On the Diversity-Multiplexing Tradeoff for Ordered SIC Receivers Over MIMO Channels

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***Abstract*—The diversity-multiplexing tradeoff for MIMO point-to-point channels and multiple access channels are first proposed and studied in [4][5]. While the optimal tradeoff curves for MIMO channels have been explicitly explored, those corresponding to some practical MIMO schemes are still open. One such example, as mentioned in [4][5], is the diversity-multiplexing tradeoff problem for ordered successive interference cancellation (SIC) receivers, which is the focus of this paper. In literature, the impact of the optimal ordering on the diversity order for V-BLAST SIC receivers is analyzed for 2-layer scenarios [2][3][6], but only conjectured for larger number of layers through numerical results [2][7]. In this paper, based on a novel geometrical analysis, we prove that under general settings, any ordering rule for a V-BLAST SIC receiver will not improve its performance regarding diversity-multiplexing tradeoff. Furthermore, extending the study to multiple access channels, we show that the two extreme points of the tradeoff curve remain unchanged regardless of ordering, which motivates us to predict that the whole tradeoff curve is the same as that of fixed-order detectors.**

Keywords-Diversity-multiplexing tradeoff, SIC, V-BLAST

I. INTRODUCTION

It is well-known that MIMO fading channels can be explored to provide either multiplexing gain or diversity gain. However, these two gains typically compete with each other, and the tradeoff between them is expressed by the diversity-multiplexing tradeoff curve proposed in the pioneering work of [4]. In [5], the discussion of diversity-multiplexing tradeoff is extended to multiple access channels, where the fundamental tradeoff among diversity gain, multiplexing gain and multiple access gain are effectively characterized. The tradeoff discussions in [4] and [5] mainly deal with the performance limits among all possible MIMO schemes, i.e. optimal joint encoding and decoding are employed so that the channel capacity is assumed to be achievable. There are also some discussions on suboptimal and practical encoding and decoding schemes, for example, the V-BLAST architecture [1], in which different layers are separately encoded [4]. The discussion on V-BLAST is naturally extended to the spatial-division multiple access (SDMA) systems (or multiple access systems) in [5], where independent transmitters are separately encoded, but can be jointly detected (through multiuser detection).

This paper is mainly focused on the impact of optimal ordering on the performance of V-BLAST and SDMA schemes with successive interference cancellation (SIC) receivers. In [4][5], the tradeoff curves of these schemes with fixed ordering are accurately derived. However, when ordering is involved, only some loose performance upper bounds are provided (using genie-aided schemes), which are shown to be still away from the optimal tradeoff curves. The difficulty lies in that, the explicit distributions of the ordered channel gains are no longer accessible: it is in general a problem of order statistics among inter-dependent random variables, an under-developed topic itself [9]. Recently, the diversity property of the ordered SIC receivers with fixed data rate ($r = 0$) is partly analyzed for some simplified scenarios, or conjectured from numerical results. In particular, in [2] [3][6], the diversity order for a two-layer V-BLAST scheme with an ordered SIC receiver is rigorously shown to be equal to that with fixed ordering; while in [2][7] numerical results are provided to show that the diversity of SIC receivers in a V-BLAST scheme with more than two layers is not increased by the ordering rule proposed in [1]. To the best of our knowledge, diversity achieved by ordered SIC receivers with general system settings has not been rigorously analyzed thus far. In this paper, using a novel geometrical approach, we rigorously prove that ordering will not improve the performance of V-BLAST SIC receivers, with respect to (w.r.t.) the diversity-multiplexing tradeoff, thus versifying the conjectures in [2][7]. Extending the study to SDMA SIC receivers, we show that the two extreme points of the diversity-multiplexing tradeoff curve, with $r = 0$ and $d = 0$ respectively, are not improved by applying optimal ordering. Therefore we predict that the whole tradeoff curve remains unchanged, although the accurate analysis for the intermediate points is still under investigation.

The rest of the paper is organized as follows. The problem formulation is provided in Section II. Our result on the diversity-multiplexing tradeoff for ordered V-BLAST SIC receivers with point-to-point channels is presented in Section III, and is extended to multiple access channels in Section IV. Finally Section V contains some concluding remarks.

II. PROBLEM FORMULATIONS

We consider a frequency non-selective block Rayleigh fading channel model, for which a point-to-point system can be expressed as:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \qquad (1)$$

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where the $N_R \times t$ matrix $\mathbf{y}$ is the received signal block; the $N_T \times t$ matrix $\mathbf{s}$ is the transmitted signal block, each row representing one separately encoded data layer with equal power allocation; $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_{N_T}]$ is an $N_R \times N_T$ channel matrix with the constraint $N_R \geq N_T$; and $\mathbf{n}$ is the background noise. Because we focus on the non-ergodic scenario, where each codeword spans one fixed fading block (i.e. in delay limited scenarios), the coding length t of each layer is actually immaterial in our study (thus can be chosen to be 1 for ease of analysis). As is known, layered one-dimensional coding can only bring coding gain but not the diversity gain.

