Large-System Spectral Efficiency of Interference-Limited MIMO Systems

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Abstract— **In this paper, the spectral efficiency of multiple-input multiple-output (MIMO) systems operating in multicell frequency-flat fading environments is studied, for situations in which co-channel interference is the dominant channel impairment instead of ambient noise. The following detectors are analyzed: a single-cell detector, the joint optimum detector, a group linear minimum-mean-square-error (MMSE) detector and its generalized version, with the focus on their large-system asymptotic (non-random) expressions. Analytical and numerical results based on these asymptotic multicell MIMO spectral efficiencies are explored to gain insights into the behavior of multicell MIMO systems.**

I. Introduction

Recent information theoretic results have suggested the remarkable capacity potential of wireless communication systems with antenna arrays at both the transmitter and receiver ends. These so-called multiple-input multiple-output (MIMO) systems have been shown to yield unprecedented capacity, which grows at least linearly with the number of antennas [3], [8], when operating in an isolated cell with white Gaussian background noise only. However, achieving this capacity in real cellular environments can be problematic. In this situation, the co-channel interference from surrounding cells is typically the dominant channel impairment and greatly diminishes MIMO system capacity [1].

In previous work the authors and Molisch showed that the performance of MIMO systems can be significantly improved in a multicell structure, through application of advanced signal processing techniques [2]. In particular, we employed a turbo space-time multiuser receiver structure for intracell communication, which essentially approaches the Shannon limit (within 1-2 dB) for an isolated cell. Furthermore, we used another level of multiuser detection (MUD) to combat the intercell interference. Simulation results indicated significant performance improvement of our approach over the well-known V-BLAST techniques with coding. Nevertheless, numerical results also indicated that there is a significant performance gap between the obtained MUD capacity with strong interference and the interference-free theoretical capacity.

In this paper, we go on to study the underlying rationale of the advantages and limitations of the receiver structures proposed in [2]. In particular, spectral efficiencies (bits/s/Hz) of these receivers are derived and compared with some other receivers of interest. We always assume optimal detection (well approximated by turbo space-time multiuser detection) for intracell communication, so the receivers are differentiated by the multiuser detection methods used to combat the intercell interference, i.e., *a single-cell detector, the joint optimum detector, a group linear MMSE detector*. We develop asymptotic results as the network dimensions grow, based on the application of analytical results on the eigenvalue distributions of large random matrices [7]. That is, we consider the limiting region where both the number of transmit antennas K and receive antennas N go to infinity, while their ratio remains constant. Besides its analytical convenience, the study of large system performance also has practical advantages: what is revealed in the asymptotic limit is fundamental in nature, which may be concealed in the finite case by random fluctuations and other transient properties of the matrix entries; moreover, the convergence to the asymptotic limit is typically rather fast as the system size grows. The asymptotic analysis has been carried out both for single-cell MIMO systems [8] and CDMA systems with random signatures [6] before, and is readily applied to this study.

This paper is organized as follows. Section II presents the system model and empirical eigenvalue distributions of some large random matrices that will be useful in the sequel. In Section III, formulas are derived for the spectral efficiencies of multicell MIMO systems with several optimum and sub-optimum detectors, together with their large-system asymptotic expressions. In Section IV, some analytical and numerical results based on these asymptotic multicell MIMO spectral efficiencies are given. Finally, Section V contains some concluding remarks.

