

A Geometrical Analysis on Transmit Antenna Selection for Spatial Multiplexing Systems with Linear Receivers

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Abstract— For MIMO diversity schemes, it is well known that antenna selection methods that optimize the post-processing signal-to-noise ratio can preserve the diversity order of the full MIMO system. On the other hand, the diversity order achieved by antenna selection in spatial multiplexing (SM) systems, especially those exploiting practical coding and decoding schemes, has not thus far been rigorously analyzed. In this paper, from a geometrical standpoint, we propose a new framework to theoretically analyze the diversity order achieved by transmit antenna selection for independently encoded SM systems with linear receivers. Our results show that a diversity order of $(N_T - 1)(N_R - 1)$ can be achieved for an $N_R \times N_T$ SM system in which $L = 2$ antennas are selected from the transmit side.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) techniques are anticipated to be widely employed in future wireless communications to address the ever-increasing capacity demands. A major problem encountered by MIMO is its increased hardware cost due to the required multiple analog/RF front-ends, which has motivated the investigation of antenna selection schemes [2]. In many scenarios, judicious antenna selection may incur little or no loss in system performance while significantly reduce system cost. MIMO systems can be exploited for spatial diversity (SD) or spatial multiplexing (SM) gains [7]. Earlier works on MIMO antenna selection mainly focus on the former, in which only one data stream is transmitted, including, e.g. selection combining (SC), hybrid selection-maximum ratio combining (HS-MRC) [2][3], and antenna selection for space-time block coding [4][5]. Essentially in these works, with independent and identically distributed (i.i.d.) Rayleigh fading, the system error performance or outage probability can be readily analyzed through order statistics [18], and it has been shown that the diversity order of the full-size system can be maintained through the signal-to-noise ratio (SNR) maximization selection criterion [2][3].

On the other hand, antenna selection for spatial multiplexing schemes receives interest only recently [1][2]. Existing few analytical results generally assume capacity-achieving joint space-time coding and optimal decoding. The capacity-maximizing receive antenna selection is analyzed in [6], and

shown to achieve the same diversity order as the full-size system. The fundamental tradeoff between diversity and spatial multiplexing, revealed in [7], is argued to hold as well for MIMO systems with antenna selection in [8]. However, in practice, multiple streams in a SM system may be uncoded or separately encoded and sub-optimally decoded due to complexity concerns. In [1], several transmit antenna selection algorithms for SM systems with linear receivers are proposed, and some conjectures on the achieved diversity orders are made from numerical results. To the best of our knowledge, the exact diversity order achieved by antenna selection for practical SM systems has not been rigorously obtained. In contrast to MIMO diversity schemes, the key challenge that hinders its accurate performance analysis is that, selection is conducted among a list of inter-dependent random quantities, which are correlated in a complex manner.

In this paper, we propose a new framework to theoretically analyze the diversity order achieved by transmit antenna selection for SM systems with independently encoded layers (i.e. the V-BLAST structure [10]) and linear receivers. In particular, we rigorously show that the optimal diversity order is $(N_T - 1)(N_R - 1)$ for an $N_R \times N_T$ SM system when $L = 2$ antennas are selected from the transmit side. This should be compared with the diversity order of a two-stream V-BLAST system without antenna selection, $N_R - 1$. Such a diversity gain can be tremendous for downlink high-data-rate communications, where there can be a (potentially) large number of transmit antennas at base stations while few receive antennas at mobiles (e.g., $N_R = 2$). Generally speaking, our results ratify and generalize some of the conjectures made in [1], thus verifying the benefits of transmit antenna selection in practical SM systems: achieving high data rates with robust error performance without complex coding in fading channels.

