

# Transmission Throughput of Decentralized Overlaid Networks with Outage Constraints

Chengzhi Li and Huaiyu Dai  
Department of Electrical and Computer Engineering  
North Carolina State University, Raleigh, NC  
email: {cli3, hdai}@ncsu.edu

**Abstract**—Overlaid networks, typically composed of primary and secondary networks, are emerging as a viable candidate to resolve the conflict between increasing demand for spectrum and spectrum shortage. The essential purpose of overlaid networks is to improve the network spectrum efficiency, which, however, may not be achieved if the secondary network is inappropriately configured. In decentralized overlaid networks, the throughput of the primary network will decrease due to the extra interference from the secondary network, and its loss may not be compensated by the throughput gained by the secondary network. This motivates us to study the overall throughput of the overlaid networks. In particular, we examine the overall transmission throughput, which is a variation of the transmission capacity, a popular metric in the study of decentralized networks. Our study provides a sufficient condition for the secondary network setting such that the overall throughput is improved over that of a stand-alone primary network. In addition both the maximal allowable secondary density and the optimal secondary density which maximizes the overall throughput are derived.

## I. INTRODUCTION

Overlaid networks have tremendous potential to alleviate the spectrum shortage problem and achieve remarkable spectrum efficiency; the inherent spectrum sharing mechanism also provides a flexible way of spectrum management. One prominent example is cognitive radio networks, where secondary (unlicensed) users are allowed to temporarily access spectrum that is not currently used by primary (licensed) users. It is generally preferable that the operation of the secondary network is transparent to the primary network, which requires that the interference incurred by secondary operations be constrained within an acceptable level.

It is of great interest to understand to what extent the secondary network can gain in transmission of its own useful data, without harming the regular operation of the primary network significantly. Relevant research in literature mainly targets either the scaling law of transport capacity [1]–[3] or transmission capacity with outage consideration [4]–[7]. The former was proposed in the seminal work [8] and defined as the maximum bit-meters per second the network can achieve in aggregate; pioneer work in the capacity study of overlaid or cognitive radio networks [1]–[3] showed that there is no performance loss for the secondary network in terms of *scaling law* of transport capacity. Nonetheless, asymptotic analysis on the scaling law only characterizes the (rough) relationship between capacity and network size, neglecting the effect of many important system parameters. As an alternative,

transmission capacity [9] quantifies the maximum allowable spectrum efficiency per unit area under some outage probability constraint, and sparks an enormous interest recently (see [10] for an overview). The outage probability of the secondary network was studied in [4], [5]. It was pointed out in [7] that transmission capacity is not very applicable in the secondary network since the maximal outages for both networks generally can not be reached simultaneously. Instead, [7] proposed the concept of the *achievable* transmission capacity, defined as the product of the spatial density and the *actual* outage probability. The achievable transmission capacity is the metric of our interest in this work, and renamed as *transmission throughput* to avoid possible confusion.

Overlaid networks aim to improve the network spectrum efficiency by exploiting the “white space” in spectrum. Nevertheless, the secondary network should be configured appropriately to achieve its goal; otherwise the overall throughput of the overlaid networks, i.e., the sum throughput of the primary and secondary network, could potentially be hurt. In this work we endeavor to quantitatively characterize the “appropriate” secondary setting by investigating the *overall transmission throughput* of *decentralized* overlaid networks with outage constraints imposed on both primary and secondary networks. Note that the primary outage constraint, which should be satisfied by both networks, only guarantees the minimal acceptable performance for the primary network, and the actual primary throughput depends on its instantaneous outage probability and transmitter densities of both networks. Intuitively the actual primary throughput may decrease due to the additional interference incurred by the secondary network, and this loss may not be compensated by the throughput gain achieved in the secondary network. In [7] the *secondary* throughput was optimized with respect to the secondary transmitter density, while the primary one was ignored. The throughput ratio of the overlaid networks to the stand-alone primary one was derived in [6], with no analysis on whether this ratio is always larger than 1. In contrast, our study yields a sufficient condition for the secondary network configuration such that the overall (transmission) throughput is boosted over that of the stand-alone primary network. Additionally both the maximal allowable secondary density and the optimal secondary density maximizing the overall throughput are derived.

The remainder of this paper is organized as follows. The system model is given in Section II, followed by discussions

on the allowable transmitter density region for the two overlaid networks in Section III. The overall transmission throughput is investigated in Section IV. Finally the conclusion and future directions are provided in Section V.