Note that for a SDMA channel with K transmitters each equipped with N_T antennas, assuming that $N_T^{All} = KN_T$, (1) still applies, except that $\mathbf{H}$ is now expressed as:

$$\mathbf{H} = [\mathbf{H}^{(1)}, \mathbf{H}^{(2)}, \cdots, \mathbf{H}^{(K)}], \tag{2}$$

where $\mathbf{H}^{(k)}$ is an $N_R \times N_T$ sub-matrix corresponding to the channel between the kth transmitter and the receiver. As in [5], here we assume identical number of transmit antennas, N_T, for the K users, i.e. the symmetric case. Also $\mathbf{s}$ becomes

$$\mathbf{s} = [\mathbf{s}_1^T,\ \mathbf{s}_2^T, \cdots, \mathbf{s}_K^T]^T, \tag{3}$$

in which the $N_T \times t$ sub-matrix $\mathbf{s}_k$ is the data from transmitter k. Data streams from one transmitter are allowed to be jointly encoded, while those from different transmitters are independent (which can be viewed as a generalized layered structure, each layer containing more than one data stream). If $N_T = 1,\ \forall k$, SDMA coincides with V-BLAST.

Given the above channel models, following the same line as in [2]~[7], we apply the nulling-SIC algorithm at the receiver, with the understanding that its diversity order analysis, a study at high SNR regimes, also applies to MMSE-SIC receivers. Specifically, in the V-BLAST scenario, it is assumed that the detection order is fixed, and that (without loss of generality) detection starts from the layer corresponding to the first transmit antenna and ends at that of the N_Tth antenna. In the lth layer we at first cancel out the interference contributed by layers 1~l-1, based on the detected data in the previous steps. Then interference from layers yet to be detected (l+1~K) can be nulled out using linear zero-forcing methods. The post-processing SNR for the lth layer, assuming perfect decision feedback, is then proportional to the square of the projection height[1] from $\mathbf{h}_l$ to the space spanned by $\mathbf{h}_{l+1} \sim \mathbf{h}_{N_T}$, denoted by $\rho_l = \frac{\rho_0}{N_T} R_{l,span\{l+1,\cdots,K\}}$, where ρ_0 stands for the average SNR per receive antenna.

The diversity order of a communication system is defined as the slope of its joint error probability $\mathrm{P}_e(\rho_0)$ in log-scale at high SNR regimes [4]:

$$d = -\lim_{\rho_0 \to \infty} \frac{\log \mathrm{P}_e(\rho_0)}{\log(\rho_0)} = \lim_{\rho_0 \to \infty} \frac{\log \mathrm{P}_e(\rho_0)}{\log(1/\rho_0)}. \tag{4}$$

Also, we adopt the operator $\doteq$ as defined in [4], to denote exponential equality, i.e. we write $f(\rho_0) \doteq \rho_0^{-b}$ to represent $-\lim_{\rho_0 \to \infty} \frac{\log f(\rho_0)}{\log(\rho_0)} = b$. Equivalently (for the convenience of analysis in this paper), we use $f(x) \doteq x^b$ to represent $\lim_{x \to 0} \frac{\log f(x)}{\log x} = b$. The operators $\dot{\leq}$, $\dot{\geq}$, are similarly defined. Note that according to our notation, $f(x) \dot{\leq} g(x)$ indicates $f(x) \geq g(x)$ for sufficiently small x. It is known [4][8] that in non-ergodic scenarios the error probability is dominated by the outage probability. As indicated in [2][4][6], the diversity property of a fixed-order V-BLAST SIC receiver is determined by the first decoding layer. Therefore its diversity order is given by

$$d = \lim_{x \to 0} \frac{\log \Pr\{R_{1,Span\{2,3,\cdots,N_T\}} \leq x\}}{\log(x)}. \tag{5}$$