The rest of the paper is organized as follows. System model and problem formulation are given in Section II. The main ideas of our approach are illustrated in Section III, Finally Section IV contains some concluding remarks.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We target a basic frequency non-selective block Rayleigh fading channel model, for which a SM system with transmit

antenna selection can be expressed as:

$$\mathbf{y} = \sqrt{\rho_0/L} \mathbf{H}_{SL} \mathbf{s} + \mathbf{n}, \quad (1)$$

where the $N_R \times t$ matrix \mathbf{y} is the received signal block; the $N_T \times t$ matrix \mathbf{s} represents the transmitted signal block; \mathbf{H}_{SL} contains the L columns selected from the original $N_R \times N_T$ channel matrix $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_{N_T}]$, and \mathbf{n} is the background noise. Both \mathbf{H} and \mathbf{n} are modeled with i.i.d. normalized complex Gaussian entries, while the independent transmitted signals (across the antennas) assume unit average energy per symbol per antenna. Therefore, ρ_0 stands for the average SNR per receive antenna. Throughout the paper we assume $N_T \geq L$ and $N_R \geq L$. Here t is the coding length in each layer, which is actually irrelevant in our study, as layered one-dimensional coding can provide coding gain but not diversity gain.

A zero-forcing (ZF) receiver is assumed, with the understanding that its diversity order analysis applies largely to MMSE receivers for independently encoded SM systems at high SNR. A space equalizer $\mathbf{G} = \mathbf{H}_{SL}^\dagger$ is then applied to the received signal \mathbf{y} to obtain an estimate of the transmitted symbol vector $\hat{\mathbf{s}} = \mathbf{G}\mathbf{y} = \sqrt{\rho_0/L} \mathbf{L} \mathbf{s} + \mathbf{G}\mathbf{n}$. The post-processing SNR of the k th substream is proportional to the squared projection height¹ from \mathbf{h}_k to the space spanned by the other $L-1$ selected column vectors [1][11]. The diversity order of a communication system is defined as the slope of its error probability $P_e(\rho_0)$ in log-scale at high SNR regimes [7]:

$$d = - \lim_{\rho_0 \rightarrow \infty} \frac{\log P_e(\rho_0)}{\log(\rho_0)} = \lim_{\rho_0 \rightarrow \infty} \frac{\log P_e(\rho_0)}{\log(1/\rho_0)}. \quad (2)$$

Also, we adopt the operator \doteq as defined in [7], to denote exponential equality, i.e. we use $f(x) \doteq x^b$ to represent $\lim_{x \rightarrow 0} \frac{\log f(x)}{\log x} = b$. The operators \leq , \geq are similarly defined. Note that according to our notation, $f(x) \leq g(x)$ indicates $f(x) \geq g(x)$ for sufficiently small x .

Lemma I: *For independently encoded spatial multiplexing systems with ZF receivers, the antenna selection method that chooses the antenna subset with the strongest weakest data link, achieves the optimal diversity order among all the antenna selection methods.*

Proof: The conditional error probability of a layered SM system with a linear ZF receiver, after transmit antenna selection, can be upper and lower bounded as

$$P_{e_max}(\mathbf{H}) \leq P_e(\mathbf{H}) \leq \sum_{l=1}^L P_{e_l}(\mathbf{H}),$$

where $P_{e_l}(\mathbf{H})$ is the error probability of the l th selected sub-

stream, and $P_{e_max}(\mathbf{H})$ represents the worst of them. It is easily seen that

$$P_e \doteq P_{e_max} = E_{\mathbf{H}} \{P_{e_max}(\mathbf{H})\},$$

and Lemma I follows. For details please see [19]. ■

For $L=2$, supposing that antennas k and j are selected, we have the following expression for the post-processing SNR:

$$\rho_k = (\rho_0/L) R_{kj} = (\rho_0/L) \|\mathbf{h}_k\|^2 \sin^2 \theta_{kj}, \quad (3)$$

where θ_{kj} is the angle between \mathbf{h}_k and \mathbf{h}_j , which is independent with $\|\mathbf{h}_k\|^2$ [13][14]. It can be shown that [11][13] $\|\mathbf{h}_k\|^2$ are i.i.d. $\chi^2(2N_R)$ distributed and the angles between any two column vectors assume a probability density function (PDF) of

$$f_\theta(\theta) = 2(N_R - 1) \sin 2\theta (\sin \theta)^{2N_R - 4}, \quad \theta \in (0, \frac{\pi}{2}). \quad (4)$$

Furthermore, $R_{kj} = \|\mathbf{h}_k\|^2 \sin^2 \theta_{kj}$ is a $\chi^2(2(N_R - 1))$ distributed random variable [11][16], so the diversity order without antenna selection is $N_R - 1$, while transmit antenna selection achieves a product gain of $N_T - 1$ as shown below.

