Medium Access Control and Power Optimizations for Sensor Networks with Linear Receivers

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***Abstract*— We consider the uplink reachback problem in sensor networks where the receiver is equipped with multiple antennas and linear multiuser detectors. Three medium access control (MAC) schemes are analyzed: round-robin, maximum-throughput scheduling and slotted ALOHA. We optimize the average number of transmissions per slot and the transmission power for two purposes: maximizing the throughput, or minimizing the effective energy (the average energy consumption per successfully received packet) subject to a throughput constraint. By comparing the optimal performance of different MAC schemes equipped with different detectors, we draw important tradeoffs involved in the sensor network design.**

***Index Terms*– Scheduling, Multiuser Diversity, Linear Multiuser Detector.**

I. Introduction

A foremost concern in sensor network design is to minimize the energy expenditure and prolong the sensor lifetime [1], [2]. Meanwhile, the sensor network should be able to maintain a certain throughput, (which is equivalent to a certain delay constraint), in order to fulfill the QoS requirement of the end user, and to ensure the stability of the network. Typically, the throughput and the energy efficiency are inconsistent, and there exists a tradeoff between the two measures. The objective of this work is to explore fundamental limits on the achievable throughput and optimal network designs that achieve the desired throughput with minimal energy consumption.

We consider the reachback problem where all sensor nodes in the sensor field transmit to a common receiver. The receiver has replenishible power supply and possesses sophisticated data reception and processing capabilities. Such a network structure conserves sensors' energy, and an example is the SENMA [3]. We assume that each node constantly has packets to transmit. The transmission is slotted and the slot length T equals the transmission time of one packet. The sensors and the receiver thus constitute a multiple access network, which has been traditionally characterized by the collision channel model, i.e., single transmission means success and simultaneous transmissions results in failure. Nevertheless, advanced signal processing techniques such as multiuser detection [4] enable correct reception of simultaneous transmitted packets at the physical layer, and consequently, Ghez *et. al.* proposed the revolutionary multi-packet reception model [5]. In this work we assume that the receiver is equipped with N antennas and a linear multiuser detector followed by single-user decoders. The packet transmission is considered successful as long as the output signal-to-interference-ratio (SIR) of the linear detector is above a certain threshold β.

Since a sensor field usually consists of hundreds or thousands of sensors, medium access control (MAC) is necessary to determine which sensors should transmit during each slot to avoid excessive multi-access interference. We consider three MAC schemes: round-robin, "maximum-throughput scheduling" and slotted ALOHA. For round-robin, the adjacent sensors form a transmission group and the groups are scheduled for access one by one. Maximum-throughput scheduling exploits the multiuser diversity [6] in a large network in a time-varying channel: in each slot, the group with the maximum number of sensors above the SIR threshold is given the chance to transmit. For slotted ALOHA, in each slot every sensor node transmits a packet (new or retransmission) with the same probability p independently. In addition to the MAC scheme, the system performance is also determined by the linear multiuser detector employed, which can be the single-user matched filter, the decorrelating detector, or the linear MMSE detector. For a given MAC scheme with a given linear detector, we assume that the number of receive antennas is fixed, and jointly optimize the transmit power and the average number of transmission per slot for two purposes: one is to maximize the throughput, and the other is to minimize the energy consumption subject to a throughput constraint.

Many work have studied the energy-delay tradeoff through scheduling and resource allocation, but most such studies are information-theoretic, and relatively few [7], [8] adopted the multipacket reception model due to suboptimal linear receivers. In [7], the authors optimize the sensor networks with multiple mobile agents employing slotted ALOHA to maximize the throughput or the energy efficiency. In [8], the performance of sensor networks using both CDMA and multiple receive antennasis is analyzed using results on large random networks in [9]. The analysis in this paper does not rely on the large network approximation.

The paper is organized as follows. In Section II, we introduce the system model and the measures of throughput and energy efficiency. In Section III, we derive the throughput and the energy efficiency for the three MAC schemes. Analytical results for the three linear detectors used in later optimizations are given section IV. Section V and VI deal with the two optimization problems respectively. Numerical results and discussions are presented in Section VII. Finally we conclude the paper in Section VIII.