II. System Model

A. MIMO System Model

For single-cell MIMO systems, we adopt the same mathematical model as in [3] and [8], given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}\,, \tag{1}$$

where $\mathbf{y}$ is the received vector corresponding to the outputs of N receive antennas, $\mathbf{x}$ contains the substreams transmitted by K transmit antennas, $\mathbf{H}$ is an $N \times K$ channel matrix that captures the channel characteristics between transmit and receive antenna arrays, and $\mathbf{n}$ is the background noise. Throughout this paper, we assume $N \ge K$, and define $\beta = K/N$ to be the system load. The entries of $\mathbf{H}$ are independent and identically distributed (i.i.d.) normalized complex Gaussian random variables, modeling a Rayleigh flat fading channel with adequate physical separation between transmit and receive antennas. The noise is assumed to be circularly symmetric Gaussian with covariance matrix $\mathbf{\Phi_n} = \sigma^2\mathbf{I}$, where $\mathbf{I}$ denotes an identity matrix. We assume the total transmitted power is constrained to be no larger than P, and is equally assigned to the independent substreams due to the lack of channel state information (CSI)

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at the transmitter, i.e., $E\{\mathbf{x}\mathbf{x}^H\} = (P/K)\mathbf{I}$. The signal-to-noise ratio (SNR) is given by $\rho = P/\sigma^2$. The channel matrix is always assumed to be known at the receiver, which is reasonable in a quasi-static fading environment, such as that arising, for example, in indoor wireless LAN applications.

B. Multicell Communication Model

In the literature [6], [12], multicell systems are often addressed with the attractive infinite linear array model (i.e., Wyner's model):

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \alpha\mathbf{H}^{-}\mathbf{x}^{-} + \alpha\mathbf{H}^{+}\mathbf{x}^{+} + \mathbf{n}\,, \qquad (2)$$

where only the adjacent-cell interference is taken into account, characterized by a single attenuation factor $0 \le \alpha \le 1$. In this paper, we adopt the more realistic planar multicell model given as follows [2]:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \sum_{i=1}^{L} \alpha_i \mathbf{H}_{ifi}\mathbf{x}_{ifi} + \mathbf{n}\,, \qquad (3)$$

where we assume without loss of generality that $0 \le \alpha_L \le \cdots \le \alpha_1 \le 1$ with L the number of effective interfering cells. In the sequel, we sometimes assign the desired cell index 0 with the understanding of $\alpha_0 = 1$ for convenience. The signals, channels and background noise for all the cells are assumed to be mutually independent and follow the same assumptions given in the subsection II.A. The signal-to-noise ratio is given by $\rho = P/\sigma^2$ as before, and the signal-to-interference ratio (SIR) is given by $\mu = 1\Big/\sum_i \alpha_i^2$. We focus mainly on the case in which all the cells are identical, and all the MIMO users operate at the same rate, which is further distributed among K substreams equally.

C. Empirical Distribution of a Random Eigenvalue

Suppose $\mathbf{A}$ is a $p \times p$ matrix with all real eigenvalues. The empirical distribution function of the eigenvalues of $\mathbf{A}$ is defined as $F^{\mathbf{A}}(x) = \frac{1}{p}\#(\lambda_{\mathbf{A}} \le x)$ with "#" denoting the cardinality, which refers to the proportion of eigenvalues of $\mathbf{A}$ that lie below x. Equivalently, $F^{\mathbf{A}}$ can be viewed as the cumulative distribution function of a uniformly randomly selected eigenvalue of $\mathbf{A}$. The following theorem is needed to calculate the asymptotic spectral efficiency of MIMO systems. This theorem requires the definition of the Stieltjes transform for any distribution function G, given as

$$m_G(z) = \int \frac{1}{\lambda - z}\, dG(\lambda) \qquad (4)$$

for $z \in C^{+} \triangleq \{z \in C : \mathrm{Im}\, z > 0\}$.

