On the Fundamentally Asynchronous Nature of Interference in Cooperative Base Station Systems

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***Abstract*— Cooperative transmission by base stations can significantly improve the spectral efficiency of multiuser, multi-cell multiple input multiple output systems. We show that in such systems the multiuser interference is asynchronous by nature, even when perfect timing-advance mechanisms ensure that the desired signal components arrive synchronously. We establish an accurate mathematical model for the asynchronism, and use it to show that the asynchronism leads to a significant performance degradation of existing linear precoding designs that assumed synchronous interference. We consider three different previously proposed precoding designs, and show how to modify them to effectively mitigate asynchronous interference.**

I. Introduction

While the spectral efficiency gains of multiple input multiple output (MIMO) systems are significant for point-to-point links [1], they are limited in multi-user cellular networks by inter-cell co-channel interference (CCI) [2], [3]. In conventional cellular systems, CCI is partially reduced by careful radio resource management techniques such as power control, frequency reuse, and spreading code assignments [4]. Recently, base station (BS) cooperation, in which different BSs together transmit signals for different mobile stations (MSs), has been shown to improve spectral efficiency considerably.

The theoretical analyses of BS cooperation often assume that the multiple BSs can be modeled as a single giant BS with more antennas, see for example [5]–[10]. While this assumption enables the well-studied single cell downlink transmission model to be applied in a straightforward manner, it also implies that both the desired and the interfering signals from different BSs arrive at each MS *simultaneously*. As we show in Sec. II, even with perfect timing-advance mechanisms, which ensure that the signals arrive at their *intended* recipients synchronously, the simultaneous arrival of both the desired and interfering signals is fundamentally unrealizable. The BSs cannot also align all the interfering signals at each MS due to the different propagation times between the BSs and MSs. As we shall see, ignoring this effect can significantly degrade the performance of the BS cooperative schemes proposed in the literature, especially at high data rates.

To the best of our knowledge, this problem of asynchronous MIMO interference has not been addressed in the literature. This paper develops a detailed mathematical framework for BS cooperation – in a multi-user multi-cell MIMO cellular network – that explicitly accounts for the asynchronous interference described above. This framework is then applied to existing linear precoding design methods to analyze and mitigate the detrimental impact of the asynchronicity. While base station cooperation can be implemented using, for example, Dirty paper coding [5], [6] or Tomlinson-Harashima precoding [11], we focus on linear precoding designs given their relatively lower complexity requirements at both the BSs and MSs [8]–[10]. They mitigate inter-cell interference, exploit macro-diversity, and can avoid capacity bottlenecks in severely spatially correlated channels [5]–[10].

Various design methods have been proposed in the literature to determine the correspondingly optimal linear precoding matrices. These include minimizing the mean square error (MSE) [12], the signal to leakage plus noise ratio (SLNR) [13], [14], and the sum rate [5], [6], [10], which, arguably, is the ultimate metric that determines spectrum utilization. We adapt these previously proposed and different methods to handle the inevitable asynchronicity in interference.

The rest of the paper is organized as follows. Section II develops a detailed model for the asynchronous interference. Section III develops the three algorithms that maximize different metrics. The numerical results are presented in Sec. IV and are followed by our conclusions in Sec. V.

II. System Model And Problem Formulation

We consider a cellular system with B BSs (each with N_T antennas) and K MSs/users (each with N_R antennas). The cooperative BSs together transmit L_k data streams to MS k. The different links are independent and undergo frequency-flat Rayleigh fading. Therefore, $\mathbf{H}_k^{(b)}$, the baseband matrix representation of the channel from BS b to MS k, has complex Gaussian elements. Let b_k denote the index of the BS closest to MS k. For any MS, the BSs cooperate and jointly transmit the signals intended for it. The transmit vector for MS k from BS b is linearly precoded by the $N_T \times L_k$ matrix $\mathbf{T}_k^{(b)}$ as $\mathbf{x}_k^{(b)}(m) = \mathbf{T}_k^{(b)}\mathbf{s}_k(m)$, where $\mathbf{s}_k(m)$ denotes the zero-mean data vector, of size $L_k \times 1$ at time m, meant for MS k. As in [5]–[10], we assume that each BS has complete channel state information (CSI) for all the channels to all the MSs. We also assume a block-fading channel model with a large enough coherence time so that the channel fading remains the same over the duration in which is $\mathbf{T}_k^{(b)}$ used. Given current CSI, in order to maximize the per-user transmission information rate, a Gaussian code book is used for the transmit data vectors,

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with normalized power such that $\mathbf{E}\left[\mathbf{s}_k(m)\mathbf{s}_k(m)^\dagger\right] = \mathbf{I}_{L_k}$. Furthermore, the code books for different users are independent of each other, i.e., $\mathbf{E}\left[\mathbf{s}_k(m)\mathbf{s}_l(m)^\dagger\right] = \mathbf{0}$, for $k \neq l$.

A. Asynchronous Interference Despite Perfect Synchronization

The CSI available at each BS also includes the knowledge of the propagation delay from each BS to each of the MSs. Assuming perfect timing synchronization among cooperative BSs, the timing-advance mechanisms can ensure that the *desired signals* for an MS that are transmitted from multiple BSs reach the MS at the same time.