According to Lemma I, to achieve the optimal diversity order, the antenna selection rule considered in this paper is to select the antenna subset with the largest minimum projection height, denoted as $R_{SL} = \max_{k \neq j \in \{1, \dots, N_T\}} \{\min\{R_{kj}, R_{jk}\}\}$. It is known [7][15] that the error probability is dominated by the outage probability at high SNR, so the diversity order can be further evaluated as

$$d = \lim_{x \rightarrow 0} \frac{-\log \Pr(R_{SL} \leq x)}{\log x}. \quad (5)$$

Note that neither the exact PDF of R_{SL} nor its polynomial expansion near zero seems tractable, which motivates us to solve the problem through tight upper and lower bounds.

III. DIVERSITY ORDER FOR $L=2$

In this section, we discuss the main idea of our geometrical approach through the $L=2$ case. We will rigorously derive the following theorem.

Theorem I: *In an $N_R \times N_T$ layered spatial multiplexing system with linear ZF/MMSE receivers satisfying $N_T \geq 2$ and $N_R \geq 2$, if $L=2$ independently encoded data streams are transmitted from two selected antennas, the optimal achievable diversity order is $(N_T - 1)(N_R - 1)$.*

By definition,

¹ Projection height refers to the norm of the error vector, i.e., the difference between a vector and its projection onto a subspace.

$$\begin{aligned} \Pr(R_{SL} \leq x) &= \Pr(\min(R_{12}, R_{21}) \leq x, \dots, \min(R_{(N_T-1)N_T}, R_{N_T(N_T-1)}) \leq x) \\ &= \Pr\left(\bigcup_{i=1}^N A_i\right), \end{aligned}$$

where the $N = 2^{N_U}$ events $\{A_i\}$ are defined as:

$$\begin{aligned} A_1 &= \{R_{12} \leq x, R_{13} \leq x, \dots, R_{1N_T} \leq x, R_{23} \leq x, \dots, R_{(N_T-1)N_T} \leq x\} \\ A_2 &= \{R_{12} \leq x, R_{13} \leq x, \dots, R_{1N_T} \leq x, R_{23} \leq x, \dots, R_{N_T(N_T-1)} \leq x\} \\ &\vdots \\ A_N &= \{R_{21} \leq x, R_{31} \leq x, \dots, R_{N_T1} \leq x, R_{32} \leq x, \dots, R_{N_T(N_T-1)} \leq x\}. \end{aligned} \quad (6)$$

Intuitively, the selection rule of Lemma I dictates that at least one element from each of the possible subsets should be in outage. Clearly we have:

$$\max \Pr(A_i) \leq \Pr(R_{SL} \leq x) \leq \sum_{i=1}^N \Pr(A_i), \quad (7)$$

which indicates $d = \lim_{x \rightarrow 0} \frac{\log \max \Pr(A_i)}{\log(x)}$. However, it is generally difficult to identify critical terms (which dominate others at high SNR) from (6), whose cardinality grows exponentially with N_T^2 . Alternatively, we take the following approach. First, we find a common upper bound for $\Pr(A_i)$, which determines a diversity order lower bound. We then evaluate the error exponential of $\Pr(A_i)$ (more precisely one of its lower bounds) and give an upper bound for d . It turns out that these two bounds coincide and represent the best achievable diversity order. The following Lemma is useful for obtaining an explicit lower bound of the diversity order.

