Asymptotic Analysis on Spatial Diversity versus Multiuser Diversity in Wireless Networks

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***Abstract*—Spatial diversity provided by multiple antennas implemented at the physical layer (PHY) can protect a wireless link from harmful fading, while opportunistic scheduling that exploits multiuser diversity at the media access control (MAC) layer can increase the system throughput with the aid of constructive fading in a multiuser wireless network. In this paper, by studying the average system capacity of opportunistic scheduling and scheduling gain (defined as the average system capacity difference between opportunistic scheduling and conventional round-robin scheduling), we investigate the cross-layer interaction between spatial diversity and multiuser diversity. Our analyses focus on the asymptotic scenarios, by allowing either the number of users or the number of antennas, or both, to go to infinity, for which some succinct closed-form expressions can be obtained and the connections among system parameters become clear.**

I. Introduction

Spatial diversity through employing multiple antennas at either the transmitter or receiver end (or both) has proven to be a key technology that makes reliable and high data rate wireless communication a reality. In a multiuser wireless network, there is another form of diversity called multiuser diversity [7][8], for which independent fluctuations of different users' channel fading are exploited to increase the system throughput, by transmitting to the user whose channel quality is on its peak (opportunistic scheduling). There exist some work on joint spatial diversity and multiuser diversity systems. In particular, [14] studies the capacity of combined multiuser diversity and antenna diversity systems with Rayleigh fading, and [13] extends the analysis to Nakagami fading. However, these results shed little light on the interaction between spatial diversity and multiuser diversity. In [2], by studying the system signal-to-noise ratio (SNR) of space-time block coding schemes with multiuser diversity, the authors argue that spatial diversity has a negative impact on multiuser diversity gain in this scenario. However, the author in [3] indicates that this is only true for open-loop spatial diversity schemes and shows that spatial diversity in general does not reduce the multiuser diversity gain by illustrating some closed-loop schemes.

Our research is different from the above in the following aspects. First, our study focuses on the asymptotic analysis, i.e., by allowing the number of antennas or users or both to go to infinity. Besides mathematical tractability, asymptotic analysis also helps reveal some fundamental relationship of key system parameters, which may be concealed in the finite case by random fluctuations and other transient properties. Moreover in many scenarios (especially with respect to the number of antennas), convergence to the asymptotic limit is rather fast (as partly shown in our numerical results). Secondly, we put emphasis on the scheduling gain in capacity rather than the overall system SNR or capacity, which is the benefit we can really obtain through opportunistic scheduling over the traditional round robin scheduling. The impact of multiple antennas on multiuser wireless networks is increasingly drawing research interest very recently. This work will focus on spatial diversity systems; some pioneer study on spatial multiplexing systems can be found in [1][9].

The main contributions of this paper are summarized below. We derive explicit expressions for average capacity of joint spatial diversity and multiuser diversity systems when the number of users goes to infinity while the number of antennas keeps fixed (which obviously hold for scheduling gain as well). As expected, the average system capacity and scheduling gain grow with K; and we contribute by providing a rather general asymptotic expression that builds an explicit connection with key system parameters and reveals their interactions, and by providing a strict proof in convergence that is in a stronger sense than what is assumed in previous study. Our second contribution lies in showing rigorously that the scheduling gain nonetheless diminishes to zero as the size of antenna arrays grows while the number of users keeps fixed, for both open-loop and closed-loop spatial diversity systems, through asymptotic study on the mean and variance of the effective link SNR for three representative systems. Finally, by allowing both the number of antennas and users grow, we reveal how the scheduling gain behaves depending on the relative growth rate between the two. In particular, we determine a critical point, only beyond which multiuser scheduling is meaningful.

The paper is organized as follows. In section II, we provide the system model with combined spatial diversity and multiuser diversity. Then we provide our asymptotic analysis corresponding to the above three scenarios in Section III, IV and V, respectively. Numerical results are given in section VI and final conclusions are made in section VII.

