DISTRIBUTED VERSUS CO-LOCATED MIMO SYSTEMS WITH CORRELATED FADING AND SHADOWING

Huaiyu Dai

Department of Electrical and Computer Engineering, NC State University, Raleigh, NC 27695
Email: Huaiyu_Dai@ncsu.edu

ABSTRACT

MIMO communications has been a highly active research area recently due to its remarkable capacity potential. The predicted capacity gain is nonetheless greatly limited in realistic propagation scenarios, especially when the number of antennas becomes large. This motivates us to investigate a generalized paradigm for multiple-antenna communications, called distributed MIMO, which has the potential to address many of the problems inherent in conventional co-located MIMO systems. In this paper, we demonstrate the advantages of distributed MIMO versus co-located MIMO in correlated fading and shadowing scenarios through asymptotic analysis.

1. INTRODUCTION

MIMO techniques are anticipated to be widely employed in future wireless networks to address the ever-increasing capacity and quality demands. The main question is whether the enormous gains predicted can be achieved in realistic deployment. In other words, if we keep adding antennas into MIMO systems, can we keep obtaining expected returns even if the increased cost in deployment, hardware and computation can be afforded? Unfortunately the answer is no, if the antennas are to be packed together with spacing of the order of wavelength in the traditional way.

The leading reason, as is already known, comes from spatial correlation due to the scattering environment and antenna configuration [1]. Typically the aperture of antenna array cannot grow without bound due to physical limitations, so packing more antennas necessarily reduces antenna spacing and increases spatial correlation. Generally, spatial correlation increases the condition number of the channel matrix (i.e., spreads out the singular value distribution), not necessarily reducing the channel rank. The loss thus incurred can be measured in two ways. The concept of effective degrees of freedom proposed in [2] discards eigenmodes with negligible capacity at a given SNR, while in [3] the loss in growth rate (spectral efficiency per antenna) is evaluated as the number of antennas goes to infinity. In the extreme case (e.g. near-zero angle spread), the channel rank is hard-limited by the few independent propagation paths, so putting more antennas can by no means increase spatial degrees of freedom, though other advantages like diversity and array gains may still be preserved.

Another reason may be less obvious. Current study of MIMO systems seldom explicitly addresses the shadow fading issues, though it is natural to expect severely diminished link quality when unfavorable shadowing is experienced. Multiple antennas sited in the same locale experience the same shadowing thus cannot improve the situation.

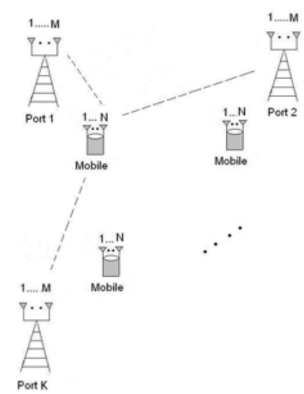

Fig. 1 Distributed MIMO Systems

Distributed MIMO (D-MIMO), originated from distributed antenna systems [4], has been proposed to remedy the problems inherent in co-located MIMO (C-MIMO) [5][6]. As depicted in Fig. 1, the key difference between D-MIMO and C-MIMO is that multiple antennas for one end of communications are *distributed* among multiple *widely-separated* radio ports, and *independent* large-scale fading is experienced for each link between a mobile-port pair. As understood in Fig. 1 from a downlink viewpoint, a D-MIMO system can be represented with a triplet of (K, M, N), while a C-MIMO can be viewed as a $(1, KM, N)$ D-MIMO. Note that although our illustration will focus on this downlink cellular scenario, our results

This work was supported in part by the National Science Foundation under Grant CCF-0515164.

will apply largely to other important applications such as cooperative MIMO mobile users or wireless sensor nodes. Intuitively, D-MIMO should significantly outperform C-MIMO under correlated fading and shadowing. This paper intends to provide some solid justifications for such intuitions, including analysis on capacity loss due to spatial correlation in Section 3 and analysis on capacity loss due to shadowing in Section 4. We start with our system model and preliminaries in the following section.