Lemma II: For any permutation of $1 \sim N_T$, denoted as $k_1 \sim k_{N_T}$, we have

$$\Pr(R_{k_1 k_2} \leq x, R_{k_2 k_3} \leq x, \dots, R_{k_{(N_T-1)k_{N_T}}} \leq x) = [\Pr(R_{k_j} \leq x)]^{(N_T-1)}, \quad \forall k \neq j.$$

Proof: We need to show that random variables in the sequence $R_{k_1 k_2}, R_{k_2 k_3}, \dots, R_{k_{(N_T-1)k_{N_T}}}$ are jointly independent. Essentially $R_{k_i k_{i+1}}$ is only a function of \mathbf{h}_{k_i} and $\mathbf{h}_{k_{i+1}}$, denoted as $R_{k_i k_{i+1}} = g(\mathbf{h}_{k_i}, \mathbf{h}_{k_{i+1}})$, therefore the conditional PDF of $R_{k_i k_{i+1}}$ given those appearing earlier in the sequence admits:

$$\begin{aligned} f(R_{k_i k_{i+1}} | R_{k_{i-1} k_i}, \dots, R_{k_1 k_2}) &= f(g(\mathbf{h}_{k_i}, \mathbf{h}_{k_{i+1}}) | g(\mathbf{h}_{k_{i-1}}, \mathbf{h}_{k_i}), \dots, g(\mathbf{h}_{k_1}, \mathbf{h}_{k_2})) \\ &= f(g(\mathbf{h}_{k_i}, \mathbf{h}_{k_{i+1}}) | g(\mathbf{h}_{k_{i-1}}, \mathbf{h}_{k_i})) = f(R_{k_i k_{i+1}} | R_{k_{i-1} k_i}), \end{aligned}$$

where the second equality holds because the states of $\mathbf{h}_{k_1} \sim \mathbf{h}_{k_{i-1}}$ do not affect \mathbf{h}_{k_i} and $\mathbf{h}_{k_{i+1}}$. Therefore, the above sequence forms a Markov chain. We are left to prove the independence between $R_{k_i k_{i+1}}$ and $R_{k_{i-1} k_i}$. Given $R_{k_i k_{i+1}} = \|\mathbf{h}_{k_i}\|^2 \sin^2 \theta_{k_{i+1} k_i}$ and $R_{k_{i-1} k_i} = \|\mathbf{h}_{k_{i-1}}\|^2 \sin^2 \theta_{k_i k_{i-1}}$, because of the independence between $\|\mathbf{h}_{k_i}\|^2$ and $\|\mathbf{h}_{k_{i-1}}\|^2$, and between

vector norms and directions (angles) [13][14], we only need to show that $\theta_{k_{i+1} k_i}$ and $\theta_{k_i k_{i-1}}$ are independent.

Following a similar rotation approach as in [11], we define $\mathbf{e}_1 \sim \mathbf{e}_{N_R}$ as a *fixed* orthonormal basis (e.g., Cartesian coordinates) of the vector space \mathbb{C}^{N_R} . We rotate $[\mathbf{h}_{k_{i-1}}, \mathbf{h}_{k_i}, \mathbf{h}_{k_{i+1}}]$ as a whole so that \mathbf{h}_{k_i} is parallel to \mathbf{e}_1 , denoted as $[\tilde{\mathbf{h}}_{k_{i-1}}, \tilde{\mathbf{h}}_{k_{i+1}}] = \mathbf{Q}(\psi_{k_i})[\mathbf{h}_{k_{i-1}}, \mathbf{h}_{k_{i+1}}]$, where ψ_{k_i} is the angle between \mathbf{h}_{k_i} and \mathbf{e}_1 , and $\mathbf{Q}(\psi_{k_i})$ is the corresponding unitary rotation matrix. Since $[\mathbf{h}_{k_{i-1}}, \mathbf{h}_{k_{i+1}}]$ is an i.i.d. Gaussian matrix (therefore the joint distribution is rotationally invariant) and is independent with ψ_{k_i} , $[\tilde{\mathbf{h}}_{k_{i-1}}, \tilde{\mathbf{h}}_{k_{i+1}}]$ is still i.i.d. Gaussian. Because $\theta_{k_{i+1} k_i}$ and $\theta_{k_i k_{i-1}}$ are unchanged after the rotation, and equal to the angles between $\tilde{\mathbf{h}}_{k_{i+1}}$ and \mathbf{e}_1 , and between $\tilde{\mathbf{h}}_{k_{i-1}}$ and \mathbf{e}_1 , respectively (see Figure 1), given the fact that $\tilde{\mathbf{h}}_{k_{i-1}}$ and $\tilde{\mathbf{h}}_{k_{i+1}}$ are independent, it is straightforward to show that $\theta_{k_{i+1} k_i}$ and $\theta_{k_i k_{i-1}}$ are independent, so are $R_{k_i k_{i+1}}$ and $R_{k_{i-1} k_i}$, and Lemma II follows. ■