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II. JOINT SPATIAL DIVERSITY AND MULTIUSER DIVERSITY SYSTEM

We consider a homogeneous downlink multiuser multi-input multi-output (MIMO) communication scenario, which is envisioned to be of crucial importance for future wireless networks. The base station is assumed to have M antennas and each of the K users is assumed to have N antennas. Throughout the paper, when asymptotic analysis with respect to the size of antenna array is pursued, we allow both M and N to go to infinity, with their ratio $\lambda = N/M$ fixed. The incorporation of the large M and fixed N scenario is relatively straightforward. We use $\mathbf{H}_k$ ($1 \le k \le K$) to denote the kth user's channel matrix. For simplicity, independent and identically distributed (i.i.d.) Rayleigh fading is considered for $\{\mathbf{H}_k\}_{k=1}^{K}$, but our analysis can be readily extended to other fading scenarios. As will be seen, only the tail behavior of the relevant probability distribution matters. The background noise is assumed to be white and Gaussian.

Assume the normalized effective *link SNR* for user k is γ_k, whose probability distribution function (PDF) and cumulative distribution function (CDF) are denoted as $f_\gamma(x)$ and $F_\gamma(x)$ respectively (same for all users). In the opportunistic scheduling scheme, the base station choose the user $k^* = \arg\max_k (\gamma_k)_{k=1}^{K}$. Thus the resultant normalized *system SNR* seen by the base station is γ_{k^*} with PDF

$$f_{\gamma_{k^*}}(x) = K f_\gamma(x) F_\gamma^{K-1}(x). \tag{1}$$

Assuming that average transmit SNR is γ_t, average system capacity obtained by opportunistic scheduling can be expressed as a function of K and M as

$$\bar{S}(K,M) = \int_0^{+\infty} \log(1+\gamma_t x) f_{\gamma_{k^*}}(x) dx. \tag{2}$$

We also define $\bar{R}(M)$, the corresponding average system capacity obtained by round-robin scheduling as a function of M as

$$\bar{R}(M) = \int_0^{+\infty} \log(1+\gamma_t x) f_\gamma(x) dx. \tag{3}$$

Finally, in order to measure the benefit brought by multiuser diversity, we define the scheduling gain $G(K,M)$ as the average capacity gain boosted by opportunistic scheduling from $\bar{R}(M)$:

$$G(K,M) = \bar{S}(K,M) - \bar{R}(M). \tag{4}$$

Spatial diversity can be realized in various forms in a multiple-antenna system. In this paper, we concretize our analysis with three spatial diversity schemes. The first employs well-known space-time block coding at the transmitter and maximum ratio combining at the receiver, coined as STBC/MRC, which does not require channel state information (CSI) at the transmit end. As user scheduling inherently requires feedback, we further explore two closed-loop diversity schemes. One of them pursues joint maximum ratio transmission and maximum ratio combining (MRT/MRC), which provides the optimal performance reference for MIMO diversity techniques. The other exploits simple antenna selection on both ends (SC/SC), trading performance for complexity. MRT/MRC and SC/SC can be viewed as the two extremes for various hybrid selection combining schemes.

In the remainder of this paper, we adopt the following notations for the limiting behaviors of two functions $f(x)$ and $g(x)$ with $\lim_{x\to\infty \text{ or } x\to 0} g(x)/f(x) = c$: $g(x) = O(f(x))$ for $0 < |c| < \infty$; $g(x) \sim f(x)$ for $c = 1$ (which is referred to as tail equivalence); $g(x) = o(f(x))$ for $c = 0$; and $g(x) = \omega(f(x))$ for $c = \infty$. When convergence of a sequence of random variables is involved, shorthand notation "*D*" stands for in distribution, "*P*" for in probability, "*r*" for in *r*th mean, and "*a.s.*" for almost surely. The user index will be omitted from relevant notations when no ambiguity is incurred.

III. ASYMPTOTIC SYSTEM CAPACITY AS K GOES TO INFINITY WHILE M KEEPS FIXED

In this section, we will examine $\lim_{K\to\infty} \bar{S}(K,M)$ with M fixed. We first summarize some results from [11][12] as the following Lemma.