2. SYSTEM MODEL

Consider a downlink one-ring composite channel model

$$\mathbf{y} = \mathbf{H}_w \mathbf{R}_T^{1/2} \mathbf{\Phi}^{1/2} \mathbf{x} + \mathbf{n}\,, \tag{1}$$

where $\mathbf{y}$ is the received vector, $\mathbf{x}$ is the transmitted vector with the total transmit power ρ equally divided among transmit antennas, $\mathbf{H}_w \mathbf{R}_T^{1/2}$ represents small-scale fading while $\mathbf{\Phi}$ captures the large-scale fading effect, and $\mathbf{n}$ is the noise vector. The entries of $\mathbf{H}_w$ and $\mathbf{n}$ are i.i.d. complex Gaussian with zero mean and unit variance. $\mathbf{R}_T$ is deterministic accounting for spatial correlation at the base station(s) (or radio port(s)). We assume a block flat-fading scenario where $\mathbf{H}_w$ changes independently from one block to another, and $\mathbf{\Phi}$ changes independently with $\mathbf{H}_w$ at a lower pace. This model includes outdoor cellular macrocell environments, where mobile terminals are surrounded by rich scatterers while antenna arrays at the base stations are elevated above urban clusters and far away from local scattering. Extensions to other scenarios are straightforward.

Our study focuses on the asymptotic scenario that both the total number of transmit and receive antennas go to infinity, with their ratio fixed. As such, we will only consider a $(M,1,N)$ D-MIMO for simplicity, and compare it with a $(1,M,N)$ C-MIMO. Besides analytical tractability through laws of large numbers and random matrix theory, the study of large system performance also has practical advantages: what is revealed in the asymptotic limit is fundamental in nature, which may be concealed in the finite case by random fluctuations and other transient properties of the matrix entries; moreover, the convergence to the asymptotic limit is typically rather fast as the system size grows. We also focus our study on the high SNR regimes.

Throughout the paper, uniform linear arrays are employed on both base stations and mobiles, which dictates a Hermitian Toeplitz structure for $\mathbf{R}_T$ for C-MIMO, i.e.,

$$\{\mathbf{R}_T\}_{ij} \triangleq \rho_{j-i} = \rho_{i-j}^{*}\,.$$

Without loss of generality, we exclude the path loss effect and assume i.i.d shadow fading for different ports of D-MIMO. Thus for C-MIMO, $\mathbf{\Phi} = \Phi \mathbf{I}_M$; while for D-MIMO $\mathbf{\Phi} = \mathrm{diag}(\Phi_1, \cdots, \Phi_M)$. The shadow fading coefficient is usually modeled as $\Phi = e^Y$, where $Y \sim \mathcal{N}(\lambda\mu_L, (\lambda\sigma_L)^2)$ is a Gaussian random variable, with μ_L (dB) the area mean, σ_L (dB) the decibel spread, and $\lambda = \ln 10/10$. The CDF of Φ is given by

$$F_\Phi(x) = 1 - Q\left(\frac{\ln x - \lambda\mu_L}{\lambda\sigma_L}\right), \tag{2}$$

where $Q(\cdot)$ is the standard Gaussian tail function. We also have

$$E\{\Phi\} = e^{\lambda\mu_L + \frac{\lambda^2\sigma_L^2}{2}}, E\{\Phi^2\} = e^{2\lambda\mu_L + 2\lambda^2\sigma_L^2}\,. \tag{3}$$

To avoid confounding effects, in Section 3 we will focus on the correlated fading case by assuming $\mathbf{\Phi} = \mathbf{I}$, while in Section 4 we will assume $\mathbf{R}_T = \mathbf{I}$ to study the impact of shadowing. A joint study on both effects constitutes our future work.