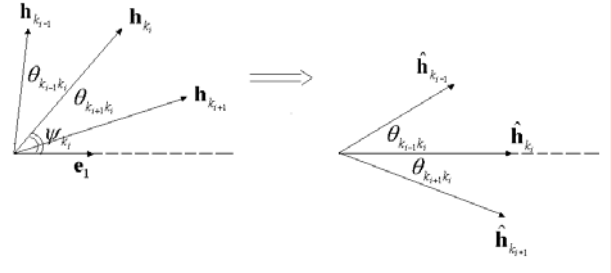


Figure 1. Independence of $\theta_{k_{i+1}k_i}$ and $\theta_{k_i k_{i-1}}$.

By applying the same rotation approach, it is straightforward to derive the following corollary:

Corollary I: $\theta_{12}, \theta_{13}, \dots, \theta_{1N_T}$ are jointly independent.

Given Lemma II, a lower bound for the optimal diversity order (or an upper bound of the error performance at high SNR) is in order.

Proposition I: The diversity order defined in (5) is lower bounded as $d \geq (N_T - 1)(N_R - 1)$.

Proof: A key observation is that in any A_i we can always find an independent subset of $N_T - 1$ random variables bearing the same form as in Lemma II. For example, in A_1 , such a subset is given by $\{R_{12}, R_{23}, \dots, R_{(N_T-1)N_T}\}$. By Lemma II and the distribution of R_{k_j} as discussed above, we can get $\max_i \Pr(A_i) \geq x^{(N_T-1)(N_R-1)}$, and Proposition I follows. ■

To find an upper bound for the optimal diversity order we choose to evaluate $\Pr(A_1)$. Intuitively A_1 contains the most correlated terms (projection heights from the same column vectors) and could potentially be one of the critical terms. The following result is obtained after some technically involved calculations, which verifies our conjecture.

Proposition II: *The diversity order defined in (5) is upper bounded as $d \leq (N_T - 1)(N_R - 1)$.*

Proof: $\Pr(A_1)$ can be expressed as

$$\begin{aligned} \Pr(A_1) &= \Pr(R_{12} \leq x, R_{13} \leq x, \dots, R_{(N_T-1)N_T} \leq x) \\ &= \Pr(\|\mathbf{h}_1\|^2 \sin^2 \theta_{12} \leq x, \|\mathbf{h}_1\|^2 \sin^2 \theta_{13} \leq x, \dots, \|\mathbf{h}_{N_T-1}\|^2 \sin^2 \theta_{(N_T-1)N_T} \leq x). \end{aligned}$$

By defining $z = \sum_{k=1}^{N_T-1} \|\mathbf{h}_k\|^2$, which is distributed as $\chi^2(2(N_T - 1)N_R)$, and $\psi_0 = (\pi/2)/(N_T - 1)$, we have:

$$\begin{aligned} \Pr(A_1) &\geq \Pr(z \sin^2 \theta_{12} \leq x, \dots, z \sin^2 \theta_{1N_T} \leq x, z \sin^2 \theta_{23} \leq x, \dots, z \sin^2 \theta_{(N_T-1)N_T} \leq x) \\ &\geq \Pr(z \sin^2 \theta_{12} \leq x, \dots, 0 < \theta_{12} < \psi_0, 0 < \theta_{13} < \psi_0, \dots, 0 < \theta_{1N_T} < \psi_0), \end{aligned} \quad (8)$$