Lemma 1: Let $X_1,...,X_K$ be i.i.d random variables with absolutely continuous CDF F_X and PDF f_X, whose derivative $f_X'(x)$ exists for all x in (x_1,∞) for some x_1. If

$$\lim_{x\to\infty} \frac{1-F_X(x)}{f_X(x)} = c \ge 0, \tag{5}$$

the standardized extreme

$$\frac{\max_{1\le k\le K} X_k - b_K}{a_K} \xrightarrow{D} \Lambda(x) = \exp(-e^{-x}), \tag{6}$$

with $b_K = F_X^{-1}(1-1/K)$, and $a_K = (K f_X(b_K))^{-1}$. Furthermore, if $c = 0$, $\max_{1\le k\le K} X_k \xrightarrow{P} b_K$, otherwise $\max_{1\le k\le K} X_k / b_K \xrightarrow{P} 1$.

From Lemma 1, we can observe that the limit b_K is determined by the tail behavior of the individual CDF or PDF. Usually in Rayleigh fading (or more general Nakagami fading) with spatial diversity transmission and reception, the tail PDF of the effective link SNR takes a form of Gamma-like functions as $\alpha x^p e^{-qx}$ ($\alpha > 0$, $p \ge 0$, and $q > 0$). By Theorem 1 introduced below, we can obtain asymptotic expressions for b_K and $\bar{S}(K,M)$.

Theorem 1: If $f_X(x) \sim \alpha x^p e^{-qx}$ with $\alpha > 0$, $p \ge 0$ and $q > 0$, then the norming constant b_K in Lemma 1 is given by (up to the second-order approximation[1])

$$b_K = \frac{1}{q}\left[\log K + p\log\log K + \log\left(\frac{\alpha}{q}\right)\right]. \tag{7}$$

Furthermore,

$$\lim_{K\to\infty}\left\{\bar{S}(K,M) - \log(1+\gamma_t b_K)\right\} = 0. \tag{8}$$

Sketch of proof: The computation of b_K is relatively straightforward and thus omitted here. In the following, we give a sketch of proof for (8). First we have [9]

[1] We defined the first order approximation when truncated at $\log K$, second order approximation when truncated at $\log\log K$.

$$P\left\{-c\log\log K \le \left(\max_{1\le k\le K}\gamma_k\right) - b_K \le c\log\log K\right\}$$
$$\ge 1 - O\left(\frac{1}{\log K}\right), \quad (9)$$

where $c = \lim_{x\to\infty}\frac{1-F(x)}{f(x)}$.

Then we can obtain a lower bound of $\bar{S}(K,M)$ through Chebyshov's inequality.

$$\bar{S}(K,M) = E\left(\log\left(1+\gamma_t\left(\max_{1\le k\le K}\gamma_k\right)\right)\right)$$
$$\ge P\left(\max_{1\le k\le K}\gamma_k \ge b_K - c\log\log K\right)$$
$$\times\log\left(1+\gamma_t\left(b_K - c\log\log K\right)\right) \quad (10)$$
$$\ge\left(1-O\left(\frac{1}{\log K}\right)\right)\times\log\left(1+\gamma_t\left(b_K - c\log\log K\right)\right)$$
$$=\log\left(1+\gamma_t b_K\right) - o(1).$$

An upper bound of $\bar{S}(K,M)$ can be obtained as follows. First notice

$$\bar{S}(K,M) = \int_0^\infty P(S(K,M) > x)dx$$
$$=\int_0^{\log(1+\gamma_t b_K)} P(S(K,M)>x)dx + \int_{\log(1+\gamma_t b_K)}^{+\infty} P(S(K,M)>x)dx \quad (11)$$

We know PDF $f(x)$ is tail equivalent to $\alpha x^p e^{-qx}$, and $\lim_{x\to\infty}\frac{1-F(x)}{f(x)} = \frac{1}{q} > 0$. Therefore we can find a positive constant c_2 and x_0, such that $1-F(x) < c_2 f(x)$, for any $x > x_0$. For sufficiently large x we have

$$P(S(K,M)>x) = 1 - P(S(K,M)\le x)$$
$$=1-F^K\left(\frac{e^x-1}{\gamma_t}\right)$$
$$=\left(1-F\left(\frac{e^x-1}{\gamma_t}\right)\right)\left(1+...+F^{(K-1)}\left(\frac{e^x-1}{\gamma_t}\right)\right) \quad (12)$$
$$\le Kc_2 f\left(\frac{e^x-1}{\gamma_t}\right).$$