Theorem II.1 [7]: Suppose $\mathbf{X}$ is an $N \times n$ matrix containing i.i.d. complex entries with unit variance, and $\mathbf{T}$ is an $n \times n$ diagonal matrix, independent of $\mathbf{X}$. Assume that, almost surely, as $n \to \infty$, $F^{\mathbf{T}}$ converges to a distribution function H, and the ratio $n/N \to c > 0$. Then, almost surely, $F^{(1/N)\mathbf{X}\mathbf{T}\mathbf{X}^H}$ converges to a nonrandom distribution function G. The Stieltjes transform m_G of G is the unique (pointwise) solution to

$$m(z) = \frac{1}{-z + c\displaystyle\int \frac{\tau}{1 + \tau m(z)}\, dH(\tau)}\,, \; z \in C^{+}\,. \qquad (5)$$

For the special case of $\frac{1}{N}\mathbf{H}\mathbf{H}^H$, where $\mathbf{H}$ is the channel matrix of our model, according to Theorem II.1, the Stieltjes transform of the limiting distribution is given by

$$m_G(z) = \frac{(-1+\beta-z) + \sqrt{-4z + (1+z-\beta)^2}}{2z} = -\frac{1}{z} - \frac{1}{4}F(-\frac{1}{z}, \beta)\,, \qquad (6)$$

where

$$F(x,z) \triangleq \left(\sqrt{x(1+\sqrt{z})^2 + 1} - \sqrt{x(1-\sqrt{z})^2 + 1}\right)^2. \qquad (7)$$

The limiting distribution admits a closed form expression in this case, whose probability density function is given by

$$f_\beta(x) = [1-\beta]^{+}\delta(x) + \frac{\sqrt{[x - a(\beta)]^{+}[b(\beta) - x]^{+}}}{2\pi x}\,, \qquad (8)$$

where $\delta(x)$ is a unit point mass at 0, $[x]^{+} = \max\{x, 0\}$, and $a(x) = (1-\sqrt{x})^2$, $b(x) = (1+\sqrt{x})^2$. Similarly, $F^{(1/K)\mathbf{H}\,\mathbf{H}^H}$ converges to the distribution function of

$$f_{1/\beta}(x) = [1-\beta]^{+}\delta(x) + \frac{\sqrt{[x - a(1/\beta)]^{+}[b(1/\beta) - x]^{+}}}{2\pi(1/\beta)x}\,. \qquad (9)$$

III. Asymptotic spectral Efficiency of Multicell MIMO Systems

For the single-cell model (1) and the associated assumptions, the optimum spectral efficiency is given by [3], [8]

$$C_{S-opt} = \log\det\left[\frac{\sigma^2\mathbf{I} + \frac{P}{K}\mathbf{H}\mathbf{H}^H}{\sigma^2\mathbf{I}}\right] = \log\det\left[\mathbf{I} + \frac{\rho}{K}\mathbf{H}\mathbf{H}^H\right]. \qquad (10)$$

In the limiting region, we can express (10) as

$$\lim_{N\to\infty}\frac{1}{N}C_{S-opt} = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\log(1+\rho\lambda_i) = E\{\log(1+\rho\lambda)\}, \qquad (11)$$

where $\{\lambda_i\}$ are the eigenvalues of $\frac{1}{K}\mathbf{H}\mathbf{H}^H$, whose limiting probability density function is given by (9). Note that $C(x) = E\{\log(1+x\lambda)\}$ is an increasing function of x with $C(0) = 0$ and $\frac{d}{dx}C(x) = E\left\{\frac{\lambda}{1+x\lambda}\right\}$. So we have

$$\begin{aligned}\lim_{N\to\infty}\frac{1}{N}C_{S-opt} &= \log e \cdot \int_0^{\rho} E\left\{\frac{\lambda}{1+x\lambda}\right\}dx \\ &= \log e \cdot \int_0^{\rho}\left(\int \frac{\lambda}{1+x\lambda} f_{1/\beta}(\lambda)\, d\lambda\right)dx \\ &= \log e \cdot \int_0^{\rho}\frac{\beta}{4x^2}F(x/\beta, \beta)\,dx = G(\rho/\beta, \beta), \end{aligned} \qquad (12)$$

with the function $F(x,z)$ given in (7), and

$$G(x,z) = z\log(1 + x - \tfrac{1}{4}F(x,z)) + \log(1 + xz - \tfrac{1}{4}F(x,z)) - \frac{\log e}{4x}F(x,z), \qquad (13)$$

after some algebra.

