fig. 2. Error exponents for detection of constant signal under average power constraint, $m=1$, $\sigma=0.5$, $\rho=0.5$, $P_{av}=1$

V. Conclusion

Distributed detection of a deterministic signal in correlated noise is studied in this paper. We demonstrate that with a specially-chosen mapping rule, MAC fusion is asymptotically optimal under the average power constraint, and outperforms PAC fusion under both average and total power constraint. We also remove the assumption of perfect synchronization over MAC, and show that the performance degradation is negligible when the phase mismatch among sensors is small.

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