Therefore when K is large enough, we have

$$\int_{\log(1+\gamma_t b_K)}^{+\infty} P(S(K,M)>x)dx$$
$$\le \int_{\log(1+\gamma_t b_K)}^{+\infty} Kc_2 f\left(\frac{e^x-1}{\gamma_t}\right)dx$$
$$=\int_{b_K}^{+\infty} Kc_2 f(x)\frac{\gamma_t}{1+x\gamma_t}dx \le K\frac{c_2\gamma_t}{1+\gamma_t b_K}\int_{b_K}^{+\infty} f(x)dx \quad (13)$$
$$=\frac{Kc_2\gamma_t}{1+\gamma_t b_K}\left(1-F(b_K)\right) = O\left(\frac{1}{\log K}\right),$$

and

$$\bar{S}(K,M) \le \log\left(1+\gamma_t b_K\right) + \int_{\log(1+\gamma_t b_K)}^{+\infty} P(S_K > x)dx$$
$$=\log\left(1+\gamma_t b_K\right) + O\left(\frac{1}{\log K}\right). \quad (14)$$

Based on (10) and (14) we can conclude

$$\lim_{K\to\infty}\left\{E\left(\log\left(1+\gamma_t\left(\max_{1\le k\le K}\gamma_k\right)\right)\right) - \log\left(1+\gamma_t b_K\right)\right\} = 0.$$

■

In the remaining part of this section, we will use Theorem 1 to compute average asymptotic capacity of some systems jointly exploiting antenna diversity and multiuser diversity.

A. STBC/MRC

In this scheme, the normalized effective link SNR for a generic user is given by $\gamma = \frac{1}{M}\sum_{i=1}^{N}\sum_{j=1}^{M}\left|h_{i,j}\right|^2$, whose PDF admits [2][10]:

$$f_\gamma^{STBC/MRC}(x) = \frac{M^{MN}}{(MN-1)!}x^{MN-1}e^{-Mx}, x\ge 0. \quad (15)$$

By Theorem 1, the corresponding asymptotic system capacity is given by $\log\left(1+\gamma_t b_K^{STBC/MRC}\right)$ with

$$b_K^{STBC/MRC} = \frac{1}{M}\left(\begin{array}{l}\log K + (MN-1)\log\log K + \\ \log\left(\frac{M^{MN-1}}{(MN-1)!}\right)\end{array}\right) \quad (16)$$
$$+O\left(\log\log\log K\right).$$

B. SC/SC

In this spatial diversity scheme, both the user and the base station will choose one optimal antenna such that the resultant channel gain is maximized. Thus the normalized effective link SNR at the receiver is $\gamma = \max_{1\le i\le N, 1\le j\le M}\left(\left|h_{i,j}\right|^2\right)$, whose PDF can be easily obtained as

$$f_\gamma^{SC/SC}(x) = MNe^{-x}(1-e^{-x})^{MN-1}, x\ge 0, \quad (17)$$

which is tail equivalent to MNe^{-x} . Therefore by Theorem 1, the limit dictates[2]

$$b_K^{SC/SC} = \log K + \log(MN). \quad (18)$$

C. MRT/MRC

In the MIMO MRT/MRC system, the base station applies the unit-norm principal right singular vector corresponding to the largest singular value $\sigma_{\max}$ of $\mathbf{H}$, $\mathbf{w}_t \in \mathbb{C}^{M\times 1}$ to the transmitted symbol, and at the receiver side the corresponding left singular vector $\mathbf{w}_r \in \mathbb{C}^{N\times 1}$ is employed.