Clearly, this interference-free theoretical limit is an upper bound for the achievable spectral efficiency of multicell MIMO systems. In the following subsections, we give the spectral efficiencies of multicell MIMO systems with several detectors of interest.

A. Single-Cell Detector

This detector ignores the structure of the intercell interference, and performs the "optimum" detection for intracell communication assuming white Gaussian background noise. Analog to the single-user detector in multiuser communication theory, this detector needs no information of the interfering signals and is easy to implement from existing technology. Meanwhile, performance degradation is anticipated due to its suboptimality in the multicell scenario. Here we are interested in the theoretical capacity limit with this receiver structure in multicell MIMO communications, which relies on the key observations in [5]. It is indicated that the spectral efficiency of this detector depends on the actual noise distribution only through its power, and thus coincides with a white Gaussian noise channel with equivalent noise power. Therefore, the spectral efficiency of the single cell detector is of the same form as (10), with the noise spectral height replaced by $\sigma^2 + P\sum_{i=1}^{L}\alpha_i^2 = \sigma^2(1+\rho/\mu)$, given as follows.

Proposition III.1: The multicell spectral efficiency of the desired-cell MIMO system with the single-cell detector is

$$C_{M-sopt} = \log\det\left[\mathbf{I} + \frac{\rho}{1+\rho/\mu}\frac{1}{K}\mathbf{H}\mathbf{H}^H\right]. \tag{14}$$

Similarly, in the limiting region we have

$$\lim_{N\to\infty}\frac{1}{N}C_{M-s} = G(\rho/(1+\rho/\mu)\beta, \beta). \tag{15}$$

B. Joint Optimum Detector

In contrast to the single-cell detectors, joint optimum detectors go to the other extreme by assuming that the receiver knows the signaling and channel information of other cells and attempts to perform joint detection. In this situation, model (3) describes a multiple-access channel. For the Gaussian multiple access channel, the capacity region is specified in [10] as (couched in the notation of the present paper)

$$\bigcap_{I\subset\{1,\ldots,K(L+1)\}}\{(R_1,\ldots,R_{K(L+1)}) : 0 \le \sum_{i\in I} R_i \le \log\det[\mathbf{I} + \frac{\rho}{K}(\mathbf{H}_E)_I(\mathbf{H}_E)_I{}^H]\} \tag{16}$$

where

$$\mathbf{H}_E = [\mathbf{H}, \alpha_1\mathbf{H}_{if1}, \ldots, \alpha_L\mathbf{H}_{ifL}], \tag{17}$$

and $(\mathbf{H}_E)_I$ denotes the $N\times|I|$ submatrix of $\mathbf{H}_E$ obtained by striking out the columns whose indices do not belong to *I*, with $|I|$ the cardinality of *I*. Here $R_1 \sim R_K$ denotes the data rates of the MIMO system in the desired cell, $R_{K+1} \sim R_{2K}$ refers to that of the first interfering cell with attenuation factor α_1, and so on. The following proposition is a specific application of (16).

Proposition III.2: The partial sum spectral efficiency of the multicell MIMO systems with the joint optimum detector is

$$SR_{M-opt}^{(J)} = \log\det[\mathbf{I} + \frac{\rho}{K}(\mathbf{H}_E)_J(\mathbf{H}_E)_J{}^H], \tag{18}$$

where $J \subset \{0,1,\ldots,L\}$ denotes the set of cells of interest (cell 0 is the desired cell), and $(\mathbf{H}_E)_J$ means the $N\times|J|K$ submatrix of $\mathbf{H}_E$ obtained by striking out the channel matrices of the interfering cells whose indices do not belong to *J*.