II. System Description

We assume that there are totally n sensors in the sensor field, and the channel states between each sensor and each receive antenna are independent, identical Rayleigh distributed variables. Sensors have no knowledge of uplink channel state information (CSI), and transmit with equal power P. If m sensors simultaneously transmit, the m sensors and N receive antennas form a virtual MIMO system, and the discrete-time model is given by

$$\mathbf{y} = \sqrt{G}\sum_{i=1}^{m} \mathbf{h}_i x_i + \mathbf{n}, \tag{1}$$

where x_i is the transmitted signal of the i-th sensor and $E[\|x_i\|^2] = P$, $\mathbf{h}_i$ is the $N \times 1$ spatial signature of the i-th sensor, whose entries are circularly-symmetric complex Gaussian variables with zero mean and unit variance, G is the common pathloss, $\mathbf{n}$ is the circularly-symmetric Gaussian noise with zero mean and covariance matrix $\sigma^2\mathbf{I}$, and $\mathbf{y}$ is the received signal vector. The average received SNR of a packet at one receive antenna is given by $\rho = \frac{PG}{\sigma^2}$. In the following we denote the matrix $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_m]$.

We assume that a feedback channel exists from the receiver to the sensor nodes, which is used for synchronization, acknowledgements, group selection and other signaling on the MAC layer. The energy consumption for receiving the signaling is assumed to be negligible throughout the paper. For simplicity we also ignore the circuit energy consumption.We measure the energy efficiency by the *effective energy* [8], defined to be the average energy consumption per successfully transmitted packet:

$$E_{\mathrm{e}} = \frac{PT}{\Pr[\mathrm{succ}]}, \tag{2}$$

where $\Pr[\mathrm{succ}]$ is the average probability of success for a transmitted packet. Denoting a as the average number of transmissions per slot, the *throughput*, defined as the average number of successful transmissions per slot is

$$\lambda = a\Pr[\mathrm{succ}]. \tag{3}$$

Throughout the paper we assume that the number of receive antennas N, the total number of sensors n, the SIR threshold β, the common pathloss G, as well as the noise variance σ^2 are fixed. Thus the optimization of the transmission power P is the same as the optimization of ρ. We further denote the probability that the received SIR of a packet is at least β when there are m transmissions and the average received SNR per antenna is ρ by

$$q(m,\rho) = \Pr\left[\mathrm{SIR} \geq \beta | m, \rho\right]. \tag{4}$$

$q(m,\rho)$ is decided by the multiuser detector used (more details are given in Section IV). In general $q(m,\rho)$ decreases with m and increases with ρ.

III. Throughput and Effective Energy of Three MAC Schemes

In this section, we introduce the three MAC schemes and derive the throughput and the effective energy for each scheme.

A. Round-Robin

Round-robin is a fair scheduling scheme and is relatively easy to implement: m sensors in close proximity form a group. For simplicity we assume that n is a multiple of m, so there are totally $K = n/m$ groups. Groups are scheduled for access one by one, and when a group is scheduled in a slot, all the sensors in that group transmit simultaneously. It is easily seen that the average number of transmissions per slot a is m, and $\Pr[\mathrm{succ}] = q(m,\rho)$. Thus the throughput of round-robin is

$$\lambda_{\mathrm{rr}}(m,\rho) = mq(m,\rho). \tag{5}$$

With $P = \rho\sigma^2/G$, the effective energy of round-robin is given by

$$E_{\mathrm{e,rr}}(m,\rho) = \frac{\rho\sigma^2 T/G}{q(m,\rho)}. \tag{6}$$

B. Maximum-Throughput Scheduling

For max-throughput scheduling, sensors are also grouped and the group size is m. At the beginning of each slot, all sensors transmit a short training sequence, and the receiver compares the SIR's of all sensors with the SIR threshold to determine the group with the maximum number of sensors above the SIR threshold, and sends a notification to that group (if multiple groups achieve the maximum, just choose any one from them).