The CDF of the normalized effective link SNR $\gamma = \sigma_{\max}^2$ is given by [4][6]:

$$F_\gamma^{MRT/MRC}(x) = \frac{\left|\mathbf{\Psi}_c(x)\right|}{\Pi_{k=1}^s\Gamma(t-k+1)\Gamma(s-k+1)}, x\ge 0, \quad (19)$$

where $s=\min(M,N)$, $t=\max(M,N)$, and $\mathbf{\Psi}_c(x)$ is an $s\times s$ Hankel matrix function with the (i,j)th entry given by $\{\mathbf{\Psi}_c(x)\}_{i,j} = \gamma(t-s+i+j-1,x)$, for $i,j=1,2,...,s$. Here $\gamma(a,\beta)$ is the incomplete Gamma function defined as

[2] This is one rare accurate expression.

$\gamma(a,\beta)=\int_0^{\beta} e^{-t}t^{a-1}dt$, and $\Gamma(a)$ is the Gamma function defined as $\Gamma(a)=\gamma(a,+\infty)$. The PDF of γ can be derived as

$$f_{\gamma}^{MRT/MRC}(x)=F_{\gamma}^{MRT/MRC}(x)tr(\mathbf{\Psi}_c^{-1}(x)\mathbf{\Phi}_c(x)), x\geq 0 \ , \quad (20)$$

where $\mathbf{\Phi}_c(x)$ is an $s\times s$ matrix whose (i,j)th entry is given by $\{\mathbf{\Phi}_c(x)\}_{i,j}=x^{t-s+i+j-2}e^{-x}$.

Though the PDF function is rather involved, fortunately we are only concerned with its tail behavior, as dictated by the following lemma. Due to space limitations, we omit the proof here.

Lemma 2: $f_{\gamma}^{MRT/MRC}(x)$ is tail equivalent to $\alpha e^{-x}x^{M+N-2}$, where $\alpha=\dfrac{1}{(M-1)!(N-1)!}$. (21)

Based on Lemma 2 and Theorem 1, the limit for MRT/MRC admits

$$\begin{aligned} b_K^{MRT/MRC} &= \log K + \left(M+N-2\right)\log\log K \\ &\quad - \log\left(\left(M-1\right)!\left(N-1\right)!\right)+O\left(\log\log\log K\right). \end{aligned} \quad (22)$$

Some interesting observations are readily in order. From (16), we can observe a tradeoff between spatial diversity and multiuser diversity for an open-loop spatial diversity system, which has also been observed by other researchers (e.g.,[2][10]). Here we give a more rigorous proof and reveal more details of the ultimate capacity relating to *M* and *N*. It is also observed that the detrimental effect of multiple transmit antennas can be avoided with closed-loop spatial diversity schemes, as seen in (18) and (22). And also from (18) and (22), we can infer that for the general hybrid spatial diversity schemes, the scaling laws should only have differences in the second order approximations.

Without doubt, $G\left(K,M\right)$ in (4) is an asymptotically increasing function of K . In the next section, we will show that $G\left(K,M\right)$ asymptotically decreases with M , no matter for open-loop or closed-loop spatial diversity systems.

IV. Asymptotic Scheduling Gain as *M* Goes to Infinity While *K* Keeps Fixed

In this section, we will examine $\lim_{M\to\infty} G\left(K,M\right)= \lim_{M\to\infty}\left(\bar{S}\left(K,M\right)-\bar{R}(M)\right)$ with K fixed. The following lemma summarizes the main result in this scenario.

Lemma 3: For STBC/MRC, SC/SC and MRT/MRC, $\lim_{M\to\infty} G(K,M)=0$ for fixed K .