Specifically, let the propagation delay from BS b to MS k be denoted by $\tau_k^{(b)}$, as illustrated in Fig. 1 for two BSs and two MSs. To guarantee simultaneous reception of $\left\{\mathbf{x}_k^{(b)}(m)\right\}_{b=1}^{B}$ at MS k, the BS b advances the time when $\mathbf{x}_k^{(b)}(m)$ is transmitted by $\Delta\tau_k^{(b)} = \tau_k^{(b)} - \tau_k^{(b_k)}$, so that $\left\{\mathbf{x}_k^{(b)}(m)\right\}_{b=1}^{B}$ all arrive at MS k with the same delay, $\tau_k^{(b_k)}$. The equivalent received baseband signal at MS k when a linear modulation with a unit energy baseband signature waveform $g(t)$ of duration T_S is used is given by

$$\mathbf{r}_k(t) = \sum_{m=0}^{\infty} g(t - mT_S - \tau_k^{(b_k)})\mathbf{H}_k\mathbf{x}_k(m) + \mathbf{n}_k(t)$$
$$+\sum_{m=0}^{\infty}\left\{\sum_{\substack{j=1\\(j\neq k)}}^{K}\sum_{b=1}^{B} g(t - mT_S - \tau_k^{(b)} + \Delta\tau_j^{(b)})\mathbf{H}_k^{(b)}\mathbf{x}_j^{(b)}(m)\right\},$$

where $\mathbf{n}_k(t)$ is the additive white Gaussian noise vector, $\mathbf{H}_k = \left[\mathbf{H}_k^{(1)}, \ldots, \mathbf{H}_k^{(B)}\right]$, and $\mathbf{x}_k(m) = \left[\mathbf{x}_k^{(1)}(m)^\dagger, \ldots, \mathbf{x}_k^{(B)}(m)^\dagger\right]^\dagger$.

At MS k, the received signal at time t, $\mathbf{r}_k(t)$, is passed through a filter matched to $g(t - mT_S - \tau_k^{(b_k)})$ to generate the sufficient statistic $\mathbf{y}_k(m)$, which is given by

$$\mathbf{y}_k(m) = \mathbf{H}_k\mathbf{T}_k\mathbf{s}_k(m) + \sum_{\substack{j=1\\(j\neq k)}}^{K}\sum_{b=1}^{B}\mathbf{H}_k^{(b)}\mathbf{T}_j^{(b)}\mathbf{i}_{jk}^{(b)}(m) + \mathbf{n}_k(m), \tag{1}$$

where $\mathbf{T}_k = \left[\mathbf{T}_k^{(1)\dagger}, \ldots, \mathbf{T}_k^{(B)\dagger}\right]^\dagger$, $\mathbf{n}_k(m)$ is the discrete noise vector at the mth interval satisfying $\mathbf{E}\left[\mathbf{n}_k(m)\mathbf{n}_k(m)^\dagger\right] = N_0\mathbf{I}_{N_R}$, and $\mathbf{i}_{jk}^{(b)}(m)$ is the asynchronous interference at MS k from the signal transmitted by BS b for MS j. It depends on the difference, $\tau_{jk}^{(b)}$, between the timing-advances used by BS b for MSs j and k:

$$\tau_{jk}^{(b)} = (\tau_k^{(b)} - \Delta\tau_j^{(b)}) - \tau_k^{(b_k)} = \Delta\tau_k^{(b)} - \Delta\tau_j^{(b)}. \tag{2}$$

In (1), the asynchronous interference term at MS k, $\mathbf{i}_{jk}^{(b)}(m)$, arises from two consecutive symbols, say with indices $m_{jk}^{(b)}$ and $m_{jk}^{(b)} + 1$, that are transmitted to MS j from BS b. Let $0 \leq \delta_{jk}^{(b)} < T_S$ denote the delay offset $\tau_{jk}^{(b)}$ modulo T_S. Then,

$$\mathbf{i}_{jk}^{(b)} = \rho(\delta_{jk}^{(b)} - T_S)\mathbf{s}_j(m_{jk}^{(b)}) + \rho(\delta_{jk}^{(b)})\mathbf{s}_j(m_{jk}^{(b)} + 1), \tag{3}$$

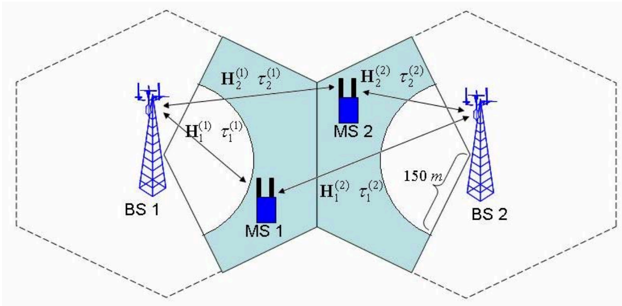

Fig. 1. A simple BS cooperation scenario with 2 BSs and 2 MSs. The MSs are placed in the shaded area in the numerical simulations.

where $\rho(\tau) = \int_0^{T_S} g(t)g(t-\tau)dt$ with $\rho(0) = 1$.