The probability that there are exactly t packets meeting the target SIR when a group of m sensors transmit is given by

$$p_t(m,\rho) = \binom{m}{t} q(m,\rho)^t \left[1 - q(m,\rho)\right]^{m-t}, \quad t = 0, 1, \cdots, m. \tag{7}$$

Denote the number of successfully received packets of each group by $t_k (k = 1, \cdots, K)$, and $t_{\max} = \max(t_1, \cdots, t_K)$. Then the distribution function of $t_{\max}$ is given by

$$\Pr[t_{\max} \leq t] = \left(\sum_{t'=0}^{t} p_{t'}(m,\rho)\right)^K, \quad t = 0, 1, \cdots, m. \tag{8}$$

Therefore the throughput of the max-throughput scheduler is

$$\lambda_{\mathrm{mt}}(m,\rho) = \sum_{t=1}^{m} t\left[\left(\sum_{t'=0}^{t} p_{t'}(m,\rho)\right)^K - \left(\sum_{t'=0}^{t-1} p_{t'}(m,\rho)\right)^K\right]. \tag{9}$$

As round-robin, we have $a = m$. The probability of success given that a packet is transmitted is $\Pr[\mathrm{succ}] = \lambda_{\mathrm{mt}}(m,\rho)/m$. Thus the effective energy is given by

$$E_{\mathrm{e,mt}}(m,\rho) = \frac{\rho\sigma^2 T/G}{\lambda_{\mathrm{mt}}(m,\rho)/m}. \tag{10}$$

Max-throughput scheduling may incur some fairness concerns when the channel fading is slow, which can be remedied by certain algorithms [10], [11] with little throughput sacrifice.

C. Slotted ALOHA

Slotted ALOHA has the advantage of simplicity and built-in fairness. Denoting the transmission probability of each user by p, the throughput of slotted ALOHA is given by

$$\lambda_{\rm sa} = \sum_{k=1}^{n} \left(\begin{array}{c} n \\ k \end{array} \right) p^k (1-p)^{n-k} k q(k,\rho).$$

The average number of transmissions per slot is $a = np$. In the case n is large and p is small, we can approximate the binomial probabilities with Poisson probabilities and obtain

$$\lambda_{\rm sa}(a,\rho) = e^{-a} \sum_{k=1}^{n} \frac{a^k}{k!} k q(k,\rho) = e^{-a} \sum_{k=1}^{n} \frac{a^k}{(k-1)!} q(k,\rho). \tag{11}$$

The average success probability is $\Pr[\text{succ}] = \lambda_{\rm sa}(a,\rho)/a$, thus the effective energy is given by

$$E_{\rm e,sa}(a,\rho) = \frac{\rho\sigma^2 T/G}{\lambda_{\rm sa}(a,\rho)/a}. \tag{12}$$

IV. Linear Multiuser Detectors in Rayleigh Fading Channels

In this section we give the expression of $q(m,\rho)$ in Rayleigh fading channels for the three linear detectors. As we will use the asymptotic value of $q(m,\rho)$ as $\rho \to \infty$ in later analysis, we also derive the expression of $q(m,\infty) \doteq \lim_{\rho\to\infty} q(m,\rho)$.

A. Matched Filter

The SIR of the i-th user after matched-filtering is given by

$$\text{SIR}_i = \frac{PG\|\mathbf{h}_i\|^4}{\sigma^2\|\mathbf{h}_i\|^2 + PG\sum_{j=1,j\neq i}^{m} \|\mathbf{h}_i^\dagger \mathbf{h}_j\|^2}, \tag{13}$$

where † denotes conjugate transpose.

Lemma 4.1: The $q(m,\rho)$ of the matched filter in the Rayleigh fading channel is given by

$$q^{\rm mf}(m,\rho) = \left\{ \begin{array}{l} 1-\Gamma(\beta/\rho\ , N),\ m=1, \\ \frac{1}{(m-2)!}\int_0^\infty \left[1-\Gamma(\beta y + \frac{\beta}{\rho}, N)\right] y^{m-2} e^{-y} dy,\ m>1. \end{array} \right. \tag{14}$$

where $\Gamma(a,x)$ is the regularized gamma function given by $\Gamma(a,x) = \frac{\int_0^x t^{a-1}e^{-t}dt}{\int_0^\infty t^{a-1}e^{-t}dt}$.