When non-zero multiplexing gain r is considered, a family of codes $\{\varsigma(\rho_0)\}$ over a block length shorter than fading coherence time is employed, one at each SNR level with the rate $R(\rho_0) = r \log \rho_0$, the analysis in [2][4][6] shows that for fixed-order V-BLAST SIC receivers, the diversity-multiplexing tradeoff is given by:

$$d(r) = (N_R - N_T + 1)(1 - r/N_T). \tag{6}$$

When the ordering rule in [1] is taken into considerations, the problem becomes more involved, as (5) is now replaced by:

$$d = \lim_{x \to 0} \frac{\log \Pr\{\max_k R_{k,Span\{\bar{k}\}} \leq x\}}{\log(x)}, \tag{7}$$

where $R_{k,Span\{\bar{k}\}}$ is the squared projection height from $\mathbf{h}_k$ to the space spanned all the other $N_T - 1$ column vectors. Since different $R_{k,Span\{\bar{k}\}}$ are inter-dependent, the exact distribution of the post-processing SNR of the first decoding layer is in general not accessible [2][4]. An exception is for the $N_T = 2$ case [2][6], for which we have

$$R_{1,2} = \|\mathbf{h}_1\|^2 \sin^2 \theta_{12}, \text{ and } R_{2,1} = \|\mathbf{h}_2\|^2 \sin^2 \theta_{12},$$

where θ_{ik} is the angle between the two vectors $\mathbf{h}_i$ and $\mathbf{h}_k$. The ordering rule is reduced to simply choosing the transmit antenna whose corresponding column vector has a larger norm, therefore the asymptotic exponential behavior of its outage probability can be explicitly explored. It is shown that in this case the diversity order is the same as that of the fixed-order case [2][6]. However, the analy-

[1] Projection height refers to the norm of the error vector, i.e., the difference between a vector and its projection onto a subspace.

sis for $N_T > 2$ scenarios, which will be rigorously derived in Section III, cannot be solved by the methods in [2][6].

For multiple access channels, we investigate the nulling-SIC detector as described in [5]. Specifically, with fixed detection order, when detecting the data streams for the *k*th transmitter, we null out the interference from transmitters *k*+1 to *K* by projecting $\mathbf{H}^{(k)}$ to the null space of the space spanned by the column vectors in $\mathbf{H}^{(k+1)} \sim \mathbf{H}^{(K)}$, after canceling the interference contributed by transmitters 1 to *k*-1 (fed back from previous detectors). According to [5], the fixed-order diversity-multiplexing tradeoff, denoting the asymptotic exponential behavior of the probability that any user is erroneously detected, can be expressed as:

$$d(r) = d^*_{N_T,\, N_R-(K-1)N_T}(r)\,. \tag{8}$$

where $d^*_{m,n}(r) = (m-r)(n-r)$ stands for the optimal diversity-multiplexing tradeoff for an $n \times m$ Rayleigh fading MIMO channel [4]. When ordering is involved, it is claimed in [5] that the diversity analysis is in general not accessible. We are able to partly solve it in Section IV, by applying the technique proposed in the following section.

III. ORDERED V-BLAST SIC RECEIVER

In this section, by applying a novel geometrical approach, we rigorously prove the following Theorem:

Theorem I: *For general $N_R \times N_T$ V-BLAST systems with SIC receivers, applying any ordering rule will not change the diversity-multiplexing tradeoff of the system.*

As indicated above, [2][6] verify Theorem I only for $N_T = 2$ case, while we will investigate the general scenario with arbitrary values of N_T. As we know, an exhaustive search among all the $N_T!$ possible permutations targeting the minimum system joint error probability represents the optimal ordering rule (and leads to the optimal diversity) [2]. However, its diversity property is generally hard to analyze. Through the following lemma we show that the ordering rule proposed in [1], although not optimal in minimizing the error probability, is optimal w.r.t. diversity order.

Lemma I: *The ordering rule in [1] that maximizes the post processing SNR at each detection step of the V-BLAST SIC receiver, is optimal with respect to the diversity order among all ordering rules.*

Proof: For any ordering rule, we define P_l as the average error probability of the *l*th decoding layer assuming perfect feedback (no interference from the previously decoded layers), P_l^J as the joint error probability of layers 1 to *l*, and P_e as the system joint error probability. Therefore, we can naturally get $P_1^J = P_1$, $P_e = P_{N_T}^J$ and

$$P_l^J = P_l(1-P_{l-1}^J) + P_{l-1}^J\,. \tag{9}$$

While in the second layer, (9) becomes

$$P_2^J = P_2(1-P_1) + P_1\,. \tag{10}$$

Since $P_2.P_1$ is an exponentially smaller term, we derive $P_2^J \doteq \max(P_1, P_2)$. By iteratively applying the above analysis in (9), we finally get $P_e \doteq \max(P_1, P_2, \cdots, P_{N_T})$. For any ordering rule, it is intuitive to have

$$\max(P_1, P_2, \cdots, P_{N_T}) = P_1,$$

because by assuming perfect feedback, together with identical noise statistics and transmit power across different layers, all the other layers face less interference than layer 1. We then have $P_e \doteq P_1$, and Lemma I follows, since the ordering rule in [1] can guarantee that the performance of the first layer is optimized.