## II. SYSTEM MODEL

We assume the primary and secondary transmitters are distributed in the same two-dimensional plane, and their positions are modeled as two stationary Poisson Point Processes; the former is denoted by  $\Pi_o^t = \{X_o(i)\} \subset \mathcal{R}^2$  with density  $\lambda_{ot}$  and the latter by  $\Pi_s^t = \{X_s(i)\} \subset \mathcal{R}^2$  with density  $\lambda_{st}$ . Primary (secondary) transmitter  $i$  ( $j$ ) is paired with a receiver located at  $Y_o(i)$  ( $Y_s(j)$ ). No cooperation between the primary and secondary network is allowed and the common assumptions about their intra-network communications are given below:

- All the primary (secondary) transmitters use the same transmission power  $P_o$  ( $P_s$ ), which supports a fixed transmission range  $l_o$  ( $l_s$ ). And the power ratio between the primary and secondary network is denoted by  $\theta = \frac{P_o}{P_s}$ . Concurrent primary and secondary transmissions are simply treated as interference. Thermal noise is assumed negligible in this interference-limited scenario.
- For both networks large-scale path loss and small-scale Rayleigh fading are considered. Particularly the channel power gain for a communication link of length  $r$  is given by  $g(r) = r^{-\alpha}u$ , where  $\alpha > 2$  is the path loss exponent, and  $u$  is exponentially distributed with *unit* mean. The Signal to Interference Ratio (SIR) at a primary receiver  $y_o$ ,  $r_o$ -distance away from its transmitter  $x_o$ , is given by

$$SIR_o(r_o) = \frac{P_o g(r_o)}{I_o + I_{so}}, \quad (1)$$

where  $I_o = \sum_{X_o(k) \in \Pi_o^t \setminus \{x_o\}} P_o g(\|X_o(k) - y_o\|)$  ( $I_{so} = \sum_{X_s(k) \in \Pi_s^t} P_s g(\|X_s(k) - y_o\|)$ ) is the sum of interference power from concurrent primary (secondary) transmitters, and  $\|\cdot\|$  is Euclidean norm. The SIR at a secondary receiver,  $SIR_s$ , is defined similarly as

$$SIR_s(r_s) = P_s g(r_s) / (I_s + I_{os}), \quad (2)$$

where  $I_s$  ( $I_{os}$ ) is the sum of interference power from concurrent secondary (primary) transmitters to a secondary receiver,  $r_s$ -distance away from its corresponding transmitter.

- The primary (secondary) transmission is successful if the  $SIR_o$  ( $SIR_s$ ) is no less than a threshold  $T_o$  ( $T_s$ ), assumed fixed in our study. The transmission rate is a deterministic function of this threshold  $R_o = f(T_o)$  ( $R_s = f(T_s)$ ).
- There are outage constraints imposed on primary and secondary transmission links, given by

$$\Pr(SIR_o(l_o) < T_o) \leq \epsilon_o, \quad (3)$$

$$\Pr(SIR_s(l_s) < T_s) \leq \epsilon_s, \quad (4)$$

respectively, where  $l_o$  and  $l_s$  are primary and secondary transmission range and  $\epsilon_o (< 0.5)$  and  $\epsilon_s (< 0.5)$  are predetermined small numbers.

- For convenience of analysis, it is typically assumed that a primary (secondary) receiver is located at the origin, which does not change the statistics of homogenous P.P.P. according to the Slivnyak's theory [11].

*Remark 1:* The settings of the primary network such as its density and power are assumed fixed without considering the accommodation of the secondary network. In contrast, the secondary configuration can be tuned to improve the network spectrum efficiency, provided the primary outage constraint is satisfied.

In this *decentralized* framework, the metric transmission capacity receives increasing interest recently, defined as the maximum density of successful transmissions subject to an outage constraint. In our study, we will mainly consider the scenario where the transmitter density of a network is given, and thus will use the term ‘‘throughput’’ to avoid possible confusion. In particular, for a Poisson network with transmitter density  $\lambda$ , a typical link length  $r$ , a pre-determined SIR threshold  $T$  and transmission rate  $R$ , the transmission throughput is defined as:

$$\bar{C}(\lambda) = R\lambda(1 - \delta(\lambda, r)), \quad (5)$$

where the outage probability  $\delta(\lambda, r) = \Pr(SIR(r) < T)$ . The transmission capacity with outage constraint  $\epsilon$  [9], [10] is defined as  $C = R\lambda_\epsilon(1 - \epsilon)$ , where  $\lambda_\epsilon$  is the maximal density satisfying the outage constraint, i.e.,  $\delta(\lambda_\epsilon, r) = \epsilon$ . In general,  $C$  is different from  $\max_\lambda(\bar{C}(\lambda))$  subject to the outage constraint  $\delta(\lambda, r) \leq \epsilon$ , though they are close to each other when  $\epsilon$  is small.