Only if the asynchronous nature of interference is neglected, does (1) simplify to the following form used in [5]–[10]:

$$\begin{aligned}\mathbf{y}_k(m) &\\ &= \mathbf{H}_k\mathbf{T}_k\mathbf{s}_k(m) + \sum_{\substack{j=1\\(j\neq k)}}^{K}\left(\sum_{b=1}^{B}\mathbf{H}_k^{(b)}\mathbf{T}_j^{(b)}\right)\mathbf{s}_j(m) + \mathbf{n}_k(m),\\ &= \mathbf{H}_k\mathbf{T}_k\mathbf{s}_k(m) + \sum_{\substack{j=1\\(j\neq k)}}^{K}\mathbf{H}_k\mathbf{T}_j\mathbf{s}_j(m) + \mathbf{n}_k(m). \end{aligned}\tag{4}$$

B. Statistics of Asynchronous Interference

We now derive the second-order statistics of the asynchronous interference, which will come in handy later.

From (3), we have $\mathbf{E}\left[\mathbf{i}_{jk}^{(b)}(m)\right] = \mathbf{0}$, for all j, k, and b, and $\mathbf{E}\left[\mathbf{i}_{j_1k}^{(b_1)}(m)\mathbf{i}_{j_2k}^{(b_2)}(m)^\dagger\right] = \mathbf{0}$, for $j_1 \neq k$, $j_2 \neq k$, and $j_1 \neq j_2$. It can be shown that, for $j \neq k$,

$$\mathbf{E}\left[\mathbf{i}_{jk}^{(b_1)}(m)\mathbf{i}_{jk}^{(b_2)}(m)^\dagger\right] = \beta_{jk}^{(b_1,b_2)}\mathbf{I}_{L_j}, \tag{5}$$

where the asynchronous interference correlation, $\beta_{jk}^{(b_1,b_2)}$, for $j \neq k$ has the following properties:
$\beta_{jk}^{(b_1,b_2)} = 0$, if $|m_{jk}^{(b_2)} - m_{jk}^{(b_1)}| > 1$;
$\beta_{jk}^{(b_1,b_2)} = \rho(\delta_{jk}^{(b_1)})\rho(\delta_{jk}^{(b_2)} - T_S)$, if $m_{jk}^{(b_2)} = m_{jk}^{(b_1)} + 1$;
$\beta_{jk}^{(b_1,b_2)} = \rho(\delta_{jk}^{(b_1)})\rho(\delta_{jk}^{(b_2)}) + \rho(\delta_{jk}^{(b_1)} - T_S)\rho(\delta_{jk}^{(b_2)} - T_S)$, if $m_{jk}^{(b_2)} = m_{jk}^{(b_1)}$; and
$\beta_{jk}^{(b_1,b_2)} = \rho(\delta_{jk}^{(b_2)})\rho(\delta_{jk}^{(b_1)} - T_S)$, if $m_{jk}^{(b_2)} = m_{jk}^{(b_1)} - 1$.
Also, $\beta_{kk}^{(b_1,b_2)} = 1$, for all BSs b_1 and b_2. Since all the K users use the same waveform, the asynchronous interference correlation values corresponding to different timing parameters can be pre-calculated and stored in a look-up table.

III. Adapting Linear Precoding Design Methods

Our goal is to jointly optimize the transmitter precoding matrices, $\{\mathbf{T}_k\}_{k=1}^{K}$, subject to a set of MS-specific power constraints:

$$\mathrm{Tr}\left\{\mathbf{T}_k^\dagger\mathbf{T}_k\right\} \leq P_k^{\mathrm{tx}}. \tag{6}$$

Note: A uniform per-MS power constraint, $P_k^{\mathrm{tx}} = P_T$, for all k, was also assumed in [6], [8], [15] to ensure "power fairness" for the different users. While the per-BS power constraint [5], [10] makes more physical sense, the advantage of the MS-specific power constraint is that it leads to analytically tractable solutions (for further discussion see [10]). Most importantly, other, more general power constraints can now be obtained numerically. For example, this can be done by an "outer loop" that adjusts P_k^{tx} iteratively until certain criteria such as per-BS power constraints or MS-specific quality-of-service constraints are fulfilled.

As mentioned, determining the linear precoding matrices, even for the synchronous scenario, is a hard problem. The nullification method [8], [10], which forces the precoding matrices to satisfy the constraint, $\mathbf{H}_k\mathbf{T}_j = \mathbf{0}$, for all $k \neq j$, is an ad hoc method that was proposed to find suitable precoding matrices. However, in the presence of asynchronous interference, this constraint can no longer annul all the interference terms in (1). Another option is to force a stronger per-BS constraint $\mathbf{H}_k^{(b)}\mathbf{T}_j^{(b)} = \mathbf{0}$, for all pairs of k and j such that $k \neq j$ [16]. While this constraint does ensure that the asynchronous interference is completely canceled, it can support only $K \leq N_T/N_R$ users, which is a severe limitation.

In this paper, we adapt to the asynchronicity in interference three methods proposed in the literature. These methods are more effective as they strive to minimize CCI to the extent required instead of canceling it out completely.

A. Three Design Methods

1) Overall Normalized Mean Square Error (NMSE): In this method, the goal is to optimize the precoding matrices $\{\mathbf{T}_k\}_{k=1}^K$ to minimize the overall NMSE metric between the received signal and its 'desired' form at each user. The functional form of the metric and the optimal solution for it are derived in Sec. III-B.