Sketch of proof: First we can show that

$$\lim_{M\to\infty}\left(\bar{R}\left(M\right)-\log\left(1+\gamma_t\mu_{\gamma}\right)\right)=0, \quad (23)$$

where

$$\mu_{\gamma}=\begin{cases} \log\left(MN\right)=\log\left(\lambda M^2\right), & for \quad SC/SC \\ N=\lambda M, & for \quad STBC/MRC \\ \left(\sqrt{M}+\sqrt{N}\right)^2=\left(1+\sqrt{\lambda}\right)^2 M, & for \quad MRT/MRC. \end{cases} \quad (24)$$

Secondly we have [11]

$$\bar{S}(K,M)\leq \log\left(1+\gamma_t\left(\mu+\frac{(K-1)\sigma}{\sqrt{2K-1}}\right)\right). \quad (25)$$

where μ and σ are the mean and standard deviation for the individual link. A key step in this proof is to show $\lim_{M\to\infty}\mu/\mu_{\gamma}=1$ and $\lim_{M\to\infty}\sigma/\mu_{\gamma}=0$ for all three schemes. Finally from (23) and (25) we can made the conclusion that $\lim_{M\to\infty} G(K,M)=0$. ■

Lemma 3 shows that the scheduling gain will diminish when the number of antennas goes to infinity for fixed K . The intuition comes from the fact that the mean of the link SNR grows at a higher-order rate than the variance with the number of antennas, therefore the capacity difference between opportunistic scheduling and round robin scheduling disappears eventually. This is reminiscent of the multiple-antenna channel hardening effect studied in [1]. Therefore, multiuser scheduling is not worthwhile in an antenna-dominant environment. It is also interesting to see the difference in $\lim_{M\to\infty}\bar{R}\left(M\right)$ for different diversity techniques. These observations are verified in Section VI.

V. Asympotic Schdeuling Gain when Both *M* and *K* Goes to Infinity

From the above discussions, we observe that multiple antennas will hurt the scheduling gain, while multiple users will increase the scheduling gain. An interesting question naturally arises: when both the number of users and antennas are allowed to grow simultaneously, how will $G(K,M)$ behave? To analyze this problem, the key step lies in obtaining the scaling law for $\max\left\{\gamma\left(k,M\right)\right\}_{k=1}^{K}$, where $\gamma\left(k,M\right)$ denotes the normalized effective link SNR for the *k*th user. Since the number of antennas also grows to infinity, the results derived in Section III do not apply. We thus take the following three steps to obtain the desired results. Note that such an approach was also taken in the relevant study of [1].

Step 1: Find some norming constants to transform $\gamma(k,M)$ into a random variable whose distribution does not depend on M as $M\to\infty$, i.e., find a_M and b_M such that $\dfrac{\gamma(k,M)-b_M}{a_M}\xrightarrow{D} w_k$, whose distribution function desirably admits an analytical form.

Step 2: Apply the results derived in section III to obtain the scaling law for $\max\left(w_k\right)_{k=1}^{K}$, assumed to be $\varphi(K)$.

Step 3: Consider $b_M+a_M\varphi(K)$ as the scaling law for $\max\left\{\gamma\left(k,M\right)\right\}_{k=1}^{K}$ when both K and M grow to infinity.

As will be seen, usually b_M can be taken to be tail equivalent to μ_{γ} in (24), therefore studying $G(K,M)$ is equivalent to studying $\log\left(1+\dfrac{a_M\varphi(K)}{b_M}\right)$. Our approach thus nicely combines the effect of multiple antennas and multiple users for convenience of analysis, and the problem boils down to the determination of the dominant factor between the two. In the following, we provide our results on the asymptotic scheduling gain for STBC/MRC, SC/SC and MRT/MRC when both the number of antennas and users go to infinity.

A. STBC/MRC

Choose $b_M = N = \lambda M$, $a_M = \sqrt{N/M} = \sqrt{\lambda}$. By Central Limit Theorem, we can get $\frac{\gamma(k,M)-b_M}{a_M} \xrightarrow{D} w_k$, whose PDF is the standard normal distribution function. We can then obtain $\varphi(K) \sim \sqrt{2\log K}$ and $\frac{a_M\varphi(K)}{b_M} \sim \frac{\sqrt{2\lambda \log K}}{\lambda M}$. Thus

$$G^{STBC/MRC}(K,M) \to \begin{cases} 0, & when \quad \log K = o\left(M^2\right) \\ c, & when \quad \log K = O\left(M^2\right) \\ +\infty, & when \quad \log K = \omega\left(M^2\right). \end{cases} \qquad (26)$$