*Remark 2:* In overlaid networks, transmission capacity may not be investigated directly since the maximal allowable outage of the two overlaid networks may not be reached simultaneously. In contrast, transmission throughput is well defined in either homogenous or heterogenous networks, while keeping the spirit of the transmission capacity.

## III. DENSITY REGION

Due to the primary and secondary outage constraints, the primary and secondary *transmitter* densities are constrained; the corresponding feasible density region is defined as  $\{(\lambda_{ot}, \lambda_{st})\}$ . Since the secondary network should remain transparent to the primary network, it can be easily derived from Lemma 3.1 in [12] that

$$\lambda_{ot} \leq \lambda_{ot,m} = \frac{-\ln(1 - \epsilon_o)}{K_\alpha T_o^{2/\alpha} l_o^2}, \quad (6)$$

where  $K_\alpha = \frac{2\pi^2}{\alpha \sin(2\pi/\alpha)}$ , regardless of the existence of the secondary network. The secondary network can be accommodated only when  $\lambda_{ot} < \lambda_{ot,m}$ ; otherwise, there is no chance for a secondary network to survive. Given  $\Delta\lambda_{ot} = \lambda_{ot,m} - \lambda_{ot} > 0$ , it is the secondary network that is responsible for satisfying the primary outage constraint, as well as its own. To satisfy the primary outage constraint, the secondary transmitter density should be bounded above by:

$$\lambda_{st} \leq \lambda_{st,m1} = \theta^{2/\alpha} \Delta\lambda_{ot}. \quad (7)$$

The derivation is given in Appendix A. Note that  $\lambda_{st,m1}$  is proportional to the power ratio  $\theta$  and the “white space”  $\Delta\lambda_{ot}$ .

In order to satisfy the secondary outage constraint (c.f. Appendix A), the secondary transmitter density should be further upper bounded by :

$$\lambda_{st} \leq \lambda_{st,m2} = \lambda_{st,m1} - \lambda_{ot}\theta^{2/\alpha}, \quad (8)$$

where  $\lambda_{st,m1} = \frac{-\ln(1-\epsilon_s)}{K_\alpha T_s^{2/\alpha} l_s^2}$ , represents the maximal allowable secondary transmitter density in a stand-alone setting (c.f. Eq. (6)), and the second term accounts for the additional constraints from the primary network. Intuitively, with increased interference from the primary network, the secondary transmitter density needs to be decreased to maintain the communication quality. From Eq. (8) an upper bound for  $\theta$  can be deduced:

$$\theta < \theta_{\max} \triangleq \left( \frac{\lambda_{st,m1}}{\lambda_{ot}} \right)^{\alpha/2}. \quad (9)$$

Considering both Eq. (7) and (8) a feasible secondary transmitter density should satisfy

$$\lambda_{st} \leq \lambda_{st,mm} \triangleq \min(\lambda_{st,m1}, \lambda_{st,m2}). \quad (10)$$

*Proposition 1:*  $\lambda_{st,mm}$  achieves the maximum  $\frac{\lambda_{st,m1}}{\lambda_{ot,m}} \Delta\lambda_{st}$  when  $\theta = \theta_{opt} \triangleq \left( \frac{\lambda_{st,m1}}{\lambda_{ot,m}} \right)^{\alpha/2}$ .