2) Signal to Leakage plus Noise Ratio (SLNR): An alternative approach is to maximize the signal-to-leakage-plus-noise-ratio (SLNR). More precisely, for an MS k, we design the precoding matrix $\mathbf{T}_k$ to maximize the SLNR, which is the ratio of the power of the desired signal received at MS k and the sum of the noise and the total interference power (leakage) due to $\mathbf{x}_k$ at other MSs. This approach minimizes the interference that stems from the data streams intended for one user instead of the interference that arrives at that MS. We note that while using the SLNR for precoding design was first suggested in [13], [14], these papers only cover the simple case of one data stream per user, and do not model asynchronous interference. The solution for it is derived in Sec. III-C.

3) Sum of Information Rates: Arguably, the most relevant metric from a system-wide spectral efficiency standpoint is to maximize the sum of the information rates over all users that is achieved by the precoding designs [3], [5], [6], [10].

As we shall see, the optimization problem is non-linear and non-convex, which makes it difficult to find analytical solutions. Brute-force numerical optimization involves searching over an extremely large space of dimension $BN_T \sum_{k=1}^K L_k$, and is practically infeasible. We develop in Sec. III-D an alternate, albeit sub-optimal, algorithm to determine the precoding matrices.

The advantage of the first two metrics is that they are amenable to analysis. The discussion below highlights the intuitive basis for the methods and their observed effectiveness; however, it must be noted that they are ad hoc in nature.

B. Joint Wiener Filtering (JWF) to Minimize NMSE

Our aim is to optimize the transmitter precoders $\{\mathbf{T}_k\}_{k=1}^K$ to minimize the NMSE between the received signal and the 'desired' signal for all the K users. To mitigate interference, we strive to make the actual input signal at the receiver of MS k as close as possible to a desired (virtual) 'cleaned' signal that mimics a single-user MIMO environment that is free of multi-user interference (MUI) and noise.

In such a clean environment, the desired signal input to the receiver would be $\mathbf{z}_k = \mathbf{H}_k\mathbf{V}_k\mathbf{s}_k$, where the matrix $\mathbf{V}_k$ is only determined by the composite channel $\mathbf{H}_k$ and the power constraint in (6). The linear precoding matrix $\mathbf{V}_k$ is taken to be the eigen-beamforming matrix with water-filling power allocation over the channel $\mathbf{H}_k$ since it maximizes the information rate in the clean interference-free environment [4, Chp. 20]. The metric is also normalized so as to emphasize the contribution of all the users. Therefore, the overall normalized MSE (NMSE) metric gets defined as

$$\text{NMSE} = \sum_{k=1}^{K} \frac{\mathbf{E}\left[\|\mathbf{y}_k - \mathbf{z}_k\|^2\right]}{\Omega_k} = \sum_{k=1}^{K} \text{NMSE}_k, \qquad (7)$$

where $\text{NMSE}_k = \frac{\mathbf{E}\left[\|\mathbf{y}_k - \mathbf{z}_k\|^2\right]}{\Omega_k}$ is the NMSE of MS k, $\Omega_k = \mathbf{E}\left[\text{Tr}\left\{\mathbf{z}_k\mathbf{z}_k^\dagger\right\}\right] = \text{Tr}\left\{\mathbf{H}_k\mathbf{V}_k\mathbf{V}_k^\dagger\mathbf{H}_k^\dagger\right\}$ is the average received power of its "desired" signal, and the expectation is over the random data vectors, $\{\mathbf{s}_k\}_{k=1}^K$, and the noise, $\{\mathbf{n}_k\}_{k=1}^K$. The optimization problem is then

$$\left\{\mathbf{T}_k^{\text{opt}}\right\}_{k=1}^K = \arg \min_{\{\mathbf{T}_k\}_{k=1}^K} \sum_{k=1}^{K} \text{NMSE}_k, \qquad (8)$$

$$\text{s. t. } \text{Tr}\left\{\mathbf{T}_k^\dagger\mathbf{T}_k\right\} = \text{Tr}\left\{\sum_{b=1}^{B} {\mathbf{T}_k^{(b)}}^\dagger \mathbf{T}_k^{(b)}\right\} \leq P_k^{\text{tx}}, \qquad (9)$$
$$\text{for } k = 1, \ldots, K.$$

The following closed-form can be shown to be the solution for the optimal linear precoding matrices. (The key steps in the derivation are given in the Appendix.)

$$\mathbf{T}_k^{\text{opt}} = \frac{1}{\Omega_k}\left[\mathbf{C}_k + \kappa_k \mathbf{I}_{N_T B}\right]^{-1} \mathbf{H}_k^\dagger \mathbf{A}_k. \qquad (10)$$

Here, $\mathbf{A}_k = \mathbf{H}_k\mathbf{V}_k$ and $\mathbf{C}_k = \begin{bmatrix} \mathbf{C}_k^{(1,1)} & \mathbf{C}_k^{(1,2)} & \cdots & \mathbf{C}_k^{(1,B)} \\ \mathbf{C}_k^{(2,1)} & \mathbf{C}_k^{(2,2)} & \cdots & \mathbf{C}_k^{(2,B)} \\ \vdots & \vdots & & \vdots \\ \mathbf{C}_k^{(B,1)} & \mathbf{C}_k^{(B,2)} & \cdots & \mathbf{C}_k^{(B,B)} \end{bmatrix}$, where $\mathbf{C}_k^{(b_1,b_2)} = \sum_{j=1}^K \frac{\beta_{kj}^{(b_1,b_2)}}{\Omega_j} {\mathbf{H}_j^{(b_1)}}^\dagger \mathbf{H}_j^{(b_2)}$. $\kappa_1, \ldots, \kappa_K$ are the Lagrange multipliers that are chosen to meet the power constraints for MSs $1, \ldots, K$, respectively.