■

We can then prove Theorem I by analyzing this specific ordering rule. In [13] we proposed a geometric analysis framework for analyzing the diversity property of transmit antenna selection for V-BLAST SIC receivers. Here a similar approach is adopted. From (7), for a fixed data rate ($r = 0$), the diversity order is determined by the outage probability of the first layer, which can be upper bounded as:

$$\Pr\{\max_k R_{k,Span\{\bar{k}\}} \le x\} \le \Pr\{R_{1,Span\{\bar{1}\}} \le x\}\,.$$

When $x \to 0$, the asymptotic exponential behavior of the above upper bound represents the diversity of non-ordered V-BLAST SIC receiver, which equals to $N_R - N_T + 1$. We then get

$$\Pr\{\max_k R_{k,Span\{\bar{k}\}} \le x\} \dot{\ge} x^{N_R-N_T+1}\,. \tag{11}$$

In the following we try to derive a tight probability lower bound bearing the same diversity property.

Lemma II: *With the above settings, we have*

$$\Pr\{\max_k R_{k,Span\{\bar{k}\}} \le x\} \dot{\le} x^{N_R-N_T+1}\,. \tag{12}$$

Proof: For the ease of illustration and without loss of generality, we analyze the $N_T = 3$ case, and the extension to general values of N_T is straightforward, as will be explained at the end of the proof. In this scenario, we can decompose $\mathbf{h}_1$ by Gram-Schmidt orthogonalization and coordinate transformation as:

$$\begin{aligned}\mathbf{h}_1 &= \alpha_3\hat{\mathbf{h}}_3 + \alpha_2\hat{\mathbf{h}}_{2,3} + \sqrt{R_{1,span\{2,3\}}}\,\hat{\mathbf{h}}_{1,span\{2,3\}} \\ &= \gamma_3\hat{\mathbf{h}}_3 + \gamma_2\hat{\mathbf{h}}_2 + \sqrt{R_{1,span\{2,3\}}}\,\hat{\mathbf{h}}_{1,span\{2,3\}},\end{aligned} \tag{13}$$

where $\hat{\mathbf{h}}_3 = \mathbf{h}_3/\|\mathbf{h}_3\|$; $\hat{\mathbf{h}}_{2,3}$ is the unit vector along the projection of $\mathbf{h}_2$ on $null(\mathbf{h}_3)$, the null space of the space spanned by $\mathbf{h}_3$; and $\hat{\mathbf{h}}_2$, $\hat{\mathbf{h}}_{1,span\{2,3\}}$ are similarly defined. α_1, α_2 and $\sqrt{R_{1,span\{2,3\}}}$, the coordinates of $\mathbf{h}_1$ on the orthonormal basis $\{\hat{\mathbf{h}}_3, \hat{\mathbf{h}}_{2,3}, \hat{\mathbf{h}}_{1,span\{2,3\}}\}$, can be shown as jointly independent [2][6]. The second equality in (13) is obtained with coordinate transformation from $\{\hat{\mathbf{h}}_3, \hat{\mathbf{h}}_{2,3}\}$ to $\{\hat{\mathbf{h}}_2, \hat{\mathbf{h}}_3\}$ in the space $span\{2,3\}$ such that $\alpha_3\hat{\mathbf{h}}_3 + \alpha_2\hat{\mathbf{h}}_{2,3} = \gamma_3\hat{\mathbf{h}}_3 + \gamma_2\hat{\mathbf{h}}_2$. Through geometrical analysis, we have

$$\gamma_2 = \frac{a_2}{\sin\theta_{23}}, \text{ and } \gamma_3 = a_3 - \frac{a_2\cos\theta_{23}}{\sin\theta_{23}}. \tag{14}$$

It is known the angle θ_{23} is independent with $R_{1,span\{2,3\}}$, as indicated in [11], so γ_2 and γ_3 are independent with $R_{1,span\{2,3\}}$. We then have the following claim:

$$\begin{aligned}
&\Pr\{\max_k R_{k,Span\{\bar{k}\}} \le x\} \\
&= \Pr\{R_{1,Span\{2,3\}} \le x, R_{2,Span\{1,3\}} \le x, R_{3,Span\{1,2\}} \le x\} \\
&\ge \Pr\{R_{1,Span\{2,3\}} \le x, R_2 \le b, R_3 \le b, \gamma_2 \ge \sqrt{b}, \gamma_3 \ge \sqrt{b}\},
\end{aligned} \tag{15}$$

where b is an arbitrary constant, $R_2 = \|\mathbf{h}_2\|^2$ and $R_3 = \|\mathbf{h}_3\|^2$. To prove this claim, we define the following events:

$$\begin{aligned}
&A: R_{1,Span\{2,3\}} \le x \\
&B: R_2 \le b, R_3 \le b, \gamma_2 \ge \sqrt{b}, \gamma_3 \ge \sqrt{b} \\
&C: R_{1,Span\{2,3\}} \le x, R_{2,Span\{1,3\}} \le x, R_{3,Span\{1,2\}} \le x.
\end{aligned} \tag{16}$$