In the case $\rho \to \infty$,

$$q^{\rm mf}(m,\infty) = \left\{ \begin{array}{l} 1, \quad m=1, \\ 1-I\left(\frac{\beta}{1+\beta}; N, m-1\right), \quad m>1, \end{array} \right. \tag{15}$$

where $I(x;a,b)$ is the regularized beta function, given by $I(x;a,b) = \frac{\int_0^x t^{a-1}(1-t)^{b-1}dt}{\int_0^1 t^{a-1}(1-t)^{b-1}dt}$.

Proof: See Appendix I.

B. Decorrelating Detector

If $\mathbf{H}^\dagger\mathbf{H}$ is invertible, the SIR of the i-th user using a decorrelating detector is given by

$$\text{SIR}_i = \frac{\rho}{\left[(\mathbf{H}^\dagger\mathbf{H})^{-1}\right]_{ii}}, \tag{16}$$

and when $\mathbf{H}^\dagger\mathbf{H}$ is singular, SIR_i is zero.

Lemma 4.2: The $q(m,\rho)$ of the decorrelator in the Rayleigh fading channel is given by

$$q^{\rm dec}(m,\rho) = \left\{ \begin{array}{l} 1-\Gamma(\beta/\rho\ ,\ N-m+1), \quad m\le N, \\ 0, \quad m>N. \end{array} \right. \tag{17}$$

When $\rho \to \infty$,

$$q^{\rm dec}(m,\infty) = \left\{ \begin{array}{l} 1, \quad m\le N, \\ 0, \quad m>N. \end{array} \right. \tag{18}$$

Proof: The proof follows from [12].

C. Linear MMSE Detector

For the linear MMSE receiver, it can be shown that the SIR of the i-th user is given by

$$\text{SIR}_i = \mathbf{h}_i^\dagger \left(\mathbf{H}_i\mathbf{H}_i^\dagger + \frac{1}{\rho}\mathbf{I}\right)^{-1} \mathbf{h}_i, \tag{19}$$

where $\mathbf{H}_i$ denotes the matrix obtained by striking out the i-th column of $\mathbf{H}$. There is no straightforward closed-form expression of $q(m,\rho)$ for the linear MMSE detector in the Rayleigh fading channel, and in this paper we use exact success probabilities obtained through simulations. Nevertheless, when $\rho \to \infty$, the success probability of the linear MMSE detector has a simple form, given by the following lemma.

Lemma 4.3: For Rayleigh fading channels

$$q^{\rm mmse}(m,\infty) = \left\{ \begin{array}{l} 1, \quad m\le N, \\ 1-I\left(\frac{\beta}{1+\beta}; N, m-N\right), \quad m>N. \end{array} \right. \tag{20}$$

Proof: See Appendix II.

V. Throughput Maximization

For a given MAC scheme with a given linear detector, the SNR ρ and the average number of transmissions per slot a (for round-robin and max-throughput scheduling, $a=m$, and for slotted ALOHA, $a=np$) can be optimized, such that the throughput is maximized. The performance of various MAC schemes with different linear detectors can then be compared in terms of the maximum throughput.

First assume that a is fixed. Since $\Pr[\text{succ}]$ increases with ρ, the maximum throughput for any fixed a is achieved when $\rho \to \infty$. Therefore the maximum throughput jointly optimized over a and ρ is obtained by letting $\rho \to \infty$, and searching for the optimal a that achieves the global maximum.

For a given MAC scheme with a given linear detector, we define the *maximum asymptotic throughput* to be the maximum throughput achievable with a given number of receive antennas as SNR ρ approaches infinity, and denote it by $\Lambda(\infty) \doteq \max_a \lambda(a,\infty)$.

Proposition 5.1: The maximum asymptotic throughput of round-robin, max-throughput scheduling and slotted ALOHA are respectively given by

$$\Lambda_{\rm rr}(\infty) = \max_m mq(m,\infty); \tag{21}$$

$$\Lambda_{\rm mt}(\infty) = \max_m \lim_{\rho\to\infty} \lambda_{\rm mt}(m,\rho); \tag{22}$$

$$\Lambda_{\rm sa}(\infty) = \max_a e^{-a} \sum_{k=1}^{n} \frac{a^k}{(k-1)!} q(k,\infty), \tag{23}$$

where $\lambda_{\rm mt}(m,\rho)$ is given in (9). The above can be evaluated for different detectors using (15), (18) and (20).