C. Joint Leakage Suppression (JLS) to Maximize SLNR

We first derive the formula for the SLNR of an MS k, then state the optimization problem, and then find the optimal precoding solution that maximizes it. To ensure analytical feasibility, we limit the search space to scaled semi-unitary matrices of the form $\mathbf{T}_k = \sqrt{\frac{P_k^{\text{tx}}}{L_k}}\mathbf{Q}_k$, where the columns of the $N_T B \times L_k$ matrix $\mathbf{Q}_k$ are orthonormal. This limitation, while sub-optimal, does improve performance as orthonormality eliminates cross-talk among the data streams that an MS receives. (Note that the power constraints are now trivially satisfied with equality.)

The desired signal component of $\mathbf{x}_k$ received by MS k is $\mathbf{x}_k = \sqrt{\frac{P_T}{L_k}}\mathbf{H}_k\mathbf{Q}_k\mathbf{s}_k$, and has a power $P_k^{\text{rx}} = \frac{P_k^{\text{tx}}}{L_k}\text{Tr}\left\{\mathbf{Q}_k^\dagger\mathbf{H}_k^\dagger\mathbf{H}_k\mathbf{Q}_k\right\}$. From (1), the (asynchronous) interference leakage at MS j from $\mathbf{x}_k$, which is meant for MS k, is $\sum_{b=1}^{B}\mathbf{H}_j^{(b)}\mathbf{T}_k^{(b)}\mathbf{i}_{kj}^{(b)}$. It has a power P_{kj}^{leak}, which is given by

$$P_{kj}^{\text{leak}} = \frac{P_k^{\text{tx}}}{L_k}\sum_{b_1=1}^{B}\sum_{b_2=1}^{B}\beta_{kj}^{(b_1,b_2)}\text{Tr}\left\{\mathbf{Q}_k^{(b_1)\dagger}\mathbf{H}_j^{(b_1)\dagger}\mathbf{H}_j^{(b_2)}\mathbf{Q}_k^{(b_2)}\right\}. \tag{11}$$

Here, the sub-matrix $\mathbf{Q}_k^{(b)}$ collects the rows in $\mathbf{Q}_k$ associated with the bth BS. Finally, the noise power seen at MS k is $N_0 N_R$. Therefore, the SLNR for MS k is

$$\text{SLNR}_k = \frac{P_k^{\text{rx}}}{\sum_{\substack{j=1\\(j\neq k)}}^{K} P_{kj}^{\text{leak}} + N_0 N_R}. \tag{12}$$

It can be shown that

$$\text{SLNR}_k = \frac{\text{Tr}\left\{\mathbf{Q}_k^\dagger\mathbf{M}_k\mathbf{Q}_k\right\}}{\text{Tr}\left\{\mathbf{Q}_k^\dagger\mathbf{N}_k\mathbf{Q}_k\right\}} = \frac{\sum_{l=1}^{L_k}\mathbf{q}_{kl}\mathbf{M}_k\mathbf{q}_{kl}^\dagger}{\sum_{l=1}^{L_k}\mathbf{q}_{kl}\mathbf{N}_k\mathbf{q}_{kl}^\dagger}, \tag{13}$$

where $\mathbf{q}_{kl}$ is the lth column of $\mathbf{Q}_k$, $\mathbf{M}_k = P_k^{\text{tx}}\mathbf{H}_k^\dagger\mathbf{H}_k$, $\mathbf{N}_k = N_0 N_R \mathbf{I}_{BN_T} + \sum_{\substack{j=1\\(j\neq k)}}^{K} P_k^{\text{tx}}\mathbf{W}_{kj}$, and

$$\mathbf{W}_{kj} = \begin{bmatrix} \beta_{kj}^{(1,1)}\mathbf{H}_j^{(1)\dagger}\mathbf{H}_j^{(1)} & \cdots & \beta_{kj}^{(1,B)}\mathbf{H}_j^{(1)\dagger}\mathbf{H}_j^{(B)} \\ \beta_{kj}^{(2,1)}\mathbf{H}_j^{(2)\dagger}\mathbf{H}_j^{(1)} & \cdots & \beta_{kj}^{(2,B)}\mathbf{H}_j^{(2)\dagger}\mathbf{H}_j^{(B)} \\ \vdots & \ddots & \vdots \\ \beta_{kj}^{(B,1)}\mathbf{H}_j^{(B)\dagger}\mathbf{H}_j^{(1)} & \cdots & \beta_{kj}^{(B,B)}\mathbf{H}_j^{(B)\dagger}\mathbf{H}_j^{(B)} \end{bmatrix}.$$

Optimizing the linear precoders to maximize SLNR is thus decoupled for different MSs. Yet, finding the optimal $\mathbf{q}_{kl}, \ldots, \mathbf{q}_{kL_k}$ is still analytically intractable. We therefore derive and maximize the following analytically tractable lower bound for SLNR_k, which follows from (13):

$$\text{SLNR}_k \geq \min_{l=1,\ldots,L_k}\frac{\mathbf{q}_{kl}^\dagger\mathbf{M}_k\mathbf{q}_{kl}}{\mathbf{q}_{kl}^\dagger\mathbf{N}_k\mathbf{q}_{kl}}. \tag{14}$$