3. CAPACITY LOSS DUE TO SPATIAL CORRELATION

In this section, we follow the approach in [3] to study the asymptotic capacity loss per spatial dimension due to fading correlation. We restrict our study to the scenario where the channel matrix is square ($M = N$), which is only known at the receiver so transmit power is equally distributed among transmit antennas, and examine

$$I^0 = \lim_{M\to\infty} \frac{I(\mathbf{H})}{M} = \lim_{M\to\infty} \frac{1}{M} \log\det\left(\mathbf{I} + \frac{\rho}{M}\mathbf{H}_w \mathbf{R}_T \mathbf{H}_w^H\right). \tag{4}$$

As is known from random matrix theory, (4) is insensitive to channel realizations for large *M*. At high SNR, the loss in I^0 can be nicely quantified as

$$\Delta I^0 = \int_0^1 \log S_T(f)\,df\,, \tag{5}$$

where $S_T(f) = \sum_i \rho_i e^{j2\pi i f}$ is the spectral density of the Toeplitz matrix $\mathbf{R}_T$. The underlying rationale is that the distribution of the eigenvalues of $\mathbf{R}_T$ asymptotically (as $M \to \infty$) approaches that of $S_T(f)$ on $[0,1]$ [7].

First let us consider a somewhat simplified model for $\mathbf{R}_T$

$$\rho_i = \eta^i \tag{6}$$

for some complex number η with $|\eta| \le 1$. In this case

$$S_T(f) = \frac{1-|\eta|^2}{\left|1-\eta e^{i2\pi f}\right|^2}\,. \tag{7}$$

A complex integral calculation reveals

$$\Delta I^0 = \log\left(1-|\eta|^2\right). \tag{8}$$

This simple model tells quite a bit about capacity loss due to spatial correlation: it is a monotonic decreasing function of the correlation coefficient η between two adjacent antenna elements, and the loss goes without bound as (8).

Next let us examine a more practical model [8]

$$\rho_i = e^{-j2\pi i\Delta\cos(\bar{\theta})} \cdot q^{i^2}, \text{ with } q = e^{-(1/2)(2\pi\Delta\sin(\bar{\theta})\sigma_\theta)^2}, \quad (9)$$

where Δ is the relative antenna spacing with respect to the wavelength, $\bar{\theta}$ is the mean angle of departure, and σ_θ is a parameter reflecting the angle spread, all for the transmit array. In this case

$$S_T(f) = \vartheta_3(\pi(f - \Delta\cos\bar{\theta}), q) \quad (10)$$

where $\vartheta_3(\cdot,\cdot)$ is the third-order theta function given in [9]. Using the following expansion

$$\vartheta_3(u,q) = \prod_{n=1}^{\infty}(1-q^{2n})((1+2q^{2n-1}\cos 2u + q^{2(2n-1)}), \quad (11)$$

we get

$$\Delta I^0 = \sum_{n=1}^{\infty}\log(1-q^{2n}) = \frac{1}{3}\log\left(\frac{1}{2}\vartheta_1'(0,q)\right) - \frac{1}{12}\log q, \quad (12)$$

where $\vartheta_1'(\cdot,\cdot)$ is the derivative of the first-order theta function. A schematic demonstration of (12) is given below.