Remark 1: With the decorrelating detector, applying (18) we immediately get $\Lambda_{\rm rr}^{\rm dec}(\infty) = N$, $\Lambda_{\rm mt}^{\rm dec}(\infty) = N$, both achieved at $m = N$, and $\Lambda_{\rm sa}^{\rm dec}(\infty) = \max_a e^{-a} \sum_{k=1}^{N} \frac{a^k}{(k-1)!}$. Note that with the decorrelator, the maximum asymptotic throughput of round-robin and max-throughput scheduling are the same, while that of slotted ALOHA can be much smaller. For example, when $N = 10$, the maximum asymptotic throughput of slotted ALOHA with the decorrelator is 5.831, which is achieved at $a = 7.297$.

Remark 2: While no straightforward closed-form expressions for maximum asymptotic throughput are available for the matched filter and the linear MMSE detector, some qualitative results are possible. For round-robin, comparing (15), (18) and (20) reveals

1) $\Lambda_{\rm rr}^{\rm mmse}(\infty) \geq \Lambda_{\rm rr}^{\rm mf}(\infty)$, with the equality held when $N = 1$.
2) $\Lambda_{\rm rr}^{\rm mmse}(\infty) \geq \Lambda_{\rm rr}^{\rm dec}(\infty)$; the equality holds if and only if the throughput of the linear MMSE with $m = N+1$ is smaller than with $m = N$. With simple manipulation, we obtain that the linear MMSE detector can support a throughput larger than the number of receive antennas N (and surpass the decorrelator) if and only if $\beta \geq \frac{1}{(N+1)^{1/N}-1}$.
3) The relative performance of the decorrelator and the matched filter depends on β. It can be shown that when $\beta \geq 1$, $\Lambda_{\rm rr}^{\rm dec}(\infty) \geq \Lambda_{\rm rr}^{\rm mf}(\infty)$.

As for the other two MAC schemes, since for all m we have $q^{\rm mmse}(m,\infty) \geq \max\{q^{\rm mf}(m,\infty), q^{\rm dec}(m,\infty)\}$, it must be true that the maximum asymptotic throughput with the linear MMSE is always the best, while it is not immediate whether the matched filter or the decorelator is the worst. Moreover, the maximum asymptotic throughput of max-throughput scheduling is larger than that of round-robin when the matched filter or the linear MMSE detector is employed.

VI. Throughput-Constrained Energy Minimization

In this section, we study the minimization of the effective energy subject to a throughput constraint $\lambda \geq \Delta$, which may arise from a QoS demand from the end user, or from a mild delay constraint to ensure the stability of the network. As discussed in Section V, there exists an upper limit on the throughput supportable by each MAC scheme with a given linear detector, which is the maximum asymptotic throughput. Therefore the optimization discussed here is meaningful only if the given throughput constraint does not exceed the maximum asymptotic throughput, otherwise the throughput constraint cannot be met. Comparing (6), (10), (12), we observe that $\sigma^2 T/G$ is a common factor and is fixed. Therefore to minimize $E_{\rm e}$ it suffices to find

$$\min_{a,\rho} \frac{a\rho}{\lambda(a,\rho)}, \tag{24}$$

subject to

$$\lambda(a,\rho) \geq \Delta. \tag{25}$$

We start with assuming a is fixed. Since the throughput is an monotone increasing function of ρ, we can find the smallest ρ that meets the throughput constraint, which is denoted by $\rho_{\min}(a) = \min\{\rho \mid \lambda(a,\rho) \geq \Delta\}$. Thus the minimum effective energy for fixed a is given by

$$E_{e,\min}(a) = \min_{\rho\geq\rho_{\min}(a)} \frac{a\rho}{\lambda(a,\rho)}. \tag{26}$$

Therefore, the joint optimization can be proceeded in two steps: first, find the minimum effective energy when a is fixed, then find the global minimum across all a.

Proposition 6.1: For a given throughput constraint Δ, if $\Delta \leq \Lambda(\infty)$, the minimum effective energy jointly optimized over a and ρ is given by

$$E_{e,\min} = \min_a E_{e,\min}(a) = \min_a \min_{\rho\geq\rho_{\min}(a)} \frac{a\rho}{\lambda(a,\rho)}, \tag{27}$$

while if $\Delta > \Lambda(\infty)$, the throughput constraint cannot be met.