*Proof:* Given  $\lambda_{ot}$ ,  $\lambda_{st,m1}$  increases with  $\theta$ , while  $\lambda_{st,m2}$  decreases with  $\theta$ . Therefore  $\lambda_{st,mm}(\lambda_{ot})$  is maximized when  $\lambda_{st,m2}(\lambda_{ot}) = \lambda_{st,m1}(\lambda_{ot})$ , which leads to  $\theta_{opt} = \left( \frac{\lambda_{st,m1}}{\lambda_{ot,m}} \right)^{\alpha/2}$ , independent of  $\lambda_{ot}$ . The corresponding maximum value is obtained after  $\theta_{opt}$  is plugged in. ■

An exemplary density region is shown in Fig. 1, where  $T_o = 3, T_s = 2, r_o = 1, r_s = 0.8, \epsilon_o = \epsilon_s = 0.05, \alpha = 4$ , and  $\theta = 2$ . We can see that the lines of  $\lambda_{st,m1}$  and  $\lambda_{st,m2}$  share the same slope  $-\theta^{2/\alpha} = -\sqrt{2}$  with respect to  $\lambda_{ot}$ , and the density region is determined by Eq. (7) in this case. With the increase of  $\theta$ , the curve of  $\lambda_{st,m1}$  rotates around the point of  $(\lambda_{ot,m}, 0)$  clockwise, while that of  $\lambda_{st,m2}$  rotates around the point of  $(0, \lambda_{st,m})$  clockwise. When  $\theta = \left( \frac{\lambda_{st,m1}}{\lambda_{ot,m}} \right)^{\alpha/2} = 3.66$ , two lines coincide (the dashed line in Fig. 1), and  $\lambda_{st,mm}$  (as well as the density region) is maximized. Note that the maximal secondary transmission density can be interpreted as the secondary transmission capacity when the (constant) transmission rate and outage probability are left out.

In the following discussion, we assume all densities of interest  $(\lambda_{ot}, \lambda_{st})$  are within the feasible density region.

#### IV. TRANSMISSION THROUGHPUT OF THE OVERLAID NETWORKS

In this section we optimize the sum transmission throughput of two overlaid networks, with respect to the secondary transmitter density. Our investigation reveals two interesting results: 1) a sufficient condition for the secondary transmitter density such that the overall throughput is larger than the throughput of a stand-alone primary network; 2) the optimal secondary transmitter density  $\lambda_{st}^{opt}$  such that the overall throughput is maximized.

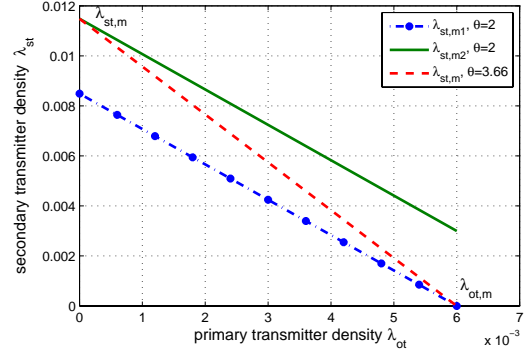


Fig. 1: An exemplary density region  $(\lambda_{ot}, \lambda_{st})$

According to Appendix A, the outage probabilities for the primary and secondary network are given by:

$$\delta_o(\lambda_{ot}, l_o) = 1 - \exp[-K_\alpha T_o^{2/\alpha} l_o^2 (\lambda_{st} \frac{1}{\theta^{2/\alpha}} + \lambda_{ot})],$$

$$\delta_s(\lambda_{st}, l_s) = 1 - \exp[-K_\alpha T_s^{2/\alpha} l_s^2 (\lambda_{ot} \theta^{2/\alpha} + \lambda_{st})].$$

Thus the overall transmission throughput is given by:

$$\begin{aligned} \bar{C}_2 &= \bar{C}_o(\lambda_{ot}) + \bar{C}_s(\lambda_{st}) \\ &= R_o \lambda_{ot} (1 - \delta_o(\lambda_{ot}, l_o)) + R_s \lambda_{st} (1 - \delta_s(\lambda_{st}, l_s)) \\ &= R_o \lambda_{ot} \exp[-K_\alpha T_o^{2/\alpha} l_o^2 (\lambda_{st} \frac{1}{\theta^{2/\alpha}} + \lambda_{ot})] \\ &\quad + R_s \lambda_{st} \exp[-K_\alpha T_s^{2/\alpha} l_s^2 (\lambda_{ot} \theta^{2/\alpha} + \lambda_{st})]. \end{aligned}$$

In the following we consider  $\bar{C}_2$  as a function of  $\lambda_{st}$ ,  $\bar{C}_2(\lambda_{st})$ , while fix all the other parameters. Note that  $\bar{C}_2(0) > 0$  is the original throughput of the primary network before the secondary network is deployed, and  $\bar{C}_2(\lambda_{st})$  ( $\lambda_{st} > 0$ ) is the sum throughput of the overlaid networks with  $\bar{C}_2(\infty) = 0$ . Our goal is to find out what conditions will guarantee that there exists a  $\lambda_{st} > 0$  such that  $\bar{C}_2(\lambda_{st}) > \bar{C}_2(0)$ .