Like (11), in the limiting region, we can rewrite (18) as

$$\lim_{N\to\infty}\frac{1}{N}SR_{M-opt}^{(J)} = E\left\{\log\left(1+\frac{\rho}{\beta}\lambda\right)\right\}, \tag{19}$$

where $\{\lambda_i\}$ are the eigenvalues of

$$\frac{1}{N}(\mathbf{H}_E)_J(\mathbf{H}_E)_J{}^H = \frac{1}{N}[\mathbf{H}_{i_1},\ldots,\mathbf{H}_{i_{|J|}}]\begin{bmatrix}\alpha_{i_1}^2 & & \\ & \ddots & \\ & & \alpha_{i_{|J|}}^2\end{bmatrix}[\mathbf{H}_{i_1},\ldots,\mathbf{H}_{i_{|J|}}]^H. \tag{20}$$

Note that (20) conforms with the conditions of Theorem II.1, so the empirical eigenvalue distribution of $\frac{1}{N}(\mathbf{H}_E)_J(\mathbf{H}_E)_J{}^H$ converges in distribution to a nonrandom distribution function *Q*, whose Stieltjes transform $m_Q(z)$ is a unique solution to

$$m_Q(z) = \frac{1}{-z + \beta\sum_{j=1}^{|J|}\frac{\alpha_{i_j}^2}{1+\alpha_{i_j}^2 m_Q(z)}}. \tag{21}$$

By (4), we have

$$m_Q(z) = E\left\{\frac{1}{\lambda - z}\right\}. \tag{22}$$

So

$$E\left\{\frac{\lambda}{1+x\lambda}\right\} = \frac{x - m_Q\left(-\frac{1}{x}\right)}{x^2}. \tag{23}$$

Therefore, using the same differentiation-integration strategy as (12), we have

$$\lim_{N\to\infty}\frac{1}{N}SR_{M-opt}^{(J)} = \log e\cdot\int_0^{\rho/\beta}\frac{x - m_Q\left(-\frac{1}{x}\right)}{x^2}dx, \tag{24}$$

where $m_Q(x)$ is an implicit solution of (21).

The exact formula of the limiting spectral efficiency for the joint optimum detector requires numerical fixed-point solutions of (21) and the definite integral of (24), which is fairly complex. So, an approximating formula of (18) in the limiting region is explored here. The key idea is to approximate $(\mathbf{H}_E)_J(\mathbf{H}_E)_J{}^H$ as

$$(\mathbf{H}_E)_J(\mathbf{H}_E)_J{}^H \approx \frac{\sum_{i\in J}\alpha_i^2}{|J|}\mathbf{H}'(\mathbf{H}')^H, \tag{25}$$

where $\mathbf{H}'$ denotes an $N\times K|J|$ random matrix with i.i.d. normalized complex Gaussian entries. Note that even though we assume $\beta\le 1$ all through the paper, we should discern here whether $\beta \le 1/|J|$, which determines whether the empirical eigenvalue distribution of $(\mathbf{H}_E)_J(\mathbf{H}_E)_J{}^H/K\sum_{i\subset J}\alpha_i^2$ has a mass point at 0 (see (9)). Finally, we get the following approximate formula for (24), which *coincides* for all values of β:

$$\lim_{N\to\infty}\frac{1}{N}SR_{M-opt}^{(J)} \approx \beta|J|G\left(\rho\sum_{i\subset J}\alpha_i^2, 1/\beta|J|\right) = G\left(\frac{\rho\sum_{i\subset J}\alpha_i^2}{\beta|J|}, \beta|J|\right). \tag{26}$$

We will see in the following that (26) gives a good enough approximation for a wide range of parameter settings. It tends to overestimate when β is small or there is great discrepancy within the set of $\{\alpha_i\}$ of interest. Even in this case, (26) roughly exhibits the same behavior as (24), and thus is still useful for theoretical analysis.