VII. Numerical Results and Discussions

In this section we present the numerical results and draw some observations on the comparative performance of different MAC schemes and different linear detectors. For all simulations, we assume the total number of sensors $n = 1000$.

Example 1: Fig. 1 shows the maximum asymptotic throughput of the three MAC schemes using different linear detectors when $\beta = 1$. Note that with the decorrelator, the two curves for round-robin and max-throughput scheduling coincide. Several observations can be made: 1) With the matched filter or the linear MMSE detector, the maximum asymptotic throughput of max-throughput scheduling is larger than that of round-robin, and the relative scheduling gain diminishes with N (the absolute gain is roughly constant). This is because as N increases, the amount of performance fluctuations among different groups is reduced, hence the scheduling gain diminishes. Such effect has been observed in [13] for the maximum information-capacity scheduling and referred to as "channel hardening". 2) The relative performance loss of slotted ALOHA with respect to round-robin is much larger with the decorrelator than with the other two detectors. 3) For each MAC scheme, the decorrelator outperforms the matched filter only when N is larger than a certain threshold, which is determined by β. 4) All three MAC schemes perform favorably with the linear MMSE detector, and can achieve a maximum asymptotic throughput greater than N when $\beta = 1$.

The following values are used in the next two examples: $N = 10$, $\beta = 1$ and $\sigma^2 T/G = 1$ (scaling factor of $E_{\rm e}$).

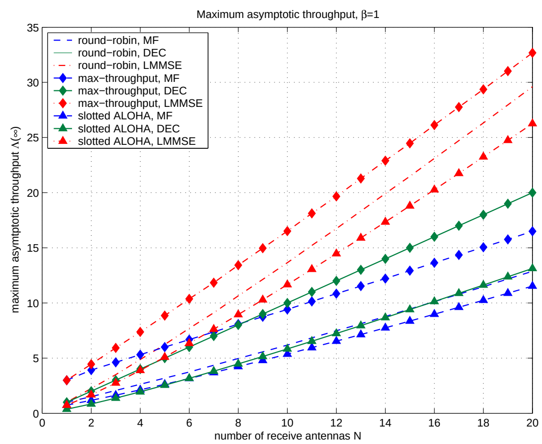

Fig. 1. Maximum asymptotic throughput of round-robin, max-throughput scheduling and slotted ALOHA with different linear detectors, $\beta = 1$

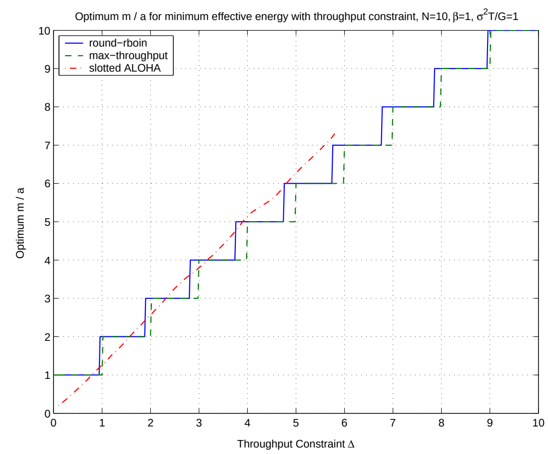

Fig. 3. Optimal m/a for minimum effective energy with throughput constraint for different MAC schemes with the decorrelator, $N = 10$, $\beta = 1$, $\sigma^2 T/G = 1$

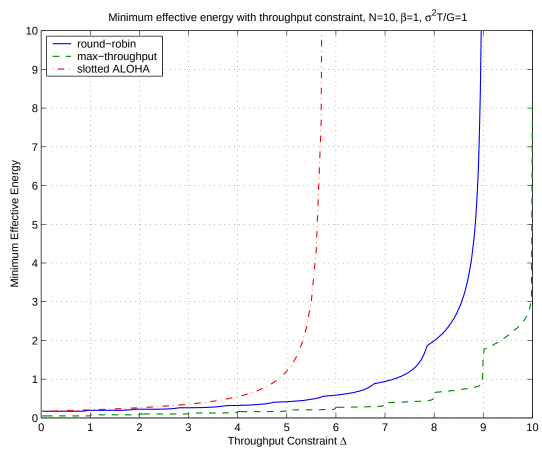

Fig. 2. Minimum effective energy with throughput constraint for different MAC schemes with the decorrelator, $N = 10$, $\beta = 1$, $\sigma^2 T/G = 1$