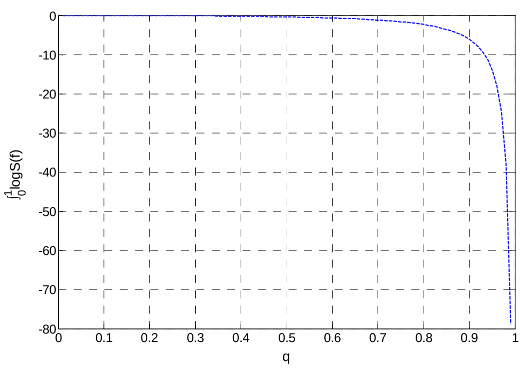

Fig. 2 Capacity Loss due to Fading Correlation

Again, it is a monotonic decreasing function of *q*, but by the definition of *q* and the above quantitative results, we will be able to directly relate the capacity loss with the environment and system parameters including the angle spread and antenna spacing. Specifically, we observe that D-MIMO simultaneously enlarges the antenna spacing Δ and angle spread σ_θ, rendering a *q* small enough to incur negligible loss. We can also calculate the critical antenna spacing, or the optimal antenna number for a given aperture and propagation environment, just to name a few applications. In this paper we focus on flat-fading channels. It is found in [8] that delay spread channels offer advantages over flat-fading channels in terms of ergodic capacity for C-MIMO systems, a result of the assumption that delay paths tend to increase the total angle spread. The proposed D-MIMO offers capacity gains over C-MIMO even for flat-fading channels, due to its inherent large angle spread and wide antenna spacing.

4. CAPACITY LOSS DUE TO SHADOWING

Properly normalized, shadow fading should not have impact on ergodic capacity for C-MIMO, but its induced variance should deteriorate the outage capacity. The impact of shadowing on the outage capacity $C^{(p)}$, corresponding to an outage probability $p = P\left(I(\mathbf{H}) < C^{(p)}\right)$ for C-MIMO has been studied in [10], where $I(\mathbf{H}) = \log\det\left(\mathbf{I} + \frac{\rho}{M}\mathbf{H}_w\mathbf{\Phi}\mathbf{H}_w^H\right)$ is the instantaneous channel mutual information. The main results are summarized below.

Theorem 1: For large M, N, ρ, with $M/N \to \beta$, the outage capacity for C-MIMO under shadow fading $C_{C,S}^{(p)}$ admits

$$E_\Phi\left\{Q\left(\frac{\min(M,N)\log\Phi + \mu - C_{C,S}^{(p)}}{\sigma}\right)\right\} = p, \quad (13)$$

where

$$\mu = \begin{cases} M\left[\log\frac{\rho}{e} + \log\frac{1-\beta}{\beta} + \frac{1}{\beta}\log\frac{1}{1-\beta}\right] & \beta < 1 \\ N\log\frac{\rho}{e} & \beta = 1 \\ N\left[\log\frac{\rho}{e} + (\beta-1)\log\frac{\beta}{\beta-1}\right] & \beta > 1 \end{cases} \quad (14)$$

and (γ is the Euler constant)

$$\sigma = \begin{cases} \log e\sqrt{\ln\frac{1}{1-\beta}} & \beta < 1 \\ \log e\sqrt{\ln N + \gamma + 1} & \beta = 1 \\ \log e\sqrt{\ln\frac{\beta}{\beta-1}} & \beta > 1. \end{cases} \quad (15)$$

In particular, for log-normal shadowing (2)

$$\begin{aligned} C_{C,S}^{(p)} &= (\mu + \min(M,N)\lambda\mu_L\log e) \\ &\quad -\sqrt{\sigma^2 + \min^2(M,N)\lambda^2\sigma_L^2\log^2 e}\,Q^{-1}(p). \end{aligned} \quad (16)$$

Remark: Result (16) should be compared with the pure Rayleigh fading case

$$C_{C,R}^{(p)} = \mu - \sigma Q^{-1}(p). \quad (17)$$

In both cases, the first term denotes the mean (ergodic) capacity due to symmetry of involved distributions, whose difference is somewhat related to the artifacts in the definition of shadowing parameters. What is of more interest is the difference in the second term, which clearly quantifies the detrimental effect of shadow fading on outage capacity.

Our main results for D-MIMO are given as follows.