The optimal precoding matrix is the solution to the following max-min problem:

$$\mathbf{Q}_k^{\text{opt}} = \arg\max_{\mathbf{Q}_k:\ \mathbf{Q}_k^\dagger\mathbf{Q}_k=\mathbf{I}_{L_k}}\ \min_{l=1,\ldots,L}\frac{\mathbf{q}_{kl}^\dagger\mathbf{M}_k\mathbf{q}_{kl}}{\mathbf{q}_{kl}^\dagger\mathbf{N}_k\mathbf{q}_{kl}}. \tag{15}$$

Its structure is given in the following Lemma. The Lemma's proof is based on the Courant-Fisher Min-Max theorem [17] and is omitted due to space constraints. The $L_k = 1$ scenario of [14] is a special case of Lemma 1.

Lemma *1:* The lower bound of SLNR_k in (14) is maximized when:

$$\mathbf{q}_{kl}^{\text{opt}} = \mathbf{v}_l(\mathbf{N}_k^{-1}\mathbf{M}_k),\ \text{for } 1 \leq l \leq L_k, \tag{16}$$

where $\mathbf{v}_l(\mathbf{N}_k^{-1}\mathbf{M}_k)$ denotes the eigenvector of the matrix $\mathbf{N}_k^{-1}\mathbf{M}_k$ corresponding to its lth largest eigenvalue.

D. Controlled Iterative Singular Value Decomposition (CISVD) to Maximize Sum Rate

The sum-rate maximization problem can be stated as:

$$\left\{\mathbf{T}_k^{\text{opt}}\right\}_{k=1}^{K} = \arg\max_{\{\mathbf{T}_k\}_{k=1}^{K}}\sum_{k=1}^{K}R_k, \tag{17}$$
$$\text{s. t. Tr}\left\{\mathbf{T}_k^\dagger\mathbf{T}_k\right\} \leq P_k^{\text{tx}}, \quad \text{for } k = 1,\ldots,K.$$

From (1), the bandwidth-normalized information rate, R_k, of MS k is given by [6][11]

$$R_k = \log\left|\mathbf{I}_{N_R} + \mathbf{\Phi}_k^{-1}\mathbf{H}_k\mathbf{T}_k\mathbf{T}_k^\dagger\mathbf{H}_k^\dagger\right|, \tag{18}$$

where $\mathbf{\Phi}_k$ is the covariance of noise plus interference in (1):

$$\mathbf{\Phi}_k = N_0\mathbf{I}_{N_R} + \sum_{\substack{j=1\\(j\neq k)}}^{K}\sum_{b_1=1}^{B}\sum_{b_2=1}^{B}\beta_{jk}^{(b_1,b_2)}\mathbf{H}_k^{(b_1)}\mathbf{T}_j^{(b_1)}\mathbf{T}_j^{(b_2)\dagger}\mathbf{H}_k^{(b_2)\dagger}.$$

Given the non-linear and non-convex nature of the problem, we propose an iterative "hill-climbing" optimization algorithm to maximize the spectral efficiency. In each step we optimize the precoding matrix for MS k, $\mathbf{T}_k$, by keeping the other precoding matrices, $\mathbf{T}_j$ $(j \neq k)$, fixed. The optimal $\mathbf{T}_k$ is then obviously the water-filling power allocation on the equivalent MIMO channel $\mathbf{\Phi}_k^{-1/2}\mathbf{H}_k$ with unit additive noise power. The iterations are initialized using the JLS solution in (16) given its simplicity, and are continued only when the target sum-rate increases by at least a certain threshold amount.

The pseudo-code for the algorithm is as follows:

1) For $k = 1,\ldots,K$, calculate $\mathbf{T}_1,\ldots,\mathbf{T}_K$ from (16), i.e., use JLS solution as the starting point.
2) For each $k = 1,\ldots,K$, fix $\mathbf{T}_j$ $(j \neq k)$ and update $\mathbf{T}_k$ to the water-filling power allocation for the MIMO channel $\mathbf{\Phi}_k^{-1/2}\mathbf{H}_k$ (with unit noise power).
3) Repeat previous step until the sum rate target function in (17) increases by less than a pre-defined threshold.

Compared with random or exhaustive search algorithms, this method iteratively optimizes one precoder in each step to improve the corresponding MS's performance, while ensuring that a relatively low level of interference is imposed on other users. (Otherwise, the iteration terminates.) While this procedure is simple and sub-optimal, we shall see that it provides good results. This algorithm falls under the general class of greedy "alternate & maximize" algorithms, e.g., [18], and is similar to the iterative water-filling algorithm in [19], which dealt with the sum rate maximization over different orthogonal sub-carriers in DSL systems with cross-talk.

IV. Numerical Results

We simulate the performance of the three design methods discussed above in the downlink scenario of Fig. 1, which consists of an urban micro-cellular network with two cells, each with 1 BS and 1 MS, i.e., $B = 2$ and $K = 2$. The inter-BS distance is 500 m. As BS cooperation results in performance gains when the signal from one BS does not completely dominate the signal from the other BS, we consider scenarios in which the MSs are uniformly distributed in the shaded area shown in Fig. 1. Thus, the MSs are at least 150 m away from their closest BSs. The path-loss coefficient for all the BS-MS channels is 2.0 (free-space propagation) up to a distance of 30 m, and increases to 3.7 thereafter. Without loss of generality, the channel path-losses are normalized with respect to the largest in-cell path-loss in the SINR calculations. In all the considered scenarios we assume $L_1 = L_2 = 2$ and $N_T = 3$ and $N_R = 2$. The symbol pulse is rectangular in shape and has a duration, T_S, of 1 μs. This corresponds to a nominal system bandwidth of 1 MHz that is shared by both BSs. (Increasing the cell size or decreasing T_S increases the asynchronicity of interference.)