Define

$$d = \frac{\theta^{2/\alpha} R_s}{K_\alpha T_o^{2/\alpha} l_o^2 \lambda_{ot} R_o \exp(\theta^{2/\alpha} a \lambda_{ot})}, \quad (11)$$

where  $a = -K_\alpha T_o^{2/\alpha} l_o^2 \frac{1}{\theta^{2/\alpha}} + K_\alpha T_s^{2/\alpha} l_s^2$ . Our main result is summarized as follows.

*Theorem 1:* For any feasible primary and secondary transmitter densities, if  $d > 1$  the sum transmission throughput is augmented over that of the stand-alone primary network. And the overall throughput is maximized by  $\lambda_{st}^{opt} = \min(\lambda_{st,mm}, \lambda_{st}^*)$ , where  $\lambda_{st}^*$  is the solution of (12), and  $\lambda_{st,mm}$  is given in (10).

*Proof:* Given that  $\bar{C}_2(\lambda_{st})$  is a continuous function of  $\lambda_{st}$ , and  $\bar{C}_2(0) > 0$  while  $\bar{C}_2(\infty) = 0$ , if there is a single stable point in the range  $\lambda_{st} \in (0, \infty)$ , this point must be a local maximum. Taking derivative of  $\bar{C}_2(\lambda_{st})$  on  $\lambda_{st}$  and setting it to zero, we get after some calculation:

$$\exp(a \lambda_{st}) = c \lambda_{st} + d, \quad (12)$$

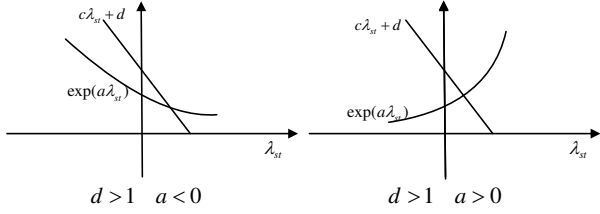


Fig. 2: when  $d > 1$ , there is only one solution.

where  $a$  is given in Eq. (11) and  $c = \frac{-T_s^{2/\alpha} l_s^2 \theta^{2/\alpha} R_s}{T_o^{2/\alpha} l_o^2 \lambda_{ot} R_o \exp(\theta^{2/\alpha} a \lambda_{ot})} < 0$ . It can be shown that Eq. (12) has one single positive solution when  $d > 1$ , as illustrated in Fig. 2. In contrast, there is no positive solution when  $d \leq 1, c \leq a$ , as illustrated in Fig. 3. The remaining case,  $d \leq 1, a < c < 0$ , is more complex, and so is not further discussed (see Fig. 4 for an illustration).

In summary,  $d > 1$  is a sufficient condition for the improvement of the overall transmission throughput per unit area. Considering the outage probability setting for both primary and secondary network, the total throughput is maximized by setting  $\lambda_{st} = \min(\lambda_{st,mm}, \lambda_{st}^*)$ . ■

Fig. 5 shows the overall throughput for different secondary transmitter densities. The common network setting for these two sub-figures is  $\epsilon_o = 0.1, \epsilon_s = 0.1, T_o = 3, T_s = 3, l_o = 1, R_o = \log(1 + T_o), R_s = \log(1 + T_s), \lambda_{ot} = 2/3 \lambda_{ot,m}, \theta = 2, l_s = 0.5$ , and  $\theta = 0.4, l_s = 2$  for the left and right sub-figure, respectively, corresponds to  $d = 23$  and  $d = 2.9$ , both larger than 1. We can see that, in both cases, the overall throughput increases with the secondary transmitter density (note  $\lambda_{st}^{opt} = \lambda_{st,mm}$  for both). Moreover the right sub-figure implies that it is feasible for the secondary network to accommodate long distance transmission ( $l_s > l_o$ ) with higher power ( $\theta < 1$ ) in practice, as long as the primary outage constraint is satisfied. In contrast, Fig. 6 demonstrates a ‘bad’ network setting where  $\epsilon_o = 0.4, \epsilon_s = 0.4, T_o = 3, T_s = 3, l_o = 1, l_s = 2, R_o = \log(1 + T_o), R_s = \log(1 + T_s), \lambda_{ot} = 2/3 \lambda_{ot,m}$  and  $\theta = 0.03$ . In this case  $d < 1$ , and it is shown in the figure that the throughput decreases with the secondary transmitter density. Although the network setting in this example may hardly be adopted in practice, it indicates that an inappropriate configuration could hurt the overall network throughput.