where for the second inequality we have further restricted the ranges of the i.i.d. random variables $\theta_{12} \sim \theta_{1N_T}$ within $(0, \psi_0)$ (c.f. Corollary I). Based on the geometric structure involved, given θ_{12} and θ_{13} , θ_{23} is constrained as $\theta_{23} \leq \theta_{12} + \theta_{13}$, where the equality holds only when $\mathbf{h}_1 \sim \mathbf{h}_3$ are linearly dependent (located in the same subspace with a dimension less than 3) [17]. Then within the range $(0, \psi_0)$, we have

$$\sin^2 \theta_{23} \leq \sin^2(\theta_{12} + \theta_{13}) \leq \sin^2 \theta_\Sigma, \quad (9)$$

where $\theta_\Sigma = \sum_{k=2}^{N_T} \theta_{1k}$ is still in the range of $(0, \pi/2)$. Similar results hold for $\sin^2 \theta_{24} \sim \sin^2 \theta_{(N_T-1)N_T}$. Therefore (8) can be further lower bounded as:

$$\begin{aligned} \Pr(A_1) &\geq \Pr(z \sin^2 \theta_\Sigma \leq x, 0 < \theta_{12} < \psi_0, 0 < \theta_{13} < \psi_0, \dots, 0 < \theta_{1N_T} < \psi_0) \\ &\doteq \Pr(z \sin^2 \theta'_\Sigma \leq x), \end{aligned} \quad (10)$$

where we define a new set of i.i.d. random variables $\theta'_{12} \sim \theta'_{1N_T}$ with PDF of

$$f_{\theta'_{1i}}(x) = \frac{f_{\theta_{1i}}(x)}{\int_0^{\psi_0} f_{\theta_{1i}}(x) dx} = \frac{f_{\theta_{1i}}(x)}{C}, \quad 0 < x < \psi_0, \quad 2 \leq i \leq N_T, \quad (11)$$

i.e., the restriction of $\theta_{12} \sim \theta_{1N_T}$ in the range of $(0, \psi_0)$, and

$$\theta'_\Sigma = \sum_{i=2}^{N_T} \theta'_{1i}.$$

Direct evaluation of (10) still seems intractable due to the involved PDF expression of θ'_Σ . Alternatively, we further simplify it with some lemmas on exponential equivalence given in Appendix, whose proofs can be found in the journal version of this paper [19]. Specifically, with $\theta'_0 = \max_k \theta'_{1k}$ and $m(x) = \sin^2(x)$ for $x \in (0, \psi_0)$, by Lemma III in Appendix, we have

$$\Pr(\sin^2 \theta'_\Sigma \leq x) \doteq \Pr(\sin^2 \theta'_0 \leq x). \quad (12)$$

Further by Lemma IV and Lemma V in Appendix, we get

$$\Pr(z \sin^2 \theta'_\Sigma \leq x) \doteq \Pr(z \sin^2 \theta'_0 \leq x) \doteq \Pr(z \sin^2 \theta_0 \leq x). \quad (13)$$

where $\theta_0 = \max_k \theta_{1k}$. We are then left to evaluate the smallest exponential in $\Pr(z \sin^2 \theta_0 \leq x)$. Note that z is a $\chi^2(2(N_T - 1)N_R)$ distributed random variable with cumulative distribution function (CDF):

$$F_z(x) = 1 - e^{-x} \sum_{k=0}^{M+N_T-2} \frac{x^k}{k!}, \quad M = (N_T - 1)(N_R - 1),$$

while the PDF of θ_0 is given by

$$f_{\theta_0}(\theta) = M[\sin^2 \theta]^{M-1} \sin 2\theta, \quad \theta \in (0, \frac{\pi}{2}).$$

After some tedious mathematical manipulations (whose detailed description can also be found in [19]), we can get the following equivalent polynomial form as $x \rightarrow 0$:

$$\Pr(z \sin^2 \theta_0 \leq x) = \left(\frac{1}{M!} - M \sum_{k=0}^{N_T-3} \frac{k!}{(M+k+1)!} \right) x^M + o(x^M), \quad (14)$$

where the coefficient of x^M in (14) is always positive, which completes the proof of Proposition II. ■

With Proposition I and II, Theorem I is proved.