Figure 2 considers the average NMSE per user (in linear scale) and compares JWF when it takes the asynchronicity of the interference into account and when it incorrectly ignores it despite it being present. Also shown is the NMSE achieved by conventional nullification [8]–[10]. Accounting for the asynchronicity in interference significantly improves the NMSE of JWF at all SNRs.

Figure 3 compares the average SLNR per user for JLS when it takes the asynchronicity of the interference into account and when it incorrectly ignores it. Also shown is the SLNR achieved by conventional nullification. Accounting for the asynchronicity in interference significantly improves the SLNR achieved by JLS; ignoring it reduces JLS's performance to that of conventional nullification.

Figure 4 compares the sum rate achieved by the three design methods. Also shown are the following benchmarks: (i) conventional eigen-beamforming, where an MS's signal is transmitted only by its serving BS, which treats all interference as additive noise [1], [3], [10]; (ii) ideal point-to-point MIMO in an interference-free single cell, and (iii) conventional nullification, which ignores the asynchronicity in interference. It can be seen that in the presence of asynchronous interference, all three designs, including the simple JLS design, outperform conventional nullification at all SNRs, with CISVD achieving the highest rate. This is aligned with the observations we made for NMSE (Fig. 2) and SLNR (Fig. 3). JWF always outperforms JLS. At low to medium SNRs, CISVD even outperforms single-cell interference-free point-to-point MIMO.

V. Conclusions

In this paper, we investigated the impact of asynchronous interference on the downlink performance of MIMO systems

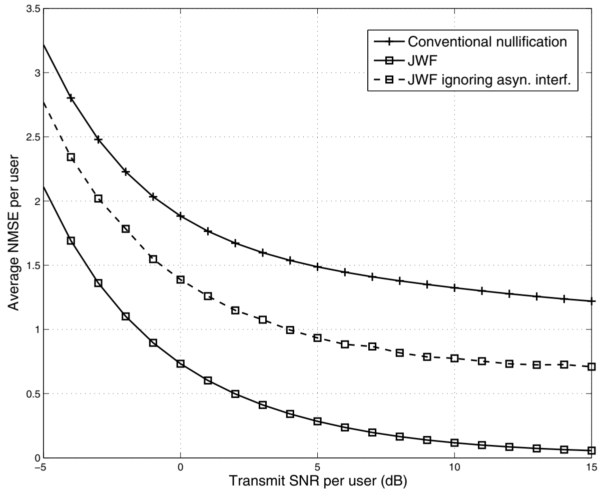

Fig. 2. Normalized MSE comparison of JWF and conventional nullification methods in the presence of asynchronous interference

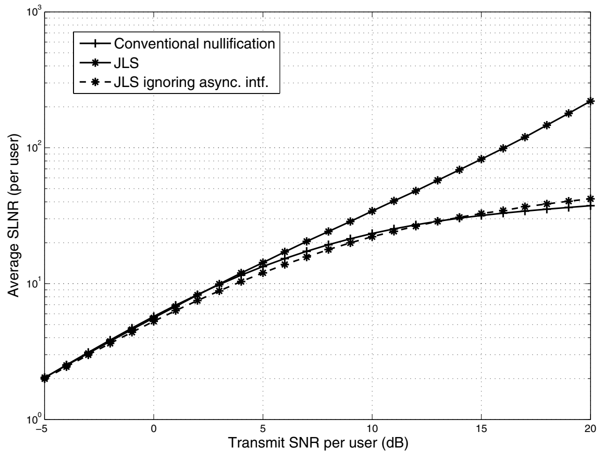

Fig. 3. SLNR comparison of JLS and conventional nullification methods in the presence of asynchronous interference

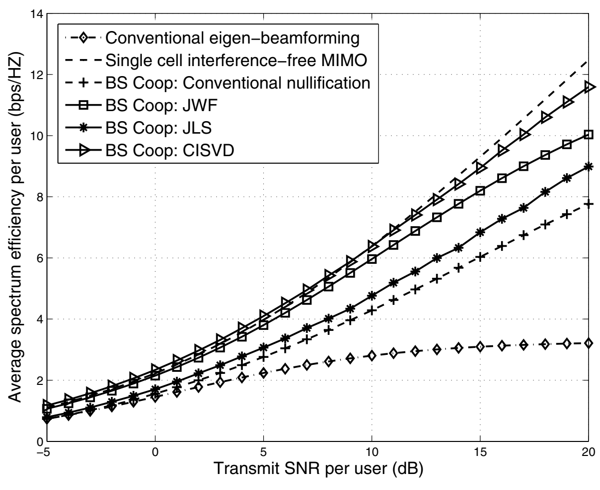

Fig. 4. Spectral efficiency comparison of JWF, JLS, CISVD, and conventional benchmark schemes in the presence of asynchronous interference