Among secondary parameters,  $l_s, T_s(R_s)$  and  $\epsilon_s$  are usually application-specific. Thus, the power ratio  $\theta$  becomes a vital parameter to determine the secondary or overall throughput. In the following, we endeavor to specify the power ratio range  $\Theta$  such that  $d > 1$ . Set  $x = \theta^{2/\alpha}$  and rearrange  $d$  as a function of  $x$ :

$$d(x) = \frac{R_s}{N \exp(-N) R_o} \frac{x}{\exp(Mx)}, \quad (13)$$

where  $N = K_\alpha T_o^{2/\alpha} l_o^2 \lambda_{ot}$  and  $M = K_\alpha T_s^{2/\alpha} l_s^2 \lambda_{ot}$ . By checking the first and second differential of  $d(x)$ , we get its properties as follows: 1)  $d(x)$  is maximized at  $x^* = \frac{1}{M}$ ; 2)  $d(x)$  is concave when  $x < 2/M$  and convex, otherwise. Therefore if  $d(x^*) > 1$  there exist  $x_1, x_2$  ( $x_1 < x^* < x_2$ )

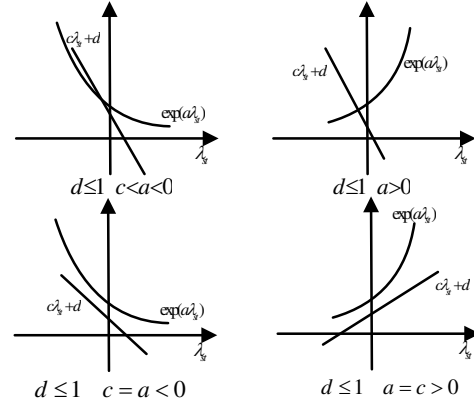


Fig. 3: when  $d \leq 1, c \leq a$ , there is no solution.

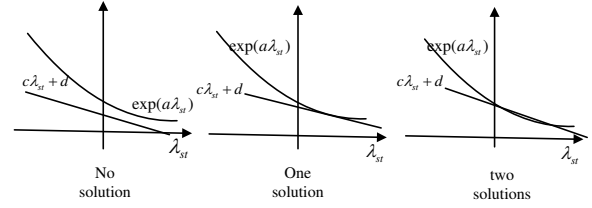


Fig. 4: when  $d \leq 1, a < c < 0$ , there is zero, one and two solutions.

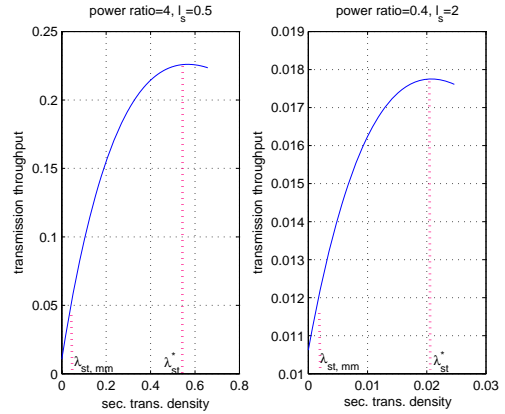


Fig. 5: Overall throughput: an appropriate secondary setting

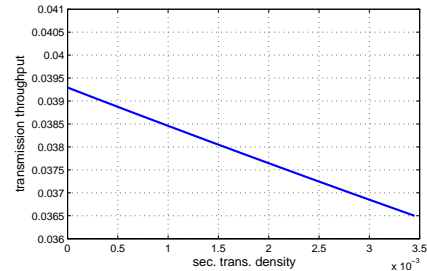


Fig. 6: Overall throughput: an inappropriate secondary setting

such that  $d(x_1) = d(x_2) = 1$  and  $d(x) > 1, \forall x \in (x_1, x_2)$ . Since  $x$  is a monotonically increasing function of  $\theta$  for  $\theta > 0$ , we can conclude that if  $d > 1$  at  $\theta^* = \frac{1}{M^{\alpha/2}}$ , there exist  $\theta_1, \theta_2$  ( $\theta_1 < \theta^* < \theta_2$ ) at which  $d = 1$ , and  $d > 1, \forall \theta \in (\theta_1, \theta_2)$ . Letting  $\theta_{\min} = \theta_1$  and noting that  $\theta_{\max} < \theta^*$  for  $\epsilon_s < 0.5$  (see Eq. (9)), we have  $\Theta = (\theta_{\min}, \theta_{\max})$  if  $\theta_{\min} < \theta_{\max}$ ; otherwise  $\Theta = \phi$ .