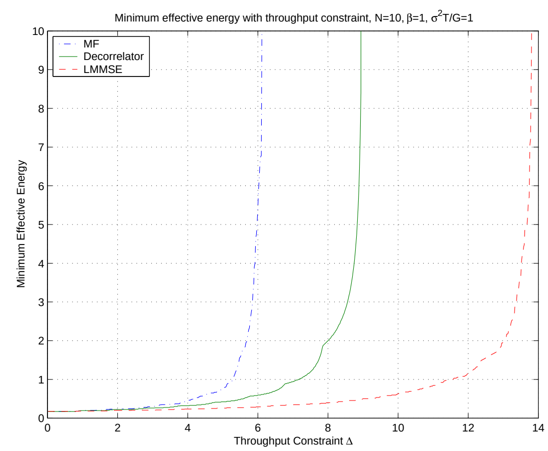

Fig. 4. Minimum effective energy with throughput constraint for round-robin with different linear detectors, $N = 10$, $\beta = 1$, $\sigma^2 T/G = 1$

Example 2: Assume the decorrelator is used, the minimum effective energy corresponding to different throughput constraints is shown in Fig. 2, and the corresponding optimal a is shown in Fig. 3. Note that the minimum effective energy curves for round-robin and max-throughput scheduling are not smooth at values of m where a jump in the optimal group size m occurs. It can be seen that the minimum effective energy increases rapidly as Δ approaches the maximum asymptotic throughput for each MAC scheme: the minimum effective energy approaches infinity for slotted ALOHA and round-robin respectively as $\Delta \to 5.831$ and as $\Delta \to 10$; for max-throughput scheduling, however, the minimum effective energy is finite (8.040) when $\Delta = 10$. Over the range of Δ, max-throughput scheduling saves more than half of the energy required for round-robin, and the saving becomes increasingly larger as Δ approaches 10. When Δ is relatively small ($\Delta \leq 3$), slotted ALOHA does not incur much energy increase compared with round-robin.

Example 3: The throughput-constrained minimum effective energy for round-robin with various linear detectors is shown in Fig. 4. When $N = 10$, the maximum asymptotic throughput of round-robin with the matched filter, the decorrelator and the linear MMSE detector are about 6.4, 10 and 13.8 respectively (c.f. Fig. 1). Again, it can be seen that the minimum effective energy approaches infinity as the throughput constraint approaches the maximum asymptotic throughput, and the linear MMSE detector assumes great superiority.

VIII. Conclusions

We have considered the uplink reachback problem with simultaneous transmissions and multiple receive antennas. Three medium access control schemes are analyzed: round-robin, maximum-throughput scheduling and slotted ALOHA. We optimize the average number of transmissions per slot a and the average received signal-to-noise ratio per receive antenna ρ, to meet two objectives: throughput maximization, and throughput-constrained effective energy minimization. We have shown that both the MAC scheme and the linear multiuser detector employed have significant impact on the system performance. The multiuser scheduling gain diminishes as the number of receive antennas increases. For each MAC scheme, the minimum effective energy grows rapidly as the throughput constraint approaches the maximum asymptotic throughput. The max-throughput scheduling greatly reduces the energy consumption as required by round-robin. For each MAC scheme, the linear MMSE detector significantly outperforms the decorrelator and the matched filter in both the throughput and the energy efficiency.