Theorem 2: For large M, N, ρ, with $M/N \to \beta \le 1$, the outage capacity for D-MIMO under shadow fading is

$$\begin{aligned} C_{D,S}^{(p)} &= \mu + ME_\Phi\{\log\Phi\} \\ &\quad -\sqrt{\sigma^2 + M(E_\Phi\{\log^2\Phi\} - E_\Phi^2\{\log\Phi\})}\,Q^{-1}(p). \end{aligned} \quad (18)$$

In particular, for log-normal shadowing (2)

$$\begin{aligned} C_{D,S}^{(p)} &= (\mu + M\lambda\mu_L\log e) \\ &\quad -\sqrt{\sigma^2 + M\lambda^2\sigma_L^2\log^2 e}\,Q^{-1}(p). \end{aligned} \quad (19)$$

Remark: Compared with (16), an improvement in outage capacity is observed for D-MIMO with shadow fading. For $\beta > 1$, we have $C_{D,S}^{(p)} = \mu_D - \sigma_D Q^{-1}(p)$, where

$$\mu_D = N\left[\log\frac{\rho}{e} + \beta E_\Phi\{\log(1+C_2\Phi)\} - \log C_2\beta\right], \quad (20)$$

and C_2 is the solution to $E_\Phi\left\{\frac{C_2\Phi}{1+C_2\Phi}\right\} = \frac{1}{\beta}$. An explicit form for σ_D is not available, other than that it is the standard deviation of $\sum_{i=1}^N \log\lambda_i$, where $\{\lambda_i\}$ are the nonzero eigenvalues of $\mathbf{H}_w\mathbf{\Phi}\mathbf{H}_w^H$. However, for $\beta\to\infty$, a case of special interest, we may equivalently study the scenario that $M\to\infty$ while N keeps fixed, for which we have (with log-normal shadowing (2))

$$C_{D,S}^{(p)} = N\left[\log\rho + \left(\lambda\mu_L + \frac{\lambda^2\sigma_L^2}{2}\right)\log e\right] - \log e\sqrt{\frac{N}{M}[(1+N)e^{\lambda^2\sigma_L^2} - N]}Q^{-1}(p). \quad (21)$$

This should be compared with

$$C_{C,R}^{(p)} = N\log\rho - \log e\sqrt{\frac{N}{M}}Q^{-1}(p), \quad (22)$$

and

$$C_{C,S}^{(p)} = N(\log\rho + \lambda\mu_L\log e) - \log e\sqrt{\frac{N}{M} + N^2\lambda^2\sigma_L^2}Q^{-1}(p) \quad (23)$$

in the same scenario. We observe that D-MIMO asymptotically (as $M\to\infty$) evens out the detrimental effect of shadowing thanks to inherent macrodiversity, which nonetheless remains significant for C-MIMO.

Due to space limitations, we only give a sketch of proof for Theorem 2 in the following.

Sketch of proof: First we can show that $I(\mathbf{H})$ is asymptotic Gaussian, following similar steps in [11][12]. So we only need to calculate its mean and variance. By large random matrix theory, it can be shown that $I(\mathbf{H})$ converges almost surely to its mean given by

$$N\left[\beta E_\Phi\left\{\log\left(1+\frac{\rho}{\beta}\eta\Phi\right)\right\} - \log\eta + (\eta-1)\log e\right], \quad (24)$$

where the parameter η (multiuser efficiency of the MMSE receiver) can be approximated at high SNR as

$$\eta(\rho) = \begin{cases} 1-\beta & \beta<1 \\ C_1\rho^{-1/2} + O(\rho^{-1}) & \beta = 1 \\ C_2\rho^{-1} + o(\rho^{-1}) & \beta>1, \end{cases} \quad (25)$$

where $C_1 = \sqrt{E_\Phi\{1/\Phi\}}$, and C_2 is the same as given in (20). Plug (25) in (24) we can obtain the expression for the mean in (18) and (20) after some manipulation. The calculation for the variance requires some algebra and is omitted here.

■

5. CONCLUSIONS AND EXTENSIONS

This work provides some quantitative results to demonstrate the benefits of employing distributed MIMO systems in correlated fading and shadowing environments. Though the results rely on asymptotic analysis, they match simulation results quite well, as observed in literature and our work [10][12]. Extensions of current work include joint study of correlated fading and shadowing, the cooperative processing potential, and the outage-scheduling gain tradeoff in D-MIMO.

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