Since $R_{2,Span\{1,3\}}$ is the magnitude of the projection from $\mathbf{h}_2$ to the null space of $span\{1,3\}$, it represents the shortest distance from vector $\mathbf{h}_2$ to any points in $span\{1,3\}$, and by choosing $\{\mathbf{h}_1, \hat{\mathbf{h}}_3\}$ as one basis of $span\{1,3\}$, together with (13), it can be upper bounded as:

$$\begin{aligned}
&R_{2,Span\{1,3\}} \\
&= \min_{\beta_{23},\beta_{21}} \left\| \sqrt{R_2}\hat{\mathbf{h}}_2 - (\beta_{23}\hat{\mathbf{h}}_3 + \beta_{21}\mathbf{h}_1) \right\|^2 \\
&= \min_{\beta_{23},\beta_{21}} \| \sqrt{R_2}\hat{\mathbf{h}}_2 - \beta_{23}\hat{\mathbf{h}}_3 \\
&\qquad - \beta_{21}[\gamma_2\hat{\mathbf{h}}_2 + \gamma_3\hat{\mathbf{h}}_3 + \sqrt{R_{1,span\{2,3\}}}\hat{\mathbf{h}}_{1,span\{2,3\}}] \|^2 \\
&\le \bar{\beta}_{21}^2 R_{1,span\{2,3\}} = \frac{R_2}{\gamma_2^2} R_{1,span\{2,3\}},
\end{aligned} \tag{17}$$

where the inequality is derived by taking specific values of β_{21} and β_{23}, so that the magnitudes in directions $\hat{\mathbf{h}}_2$ and $\hat{\mathbf{h}}_3$ are zeros, i.e. $\bar{\beta}_{21} = \sqrt{R_2}/\gamma_2$ and $\bar{\beta}_{23} = -\gamma_3\sqrt{R_2}/\gamma_2$. In another word, we upper bound $R_{2,Span\{1,3\}}$ by (non-orthogonal) projecting $\mathbf{h}_2$ onto $span\{1,3\}$ along the direction $\hat{\mathbf{h}}_{1,span\{2,3\}}$. Similarly, we have

$$R_{3,Span\{1,2\}} \le \frac{R_3}{\gamma_3^2} R_{1,span\{2,3\}}. \tag{18}$$

It is then easy to see that given the event $A \cap B$, event C is true, which proves the claim (15). Also note that the random variables R_2, R_3, γ_2 and γ_3 are independent with $R_{1,Span\{2,3\}}$, thus A and B are independent with each other, and the lower bound in (15) satisfies:

$$\begin{aligned}
&\Pr\{R_{1,Span\{2,3\}} \le x, R_2 \le b, R_3 \le b, \gamma_2 \ge \sqrt{b}, \gamma_3 \ge \sqrt{b}\} \\
&= \Pr\{R_{1,Span\{2,3\}} \le x\}\Pr\{R_2 \le b, R_3 \le b, \gamma_2 \ge \sqrt{b}, \gamma_3 \ge \sqrt{b}\} \\
&\doteq \Pr\{R_{1,Span\{2,3\}} \le bx\} \doteq x^{N_R - N_T + 1},
\end{aligned} \tag{19}$$

since $\Pr\{R_2 \le b, R_3 \le b, \gamma_2 \ge \sqrt{b}, \gamma_3 \ge \sqrt{b}\}$ will only introduce a constant factor.

Therefore, we get $\Pr\{\max_k R_{k,Span\{\bar{k}\}} \le x\} \dot{\le} x^{N_R - N_T + 1}$ for $N_T = 3$ case. From the derivations above, it is not difficult to see that the extension to the general N_T scenario is straightforward. Generally speaking, by the theorem of coordination transformation [10], (14) can be re-shaped as:

$$\boldsymbol{\gamma} = \mathbf{Q}\boldsymbol{\alpha},$$

where $\boldsymbol{\gamma}$ is the coordinates of $\mathbf{h}_1$'s projection on $span\{\bar{1}\}$ along the basis $\{\hat{\mathbf{h}}_2, \hat{\mathbf{h}}_3, ..., \hat{\mathbf{h}}_{N_T}\}$, $\boldsymbol{\alpha}$ is its coordinates along the orthonormal basis, and the transfer matrix $\mathbf{Q}$ is only associated with the directional relationships among $\{\hat{\mathbf{h}}_2, \hat{\mathbf{h}}_3, ..., \hat{\mathbf{h}}_{N_T}\}$. Both $\boldsymbol{\alpha}$ and $\mathbf{Q}$ are then independent with $R_{1,span\{\bar{1}\}}$ [6][11], so is $\boldsymbol{\gamma}$. Therefore we can build up a probability lower bound as in (15), whose proof follows the same line as in (17)~(19).