B. SC/SC

According to the results in section III, if we choose $b_M = \log(MN)$, $a_M = 1$, then $\gamma(k,M) - b_M \xrightarrow{D} w_k$, whose CDF is $\exp\left(-e^{-x}\right)$. We can then obtain $\varphi(K) \sim \log K$ and $\frac{a_M\varphi(K)}{b_M} \sim \frac{\log K}{\log(MN)} = \frac{\log K}{\log(\lambda M^2)}$. Therefore

$$G^{SC/SC}(K,M) \to \begin{cases} 0, & when \quad \log K = o(\log M) \\ c, & when \quad \log K = O(\log M) \\ +\infty, & when \quad \log K = \omega(\log M). \end{cases} \qquad (27)$$

C. MRT/MRC

The norming constants a_M and b_M have already been derived in [15] as

$$b_M = \left(\sqrt{M} + \sqrt{N}\right)^2 = M\left(1+\sqrt{\lambda}\right)^2, \qquad (28)$$

$$a_M = M^{1/3}\left(1+\sqrt{\lambda}\right)^{4/3} / \sqrt{\lambda} . \qquad (29)$$

Further

$$\frac{\gamma(k,M) - b_M}{a_M} \xrightarrow{D} w_k , \qquad (30)$$

where w_k is a random variable whose distribution follows the Tracy-Widom law of order 2, defined by

$$F_2(s) = \exp\left\{\int_s^{\infty} (x-s) q^2(x) dx\right\}, s \in \mathbb{R} , \qquad (31)$$

where $q(x)$ solves the nonlinear Painleve II differential equation

$$q''(x) = xq(x) + 2q^3(x), \text{ as } x \to \infty , \qquad (32)$$

with $q(x) \sim 2^{-1}\pi^{-1/2}x^{-1/4}e^{\frac{-2}{3}x^{3/2}}$.

From (31), we can obtain

$$f_2(s) = dF_2(s)/ds \sim \int_s^{\infty} q^2(x)dx \sim (8\pi s)^{-1} e^{\frac{-4}{3}s^{3/2}} . \qquad (33)$$

After some algebra, we can obtain $\varphi(K) \sim (\log K)^{2/3}$ and $\frac{a_M\varphi(K)}{b_M} \sim \left(\frac{\log K}{M}\right)^{2/3}$. Therefore

$$G^{MRT/MRC}(K,M) \to \begin{cases} 0, & when \quad \log K = o(M) \\ c, & when \quad \log K = O(M) \\ +\infty, & when \quad \log K = \omega(M). \end{cases} \qquad (34)$$

In comparisons of (26), (27) and (34), we can observe that, for the open-loop spatial diversity scheme STBC/MRC, much more users are required to attain benefits brought by the opportunistic scheduling. This is because for STBC/MRC, the variance of the effective link SNR shrinks fastest relative to the mean, while the variance is just what is exploited by opportunistic scheduler.

VI. Numerical Results

In this section, we provide some numerical results to verify the above analysis.

Fig.1 shows the average throughput of round robin scheduling with the number of antennas. The agreement between the asymptotic results and simulation results verifies (23). Fig. 2 verifies that the scheduling gain decreases with the number of antennas.

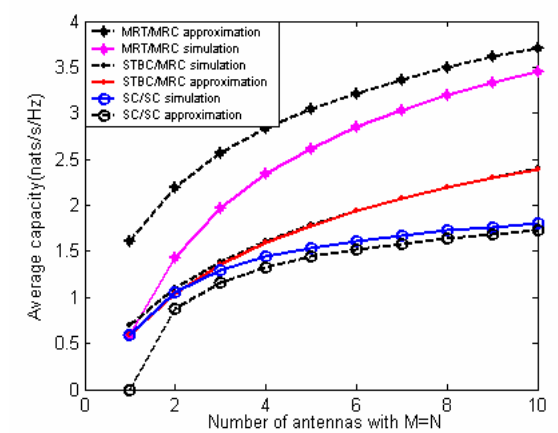

Figure 1. Average throughput of round robin scheduling (SNR=0 dB)