The lower bound of  $\Theta, \theta_{\min}$ , does not admit a closed-form expression. In the following we derive two analytical upper bounds  $\theta'_{\min}$  and  $\theta''_{\min}$ , for  $\theta_{\min}$ .

**Proposition 2:**  $\theta_{\min} < \theta'_{\min} < \theta''_{\min}$ , where  $\theta'_{\min} = \left(\frac{N \exp(-N)R_o}{(1-\epsilon_s)R_s}\right)^{\alpha/2}$ , and  $\theta''_{\min} = \left(\frac{-\ln(1-\epsilon_o)(1-\epsilon_o)R_o}{R_s(1-\epsilon_s)}\right)^{\alpha/2}$ .

*Proof:* It is desired that

$$d > 1 \Leftrightarrow N \exp(-N) \exp(M\theta^{2/\alpha})R_o < R_s\theta^{2/\alpha},$$

where  $N$  and  $M$  are given after Eq. (13). According to Eq. (15) the secondary outage probability  $\delta(0, l_s) < \delta(\lambda_{st}, l_s) \leq \epsilon_s$  for  $\theta \leq \theta_{\max}$ , which leads to  $\exp(M\theta^{2/\alpha}) < \frac{1}{1-\epsilon_s}$ .  $\theta'_{\min}$  is thus obtained. Consider the function  $y(N) = N \exp(-N)$ . It is easy to check that  $y(N)$  monotonically increases with  $N$  when  $N \in (0, 1)$ . According to Eq. (6) and its following discussion,  $N < -\ln(1-\epsilon_o) < 1$  for  $\epsilon_o < 0.5$ , which results in  $y(N) < y(-\ln(1-\epsilon_o))$ . Then  $\theta'_{\min} < \theta''_{\min}$  is proved. ■

In summary, in order to design an optimal secondary network, in terms of overall throughput, the following procedures may be taken:

- 1) choose  $l_s, T_s$  and  $\epsilon_s$  so that  $d > 1$  at  $\theta = \theta_{\max}$ ;
- 2) choose  $\theta \in [\theta'_{\min}, \theta_{\max})$ ;
- 3) find the optimal transmitter density according to Theorem 1, and calculate the corresponding maximal throughput;
- 4) repeat 2) and 3) until the maximal throughput is obtained; an exhaustive search on power ratio may be used if applicable.

## V. CONCLUSIONS AND FUTURE WORK

We have made some quantitative study on the transmission throughput of the overlaid networks subject to the outage constraints on both the legacy network and the secondary network. The feasible region of the primary and secondary transmitter densities is explored and the maximal allowable secondary transmitter density is derived. As our main contribution in this paper, a sufficient condition is obtained for the secondary setting such that the sum throughput of the primary and secondary network is improved over that of the stand-alone primary network. As part of our future work, we plan to explore multi-hop transmission throughput in secondary networks.

### APPENDIX A DERIVATION OF OUTAGE PROBABILITY

Overlaid with the secondary network, the probability of a successful primary transmission,  $\Pr(SNR_o(l_o) \geq T_o)$ , is

given by:

$$\begin{aligned} \Pr\left(\left(\frac{P_o l_o^{-\alpha} u}{I_o + I_{so}}\right) \geq T_o\right) &= \Pr\left(u \geq \frac{T_o(I_o + I_{so})}{P_o l_o^{-\alpha}}\right) \\ &= \int_0^\infty \exp\left(-\frac{T_o}{P_o l_o^{-\alpha}} i\right) f_I(i) di \\ &= \psi_I\left(\frac{T_o l_o^{-\alpha}}{P_o}\right), \end{aligned}$$

where  $I = I_o + I_{so}$ , with pdf  $f_I$ , and  $\psi_I(i)$  is the Laplace transform of  $f_I$ . Due to the independence of  $I_o$  and  $I_{so}$ ,