■

Finally by combining (11) and (12), it is shown that the diversity of the ordered V-BLAST SIC receiver is the same as the non-ordered case. If $r > 0$, the outage is now defined as the event that the instant code rate $R(\rho_0) = r\log\rho_0$ is larger than the channel capacity conditioned on the current channel state. Therefore, to derive a tradeoff curve $d(r)$, we only need to replace (7) by

$$d(r) = \lim_{\rho_0 \to \infty} \frac{\log\Pr\{\max_k R_{k,Span\{\bar{k}\}} \le N_T\rho_0^{-(1-\frac{r}{N_T})}\}}{\log(1/\rho_0)}$$

and we can easily derive the same result as (6) for the ordered case, so that Theorem I is proved.

IV. Ordered SDMA SIC Receiver

In multiple access channels, when each transmitter is equipped with more than one antenna, multiple data streams for one user are allowed to be jointly encoded in space and time, and the diversity analysis with ordering is much more complex than in V-BLAST. It is easily shown that Lemma I can be extended to the symmetric

SDMA case, which says that the ordering rule maximizing the per-user capacity at each detection stage (or general layer) is optimal with respect to diversity order. For user *k*, the equivalent channel after nulling can be expressed by

$$\mathbf{H}_{Equ}^{(k)} = [\mathbf{h}_{1,span\{\bar{\mathbf{H}}^{(k)}\}}^{(k)}, \mathbf{h}_{2,span\{\bar{\mathbf{H}}^{(k)}\}}^{(k)}, ..., \mathbf{h}_{N_T,span\{\bar{\mathbf{H}}^{(k)}\}}^{(k)}], \qquad (20)$$

where $\mathbf{h}_{i,span\{\bar{\mathbf{H}}^{(k)}\}}^{(k)}$ is the projection from the *i*th column in $\mathbf{H}^{(k)}$ to the null space of the space spanned by the channel matrices for all the other users. Then the ordering rule tells us that the system diversity-multiplexing tradeoff equals to:

$$d(r) = \lim_{\rho_0 \to \infty} \frac{\log \Pr\{\max_k C(\mathbf{H}_{Equ}^{(k)}, \rho_0) \le r \log \rho_0\}}{\log(1/\rho_0)}, \quad (21)$$

where $C(\mathbf{H}_{Equ}^{(k)}, \rho_0)$ is the capacity of the channel $\mathbf{H}_{Equ}^{(k)}$ given SNR ρ_0. However, different capacities are correlated in a complex manner, making the analysis of (21) difficult.

To get around this problem, we focus on the extreme points of the tradeoff curve. First we investigate the diversity bounds when $r = 0$ (fixed rate). As we know, given data rate *R*:

$$\begin{aligned} &\Pr\{\max_k C(\mathbf{H}_{Equ}^{(k)}, \rho_0) \le R\} \\ &\le \Pr\{C(\mathbf{H}_{Equ}^{(1)}, \rho_0) \le R\} \doteq \rho_0^{-N_T[N_R-(K-1)N_T]}, \end{aligned} \qquad (22)$$

so $d(0) \ge N_T[N_R - (K-1)N_T]$ and we are left to show a tight lower bound of the outage probability:

***Lemma III**: With the above settings, the diversity order with a fixed data rate can be upper bounded by:*

$$d(0) \le N_T[N_R - (K-1)N_T]. \qquad (23)$$

Proof: Assume the data of user k_0 is selected to be detected in the first stage of the SIC receiver, following the capacity-maximizing ordering rule mentioned above. The achievable diversity order of its channel matrix in (20) with fixed data rate, can be upper bounded by the full diversity schemes at both the transmitter and receiver (see [12] and references therein), e.g. orthogonal space-time block coding, selection diversity and/or maximum ratio combining (MRC), maximum ratio transmission (MRT)/MRC. After diversity combining, there is one equivalent spatial data stream whose outage probability admits [12]:

$$P_{k_0\_out}^{Full\ Div} \doteq \Pr(\left\|\mathbf{H}_{Equ}^{(k_0)}\right\|^2 \le x) \dot{\ge} x^{d(0)}, \qquad (24)$$

where $x = 1/\rho_0$. Then it is easily seen that by transmitting and detecting one data stream with a full-diversity scheme for each user, if user k_1 is selected at the first stage such that

$$\left\|\mathbf{H}_{Equ}^{(k_1)}\right\|^2 = \max_{k=1..K} \left\|\mathbf{H}_{Equ}^{(k)}\right\|^2, \qquad (25)$$

its diversity order, denoted as $d'(0)$, represents an diversity upper bound of $d(0)$, i.e.