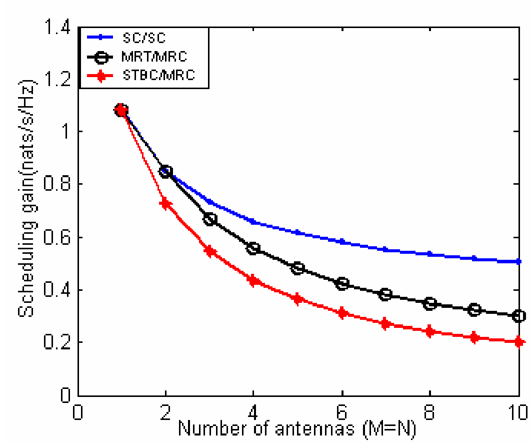

Figure 2. Scheduling gain (SNR=0 dB, K=50)

To verify the conclusions made in Section V, we consider two relative growth rate between K and M , with

$N/M=1$. First, we assume $K=M$ in Fig.3, which shows that the scheduling gain for SC/SC has a tendency towards saturation (which verifies the second equation of (27)). For MRT/MRC and STBC/MRC, the scheduling gains asymptotically decrease (which verifies the first equation of (26) and (34)). In Fig.4, we assume $K=e^M$, which shows the scheduling gain for MRT/MRC almost saturates as M grows (which verifies the second equation of (34)), while the scheduling gain asymptotically increases for SC/SC and decreases for STBC/MRC (which verifies the third equation of (27) and first equation of (26)). Due to computation constraints, it's difficult to verify the second and third equations of (26), but our results already show that for open-loop spatial diversity schemes like STBC/MRC, much more users are required to make opportunistic scheduling beneficial.

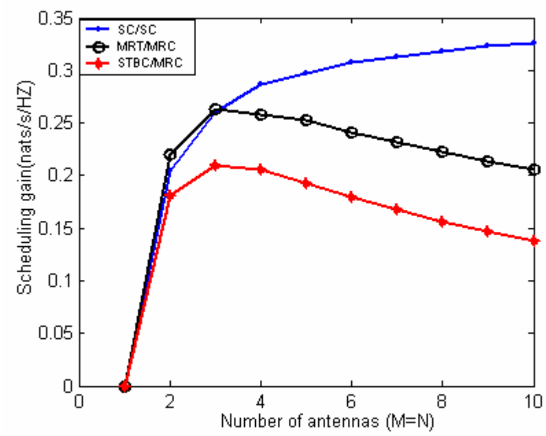

Figure 3. Scheduling gain when $K=M$ (SNR=0dB)

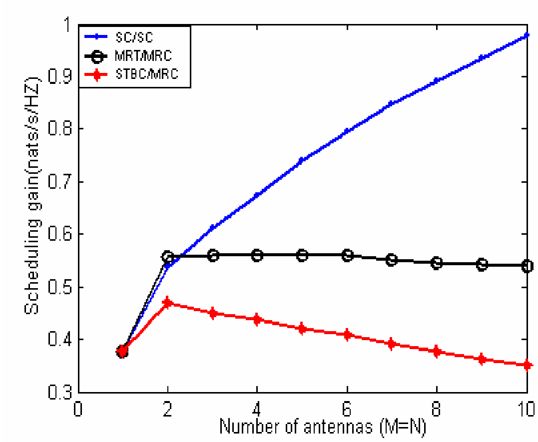

Figure 4. Scheduling gain when $K=e^M$ (SNR=0dB)

VII. Conclusion

In this paper, we present asymptotic analysis on the interaction between spatial diversity and multiuser diversity in wireless networks. Explicit expressions of scheduling gain and average system capacity in various scenarios that reveal inter-connections and fundamental tradeoffs among key system parameters are given, which afford us some insights in real system design.

Our future work includes study of the interaction between spatial diversity and multiuser diversity in a correlated fading scenario, and extension to the situations when users' channels are heterogeneous, together with the associated fairness issues. The interaction of multiuser diversity and the diversity-multiplexing tradeoff in MIMO systems also deserves further study.

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