$$\begin{aligned} \psi_I\left(\frac{T_o l_o^{-\alpha}}{P_o}\right) &= \psi_{I_o}\left(\frac{T_o l_o^{-\alpha}}{P_o}\right) \psi_{I_{so}}\left(\frac{T_o l_o^{-\alpha}}{P_o}\right) \\ &= \exp\left[-K_\alpha T_o^{2/\alpha} l_o^2 \left(\lambda_{st} \left(\frac{P_s}{P_o}\right)^{2/\alpha} + \lambda_{ot}\right)\right], \end{aligned} \quad (14)$$

where  $\psi_{I_o}(i) = \exp[-K_\alpha \lambda_{ot} (P_o i)^{2/\alpha}]$ ,  $\psi_{I_{so}}(i) = \exp[-K_\alpha \lambda_{st} (P_s i)^{2/\alpha}]$  ( $K_\alpha$  is defined after (6)). Eq. (7) is obtained by solving  $\Pr(SNR_o(l_o) \geq T_o) \geq 1 - \epsilon_o$ .

Following the same line above, the outage probability of the secondary network can be calculated as follows:

$$\begin{aligned} \delta(\lambda_{st}, r) &= \Pr\left(\frac{P_s u r^{-\alpha}}{I_s + I_{os}} \leq T_s\right) \\ &= 1 - \exp\left[-K_\alpha T_s^{2/\alpha} r^2 \left(\lambda_{ot} \left(\frac{P_o}{P_s}\right)^{2/\alpha} + \lambda_{st}\right)\right], \end{aligned} \quad (15)$$

where  $r$  is the transmission distance between a secondary transmitter and its corresponding receiver.

## REFERENCES

- [1] M. Vu, N. Devroye, M. Sharif, and V. Tarokh, "Scaling laws of cognitive networks," in *Proc. Crowncom*, Aug. 2007.
- [2] S. Jeon, N. Devroye, M. Vu, S. Chung, and V. Tarokh, "Cognitive networks achieve throughput scaling of a homogeneous network," *IEEE Trans. Info. Theory*, 2009, submitted. Available at <http://arxiv.org/abs/0801.0938>.
- [3] C. Yin, L. Gao, and S. Cui, "Scaling laws for overlaid wireless networks: A cognitive radio network vs. a primary network," *IEEE Trans. Networking*, 2009, to appear. Available at <http://arxiv.org/abs/0805.1209>.
- [4] K. Huang, V. K. N. Lau, and Y. Chen, "Spectrum sharing between cellular and mobile ad hoc networks: transmission capacity trade off," *IEEE J. Sel. areas in Commun., Special Issue on Stochastic Geometry and Random Graphs for Wireless Networks*, Sep. 2009.
- [5] C. Lee and M. Haenggi, "Interference and outage in doubly poisson cognitive networks," in *ISIT*, 2010.
- [6] C. Yin, C. Chen, T. Liu, and S. Cui, "Generalized results of transmission capacities for overlaid wireless networks," in *IEEE International Symposium on Information Theory (ISIT)*, 2009.
- [7] J. Lee, S. Lim, J. G. Andrews, and D. Hong, "Achievable transmission capacity of secondary system in cognitive radio networks," in *ICC*, 2010.
- [8] P. Gupta and P. Kumar, "The capacity of wireless networks," *IEEE Trans. Inform. Theory*, vol. 46, no. 2, 2000.
- [9] S. Weber, X. Yang, J. G. Andrews, and G. de Veciana, "Transmission capacity of wireless ad hoc networks with outage constraint," *IEEE Trans. Info. theory*, vol. 51, no. 12, pp. 4091–4102, Dec. 2005.
- [10] S. Weber, J. G. Andrews, and N. Jindal, "An overview of the transmission capacity of wireless networks," *IEEE Trans. Comm.*, 2010, to appear. Available at <http://arxiv.org/abs/0809.0016>.
- [11] D. Stoyan, W. S. Kendall, and J. Mecke, *Stochastic Geometry and its Applications*. Chichester: John Wiley & Sons, 1995.
- [12] F. Baccelli, B. Błaszczyszyn, and P. Muhlethaler, "An aloha protocol for multiphopped mobile wireless network," *IEEE Trans. Inform. Theory*, vol. 5, no. 2, Feb. 2006.