$$\Pr(\max_{k=1..K} \left\|\mathbf{H}_{Equ}^{(k)}\right\|^2 \le x) \doteq x^{d'(0)} \dot{\ge} P_{k_0\_out}^{Full\ Div} \dot{\ge} x^{d(0)}. \quad (26)$$

Considering the length limitation on this paper and without loss of generality, we investigate the simplest case, where $K = 2$, $N_T = 2$ and the channel matrices are denoted by: $\mathbf{H}^{(1)} = [\mathbf{h}_1, \mathbf{h}_2]$, and $\mathbf{H}^{(2)} = [\mathbf{h}_3, \mathbf{h}_4]$. So after nulling we get the equivalent channels as:

$$\begin{aligned} \mathbf{H}_{Equ}^{(1)} &= [\mathbf{h}_{1,span\{3,4\}}, \mathbf{h}_{2,span\{3,4\}}] \\ \mathbf{H}_{Equ}^{(2)} &= [\mathbf{h}_{3,span\{1,2\}}, \mathbf{h}_{4,span\{1,2\}}]. \end{aligned} \qquad (27)$$

The outage probability in (26) can be rewritten by:

$$\begin{aligned} \Pr(\max_{k=1..K} \left\|\mathbf{H}_{Equ}^{(k)}\right\|^2 \le x) = \Pr(&R_{1,span\{3,4\}} + R_{2,span\{3,4\}} \le x, \\ &R_{3,span\{1,2\}} + R_{4,span\{1,2\}} \le x). \end{aligned} \qquad (28)$$

Similar as in (13), we decompose $\mathbf{h}_1$ and $\mathbf{h}_2$ by:

$$\begin{aligned} \mathbf{h}_1 &= \gamma_{13}\hat{\mathbf{h}}_3 + \gamma_{14}\hat{\mathbf{h}}_4 + \sqrt{R_{1,span\{3,4\}}}\hat{\mathbf{h}}_{1,span\{3,4\}} \\ \mathbf{h}_2 &= \gamma_{23}\hat{\mathbf{h}}_3 + \gamma_{24}\hat{\mathbf{h}}_4 + \sqrt{R_{2,span\{3,4\}}}\hat{\mathbf{h}}_{2,span\{3,4\}}. \end{aligned} \qquad (29)$$

So following the same method as in (17), we get

$$\begin{aligned} &R_{3,Span\{1,2\}} \\ &= \min_{\beta_{31},\beta_{32}} \left\|\sqrt{R_3}\hat{\mathbf{h}}_3 - (\beta_{31}\mathbf{h}_1 + \beta_{32}\mathbf{h}_2)\right\|^2 \\ &= \min_{\beta_{31},\beta_{32}} \| \sqrt{R_3}\hat{\mathbf{h}}_3 \\ &\qquad - \beta_{31}[\gamma_{13}\hat{\mathbf{h}}_3 + \gamma_{14}\hat{\mathbf{h}}_4 + \sqrt{R_{1,span\{3,4\}}}\hat{\mathbf{h}}_{1,span\{3,4\}}] \\ &\qquad - \beta_{32}[\gamma_{23}\hat{\mathbf{h}}_3 + \gamma_{24}\hat{\mathbf{h}}_4 + \sqrt{R_{2,span\{3,4\}}}\hat{\mathbf{h}}_{2,span\{3,4\}}] \|^2 \\ &\le \left\|\bar{\beta}_{31}\sqrt{R_{1,span\{3,4\}}} + \bar{\beta}_{32}\sqrt{R_{2,span\{3,4\}}}\right\|^2 \\ &\le \bar{\beta}_{31}^2 R_{1,span\{3,4\}} + \bar{\beta}_{32}^2 R_{2,span\{3,4\}}, \end{aligned} \qquad (30)$$

where the last step comes from the triangle inequality, and $\bar{\beta}_{31}$ and $\bar{\beta}_{32}$ are the solutions for zero coefficients of $\hat{\mathbf{h}}_3$ and $\hat{\mathbf{h}}_4$ in the second equality of (30), expressed by:

$$\begin{bmatrix} \bar{\beta}_{31} \\ \bar{\beta}_{32} \end{bmatrix} = \begin{bmatrix} \gamma_{13} & \gamma_{23} \\ \gamma_{14} & \gamma_{24} \end{bmatrix}^{-1} \begin{bmatrix} \sqrt{R_3} \\ 0 \end{bmatrix}. \qquad (31)$$