C. Group Linear MMSE Detector

While achieving the optimum performance, joint maximum likelihood detection for multicell MIMO systems is impractical for most current applications due to its complexity [2]. With no intention of detecting the data from the interfering cells, group linear MMSE MUD is one of the most favorable techniques to suppress the intercell interference. The detection process is to first apply the weight matrix

$$\mathbf{W} = \left(\mathbf{H}\mathbf{H}^H + \sum_i \alpha_i^2 \mathbf{H}_{ifi}\mathbf{H}_{ifi}{}^H + \frac{K}{\rho}\mathbf{I} \right)^{-1} \mathbf{H} \qquad (27)$$

to the received signal (3) to combat the intercell interference, and then to *optimally* detect the data of the desired cell. The following result has been proved in [2].

Proposition III.3: The multicell spectral efficiency of the desired-cell MIMO system with the group linear MMSE detector is given asymptotically as

$$C_{M-mmse} \xrightarrow{N\to\infty} \log\det[\mathbf{I} + \frac{P}{K}\mathbf{H}\mathbf{H}^H\mathbf{\Sigma}^{-1}], \qquad (28)$$

where

$$\mathbf{\Sigma} = \sum_{i=1}^{L} \alpha_i^2 \frac{P}{K}\mathbf{H}_{ifi}\mathbf{H}_{ifi}{}^H + \sigma^2\mathbf{I} \cdot \qquad (29)$$

From Proposition III.3, we have the following consequence.

Corollary III.1: The multicell spectral efficiency of the desired-cell MIMO system with the group linear MMSE detector is asymptotically related to the partial sum spectral efficiency of the multicell MIMO systems with the joint optimum detector as

$$C_{M-mmse} \xrightarrow{N\to\infty} SR_{M-opt}^{\{0,1,\ldots,L\}} - SR_{M-opt}^{\{1,\ldots,L\}}. \qquad (30)$$

Proof: By (29),

$$\mathbf{\Sigma} = \sum_{i=1}^{L} \alpha_i^2 \frac{P}{K}\mathbf{H}_{ifi}\mathbf{H}_{ifi}{}^H + \sigma^2\mathbf{I} = \frac{P}{K}(\mathbf{H}_E\mathbf{H}_E{}^H - \mathbf{H}\mathbf{H}^H) + \sigma^2\mathbf{I}.$$

So,

$$\frac{\rho}{K}(\mathbf{H}_E\mathbf{H}_E{}^H - \mathbf{H}\mathbf{H}^H) = \frac{1}{\sigma^2}\mathbf{\Sigma} - \mathbf{I},$$

and

$$\frac{\rho}{K}\mathbf{H}_E\mathbf{H}_E{}^H = \frac{\rho}{K}\mathbf{H}\mathbf{H}^H + \frac{1}{\sigma^2}\mathbf{\Sigma} - \mathbf{I}.$$

Therefore,

$$\mathbf{I} + \frac{P}{K}\mathbf{H}\mathbf{H}^H\mathbf{\Sigma}^{-1} = \frac{\mathbf{I} + \frac{\rho}{K}\mathbf{H}_E\mathbf{H}_E{}^H}{\mathbf{I} + \frac{\rho}{K}(\mathbf{H}_E\mathbf{H}_E{}^H - \mathbf{H}\mathbf{H}^H)}. \qquad (31)$$

On comparing (18) and (28), (30) follows. □

With (30) and (26), we readily have

.

$$\lim_{N\to\infty}\frac{1}{N}C_{M-mmse} \approx G\left(\frac{\rho(1+1/\mu)}{\beta(L+1)}, \beta(L+1)\right) - G\left(\frac{\rho(1/\mu)}{\beta L}, \beta L\right) \qquad (32)$$

In general, if we partition the cells in two groups, applying the linear MMSE detection to one of them to suppress the interference of the other, followed by *optimal* detection within the set of cells of interest, the sum spectral efficiency is exactly analogous to (30). Thus, we have the following.