For a simple example of $L = 2$, $N_T = 3$, $N_R = 3$, from (14) we get as $x \rightarrow 0$

$$\Pr(z \sin^2 \theta_0 \leq x) = \frac{x^4}{120} + o(x^5).$$

Figures 2 presents relevant quantities, verifying a diversity order of 4.

Remark: A conjecture on the diversity order of independently encoded SM systems with transmit antenna selection and ZF receivers was made in [1] based on numerical results, which actually has motivated our research:

Conjecture I [1]: *For ZF receivers, when $N_R = L$, the achievable diversity order is $N_T - L + 1$.*

Our results prove its correctness for $L=2$ and further extend it to general N_R scenarios.

IV. CONCLUSIONS

In this paper, we propose a geometrical framework to analyze the diversity order achieved by transmit antenna selection for practical spatial multiplexing systems with linear receivers, and rigorously derive the diversity order for the $L=2$ case. Our results prove and extend the previous conjectures in literature drawn from simulations, and verify the predicted potential of antenna selection for practical spatial multiplexing systems. The extension to general L scenarios and decision feedback receivers can be found in the journal version of this paper [19].

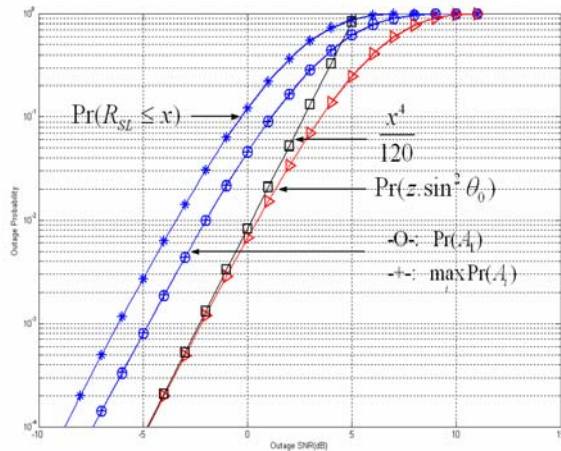


Figure 2. The Exponential Behavior of $\Pr(A_1)$ for the $N_T = 3, N_R = 3, L = 2$ Scenario.

Appendix: Some Lemmas on Exponential Equivalence in the Proof of Proposition II.

Lemma III: Let $m(\theta)$ be a positive function of θ and monotonically increases with θ , satisfying $m(0) = 0$ and $m^{-1}(x) \doteq x^{n_0}$. If $\theta_1, \dots, \theta_K$ are independent positive random variables, whose cumulative distribution functions admit $F_{\theta_k}(x) \doteq x^{n_k}$, we have

$$\Pr \left[m \left(\sum_{k=1}^K \theta_k \right) \leq x \right] \doteq \Pr \left[m \left(\max_k \theta_k \right) \leq x \right] \doteq x^{n_0 \sum_{k=1}^K n_k}.$$

Lemma IV: For independent continuous random variables a , b_1 and b_2 satisfying $a \geq 0$ with $\Pr(a \leq x) \doteq x^{n_a}$, and $0 \leq b_1, b_2 \leq 1$ with $\Pr(b_1 \leq x) \doteq \Pr(b_2 \leq x) \doteq x^{n_b}$, we have

$$\Pr(ab_1 \leq x) \doteq \Pr(ab_2 \leq x) \leq x^{n_a}.$$

Lemma V: Given the corresponding definitions, assuming $\theta_0 = \max_k \theta_{1k}$ we have

$$\Pr(z \sin^2 \theta'_0 \leq x) \doteq \Pr(z \sin^2 \theta_0 \leq x).$$

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