As what we argued in Section III (see (14)), since γ_{13} and γ_{14} are independent with $R_{1,span\{3,4\}}$, γ_{23} and γ_{24} are independent with $R_{2,span\{3,4\}}$, and R_3 is independent with any of the projection heights, we have that $\bar{\beta}_{31}$ and $\bar{\beta}_{32}$ are independent with $R_{1,span\{3,4\}} + R_{2,span\{3,4\}}$. With the same procedure, we can further derive:

$$R_{4,Span\{1,2\}} \le \bar{\beta}_{41}^2 R_{1,span\{3,4\}} + \bar{\beta}_{42}^2 R_{2,span\{3,4\}}, \qquad (32)$$

where

$$\begin{bmatrix} \overline{\beta}_{41} \\ \overline{\beta}_{42} \end{bmatrix} = \begin{bmatrix} \gamma_{14} & \gamma_{24} \\ \gamma_{13} & \gamma_{23} \end{bmatrix}^{-1} \begin{bmatrix} \sqrt{R_4} \\ 0 \end{bmatrix}, \quad (33)$$

which is also independent with $R_{1,span\{3,4\}} + R_{2,span\{3,4\}}$. So if the following event E is true:

$$E: \{R_{1,span\{3,4\}} + R_{2,span\{3,4\}} \le x/a; \\ \overline{\beta}_{31}^2, \overline{\beta}_{32}^2, \overline{\beta}_{41}^2, \overline{\beta}_{42}^2 \le a/2;\ a \ge 1\},$$

from (30) and (32), we get that $R_{1,span\{3,4\}} + R_{2,span\{3,4\}} \le x$ and $R_{3,span\{1,2\}} + R_{4,span\{1,2\}} \le x$ are both fulfilled. So we get the following lower bound of the outage probability in (28):

$$\Pr(R_{1,span\{3,4\}} + R_{2,span\{3,4\}} \le x, \\ R_{3,span\{1,2\}} + R_{4,span\{1,2\}} \le x) \ge \Pr(E). \quad (34)$$

Given

$$\Pr(E) \doteq \Pr(R_{1,span\{3,4\}} + R_{2,span\{3,4\}} \le x/a) \\ \doteq \Pr(\left\| \mathbf{H}_{Equ}^{(1)} \right\|^2 \le x) \doteq x^{N_T[N_R-(K-1)N_T]}, \quad (35)$$

from (26), we get

$$d(0) \le d'(0) \le N_T[N_R - (K-1)N_T]. \quad (36)$$

From the analysis in (27)~(36), it is easily seen that the extension to general values of K and N_T is straightforward except for more complicated mathematical expressions.

■

By combining (22) and (23), we finally get

$$d(0) = N_T[N_R - (K-1)N_T], \quad (37)$$

which means that under fixed data rate, the optimal ordering will not increase the diversity order of the joint error probability among the K users deploying SIC receivers. In another word, the diversity-multiplexing tradeoff curve at $r = 0$ is unchanged from the non-ordering case in [5]. At the other extreme point of the tradeoff curve where $d(r) = 0$, since $\mathbf{H}_{Equ}^{(1)} \sim \mathbf{H}_{Equ}^{(K)}$ are of the equivalent dimension $[N_R - (K-1)N_T] \times N_T$ [5], no matter which user is selected for detection at the first stage, we always have $r(0) = \min[N_R - (K-1)N_T, N_T]$, which equals to that in [5] for the non-ordering case. Therefore we predict that the whole diversity-multiplexing tradeoff curve will not be improved by optimal ordering, although the accurate analysis is still challenging for the intermediate points with $r > 0$.

V. CONCLUSIONS

In this paper, we propose a novel geometry-based method for analyzing the diversity-multiplexing tradeoff for SIC receivers implemented in point-to-point V-BLAST and SDMA systems. Our results rigorously show that the tradeoff curve for V-BLAST SIC receivers is not changed by ordering; while for SDMA, the diversity order with fixed data rate is not changed, and we predict that its whole tradeoff curve is the same as that of fixed-order receivers. Finally we stress that the diversity-multiplexing tradeoff is sometimes a loose indication of the error probability performance, since it only characterizes the exponential behavior. Although optimal ordering does not improve diversity order of SIC receivers, it still provides an SNR gain (coding gain), compared with fixed-order SIC receivers.

The topics of non-symmetric SDMA and the tradeoff curve with general values of *r* direct our future research.

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