Corollary III.2: The partial sum spectral efficiency of multicell MIMO systems with the *generalized* group linear MMSE detector is asymptotically given as

$$SR_{M-gmmse}^{(J)} \xrightarrow{N\to\infty} SR_{M-opt}^{\{0,1,\ldots,L\}} - SR_{M-opt}^{(\bar{J})}, \qquad (33)$$

where $J \subset \{0,1,\ldots,L\}$ denotes the set of cells of interest (cell 0 is the desired cell), and $\bar{J}$ is the complement of J in $\{0,1,\ldots,L\}$.

Comments: Note that $SR_{M-gmmse}^{\{0\}} = C_{M-mmse}$, while $SR_{M-gmmse}^{\{0,1,\ldots,L\}} = SR_{M-opt}^{\{0,1,\ldots,L\}}$.

IV. SOME ANALYTICAL AND NUMERICAL RESULTS

In this section, some analytical and numerical results are given as applications of the above derived formulas, from which we can gain some insights into the behavior of multicell MIMO systems. We assume the following parameters for (3): $L = 4$, and

$$\alpha_1^2 = \frac{1}{\mu}\frac{\gamma}{1+\gamma}\frac{\beta_1}{1+\beta_1}, \alpha_2^2 = \frac{1}{\mu}\frac{\gamma}{1+\gamma}\frac{1}{1+\beta_1},$$
$$\alpha_3^2 = \frac{1}{\mu}\frac{1}{1+\gamma}\frac{\beta_2}{1+\beta_2}, \alpha_4^2 = \frac{1}{\mu}\frac{1}{1+\gamma}\frac{1}{1+\beta_2}, \qquad (34)$$

with $\gamma = 4$, $\beta_1 = 1$, and $\beta_2 = 1$ [2]. Recall that μ is the SIR.

Clearly, the single-cell detector is interference limited, which can be verified through (15) with

$$\lim_{\rho\to\infty} G(\rho/_{(1+\rho/\mu)\beta}, \beta) = G(\mu/_{\beta}, \beta). \qquad (35)$$

Even though for $\beta = 1$, the group linear MMSE detector is also interference limited [2], we noted that this is due to the lack of sufficient degrees of freedom at the receiver to suppress the co-channel interference. We believe that if β is sufficiently small, the group linear MMSE detector is *not* interference limited. This is verified in Figs. 1 (a)-(c). Here the SIR is set to be 0 dB, indicating a strong interference environment. It is observed in these figures that the spectral efficiency of the single cell upper bound (see (12)) and that of the single-cell detector (see (15)) decrease as the system load β decreases. The single-cell detector is interference-limited, with the limiting value given by (35). The spectral efficiency of the group linear MMSE detector, both the exact (see (30) and (24)) and the approximate (see (32)), however, increases as β decreases, when the SNR is sufficiently large. Furthermore, when the system load decreases to 1/5, the group linear MMSE detector is not interference limited. Comparing Figs. 1(a), Fig. 1(c) with Fig. 1 (d), where a more favorable SIR = 5 dB is experienced, we can see that all the multicell spectral efficiencies increase as SIR increases. However, due to the interference-limited nature, the spectral efficiency of the group linear MMSE detector with system load 1 is outperformed by the same detector with system load of 1/5 at a much worse SIR, when the SNR is sufficiently large. This observation is helpful for multicell MIMO system design. Finally, we observe that the approximate formula well matches the exact one, thus providing a valuable tool for analysis.

Let us turn to the normalized approximate formula for the group linear MMSE detector, given as

$$\lim_{N\to\infty} C_{M-mmse}/N \approx R(\rho,\mu,\beta)$$
$$\triangleq G\left(\frac{\rho(1+1/\mu)}{\beta(L+1)}, \beta(L+1)\right) - G\left(\frac{\rho(1/\mu)}{\beta L}, \beta L\right), \qquad (36)$$

to study the interference-limited behavior of the group linear MMSE detector. It can be shown that

$$\lim_{\rho\to\infty} R(\rho,\mu,\beta) = \log(1+\mu) + (1-\beta(L+1))\log\left(1-\frac{1}{\beta(L+1)}\right) + (\beta L-1)\log\left(1-\frac{1}{\beta L}\right), \text{ when } \beta > \frac{1}{L}, \qquad (37)$$

and

$$\lim_{\rho\to\infty} R(\rho,\mu,\beta) = \infty \quad \text{when} \quad \beta \le \frac{1}{L+1}. \qquad (38)$$

Due to the space limitation, the derivation of (37) and (38) is omitted here.