with BS cooperation. We showed that when cooperative BSs jointly transmit to multiple users, the data streams intended for the multiple users inevitably interfere asynchronously due to the different propagation delays. This is so even when perfect timing-advance is used to synchronize the reception of the desired signal components. We considered three linear precoding design methods previously proposed in the literature, which optimize different metrics, and came up with corresponding new precoding methods – JWF, JLS, and CISVD. When redundant spatial dimensions for diversity are available, i.e., when $N_T B > \sum_{k=1}^{K} L_k$, all the three methods markedly outperformed conventional designs that did not account for the asynchronicity in interference. CISVD realized significant gains, especially at high SNR, JLS achieved a good trade-off between asynchronous interference mitigation and algorithmic complexity, and JWF outperformed JLS at all SNRs. Asynchronous interference may even improve performance, instead of degrading it, if the receivers are aware of its time structure and are capable of exploiting it [20]. Essentially, the paper moves a step closer to realizing the great potential of BS cooperation in practical implementations of interference-limited multi-user MIMO systems.

APPENDIX

A. Optimal Linear Precoding for JWF: Key Derivation Steps

Denoting the MUI term in (1) as $\mathbf{J}_k = \sum_{\substack{j=1 \\ (j \neq k)}}^{K} \sum_{b=1}^{B} \mathbf{H}_k^{(b)} \mathbf{T}_j^{(b)} \mathbf{i}_{jk}^{(b)}(m)$, NMSE_k takes the form

$$\text{NMSE}_k = \frac{1}{\Omega_k} \mathbf{E}\left[\left(\mathbf{H}_k \mathbf{T}_k \mathbf{s}_k - \mathbf{A}_k \mathbf{s}_k + \mathbf{J}_k + \mathbf{n}_k \right)^\dagger \times \left(\mathbf{H}_k \mathbf{T}_k \mathbf{s}_k - \mathbf{A}_k \mathbf{s}_k + \mathbf{J}_k + \mathbf{n}_k \right) \right]. \quad (19)$$

Using the results of Sec. II-B and the identity $\mathbf{E}\left[\mathbf{J}_k^\dagger \mathbf{J}_k\right] = \text{Tr}\left\{\mathbf{E}\left[\mathbf{J}_k \mathbf{J}_k^\dagger\right]\right\}$, the above equation can be simplified to

$$\begin{aligned} \text{NMSE}_k &= \frac{1}{\Omega_k} \text{Tr}\left\{ \sum_{b_1=1}^{B} \sum_{b_2=1}^{B} \mathbf{H}_k^{(b_1)} \mathbf{T}_k^{(b_1)} \mathbf{T}_k^{(b_2)\dagger} \mathbf{H}_k^{(b_2)\dagger} + \mathbf{A}_k \mathbf{A}_k^\dagger \right\} \\ &- \frac{1}{\Omega_k} \text{Tr}\left\{ \mathbf{A}_k \sum_{b=1}^{B} \mathbf{T}_k^{(b)\dagger} \mathbf{H}_k^{(b)\dagger} + \sum_{b=1}^{B} \mathbf{H}_k^{(b)} \mathbf{T}_k^{(b)} \mathbf{A}_k^\dagger - N_0 \mathbf{I}_{N_R} \right\} \\ &+ \frac{1}{\Omega_k} \text{Tr}\left\{ \sum_{\substack{j=1 \\ (j \neq k)}}^{K} \sum_{b_1=1}^{B} \sum_{b_2=1}^{B} \beta_{jk}^{(b_1,b_2)} \mathbf{H}_k^{(b_1)} \mathbf{T}_j^{(b_1)} \mathbf{T}_k^{(b_2)\dagger} \mathbf{H}_k^{(b_2)\dagger} \right\}, \end{aligned}$$

To solve (8) in closed-form, we minimize the following Lagrange objective function:

$$f\left(\{\mathbf{T}_k\}_{k=1}^{K}\right) = \sum_{k=1}^{K} \text{NMSE}_k + \sum_{k=1}^{K} \kappa_k \left(\text{Tr}\left\{ \sum_{b=1}^{B} \mathbf{T}_k^{(b)\dagger} \mathbf{T}_k^{(b)} \right\} - P_k^{\text{tx}} \right), \quad (20)$$

where κ_k are the Lagrange multipliers associated with the power constraints for MSs $1, \ldots, K$, respectively. Using matrix calculus, it can be shown that the optimal form of $\mathbf{T}_k$ is given by (10), and that κ_k is one of the roots of the equation $\sum_{i=1}^{N_T B} \frac{b_{ki}}{(x+\lambda_{ki})^2} = P_k^{\text{tx}} \Omega_k^2$, where $\mathbf{C}_k$ has the eigenvalue decomposition $\mathbf{U}_k \mathbf{\Lambda}_k \mathbf{U}_k^\dagger$, $\mathbf{\Lambda}_k = \text{diag}\left\{\lambda_{k1}, \lambda_{k2}, \ldots, \lambda_{k(N_T B)}\right\}$, and $b_{ki} = \left[\mathbf{U}_k^\dagger \mathbf{H}_k^\dagger \mathbf{A}_k \mathbf{A}_k^\dagger \mathbf{H}_k \mathbf{U}_k\right]_{ii}$. Since these equations may have multiple solutions, $\kappa_1, \ldots, \kappa_K$ are jointly chosen to minimize the overall NMSE. Note that the optimal solution satisfies power constraints with equality at all the MSs.

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