The analytical results of (37) and (38) agree with the numerical results of Fig. 1 very well. Thus the "magic" number 1/5 is not found by luck but rather is determined by the system behavior.

V. Conclusions

In this paper, the spectral efficiencies of multicell MIMO systems with several MUD detectors have been studied, among which are single-cell detectors, joint optimum detectors, and group linear MMSE detectors. The large-system asymptotic expressions for these spectral efficiencies have also been explored. Simple relations have been found among these capacity formulas, and all of them can be well approximated with standard functions, which makes theoretical analysis of multicell MIMO systems more expedient.

Further, conditions for non-interference-limited behavior of the group linear MMSE detector have been found. Based on this result, it is suggested that with sufficiently low system load $\beta \le \frac{1}{L+1}$, where L is the number of effective interfering cells, better performance than that of the fully loaded system may be attained in the strong interference environment with sufficiently large signal power.

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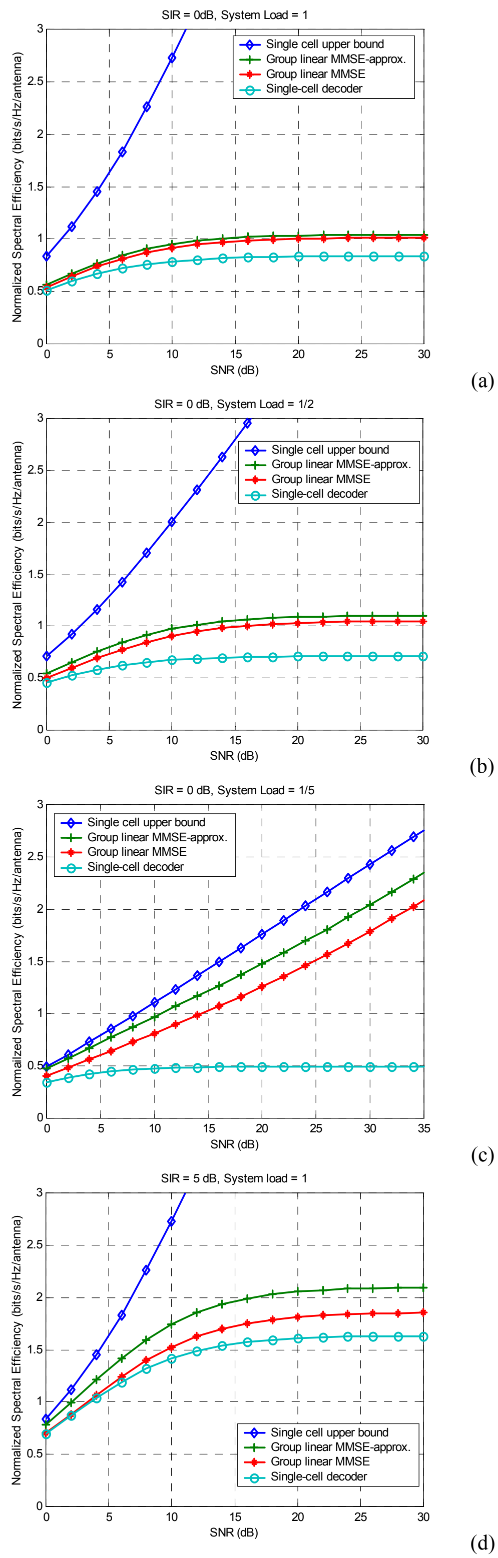

Fig. 1 Study of interference-limited behavior of various multi-cell MIMO spectral efficiencies