APPENDIX I
PROOF OF LEMMA 1

For the matched filter, when $m = 1$, we have $\mathrm{SIR}_i = \rho\|\mathbf{h}_i\|^2$, where $\|\mathbf{h}_i\|^2 \sim \chi^2_{2N}$. Thus

$$\Pr\left[\mathrm{SIR}_i \geq \beta\right] = \Pr\left[\|\mathbf{h}_i\|^2 \geq \frac{\beta}{\rho}\right] = 1 - \Gamma\left(\frac{\beta}{\rho}, N\right),$$

where $\Gamma(a,x)$ is the regularized gamma function given by $\Gamma(a,x) = \int_0^x t^{a-1}e^{-t}dt / \int_0^\infty t^{a-1}e^{-t}dt$. When $m > 1$, we can write the SIR in (13) as

$$\mathrm{SIR}_i = \frac{\|\mathbf{h}_i\|^2}{\frac{1}{\rho} + \frac{\mathbf{h}_i^\dagger\left(\mathbf{H}_i\mathbf{H}_i^\dagger\right)\mathbf{h}_i}{\|\mathbf{h}_i\|^2}},$$

where $\mathbf{H}_i$ denotes the matrix obtained by deleting the i-th column of $\mathbf{H}$. $\mathbf{H}_i\mathbf{H}_i^\dagger$ has a complex central Wishart distribution with $m-1$ degrees of freedom and covariance matrix $\mathbf{I}_{m-1}$, denoted as $\mathbf{H}_i\mathbf{H}_i^\dagger \in CW_N(m-1, \mathbf{I}_{m-1})$. Since $\mathbf{h}_i$ and $\mathbf{H}_i$ are independent, according to Theorem 3.2.8 in [14] we have

$$Y \doteq \frac{\mathbf{h}_i^\dagger\left(\mathbf{H}_i\mathbf{H}_i^\dagger\right)\mathbf{h}_i}{\|\mathbf{h}_i\|^2} \sim \chi^2_{2(m-1)},$$

and Y is independent of $\mathbf{h}_i$. Noting that $X \doteq \|\mathbf{h}_i\|^2 \sim \chi^2_{2N}$. Therefore, the probability of success is

$$\begin{aligned}\Pr\left[\mathrm{SIR}_i \geq \beta\right] &= \Pr\left[X \geq \beta\left(Y + \frac{1}{\rho}\right)\right] \\ &= \frac{1}{(m-2)!}\int_0^\infty \left[1 - \Gamma(\beta y + \beta/\rho), N)\right] y^{m-2}e^{-y}dy.\end{aligned}$$

As $\rho \to \infty$, when $m > 1$, we have

$$\mathrm{SIR}_i = \frac{X}{Y} = \frac{N}{m-1}\ \frac{X/2N}{Y/2(m-1)} \doteq \frac{N}{m-1}\ F,$$

where $F = \frac{X/2N}{Y/2(m-1)}$ has an $F_{2N,\ 2(m-1)}$ distribution. Therefore,

$$\Pr\left[\mathrm{SIR}_i \geq \beta\right] = \Pr\left[F \geq \beta\ \frac{m-1}{N}\right] = 1 - I(\frac{\beta}{1+\beta}; N, m-1),$$

where $I(x;a,b)$ is the regularized beta function, given by $I(x;a,b) = \frac{\int_0^x t^{a-1}(1-t)^{b-1}dt}{\int_0^1 t^{a-1}(1-t)^{b-1}dt}$. It is obvious that when $\rho \to \infty$, $\Pr\left[\mathrm{SIR}_i \geq \beta\right] = 1$ for $m = 1$.

APPENDIX II
PROOF OF LEMMA 3

When $m < N$, the linear MMSE detector converges to the decorrelator as $\rho \to \infty$. Thus $\Pr\left[\mathrm{SIR}_i \geq \beta\right] = \lim_{\rho\to\infty} 1 - \Gamma\left(\frac{\beta}{\rho},\ N - m + 1\right) = 1$.

When $m > N$, $\mathbf{H}_i\mathbf{H}_i^\dagger$ is invertible, so as $\rho \to \infty$,

$$\mathrm{SIR}_i \to \mathbf{h}_i^\dagger\left(\mathbf{H}_i\mathbf{H}_i^\dagger\right)^{-1}\mathbf{h}_i.$$

Since $\mathbf{h}_i$ and $\mathbf{H}_i$ are independent, and $\mathbf{H}_i\mathbf{H}_i^\dagger \in CW_N(m-1, \mathbf{I}_{m-1})$, using Theorem 3.2.12 in [14] we obtain

$$Z \doteq \frac{\|\mathbf{h}_i\|^2}{\mathbf{h}_i^\dagger\left(\mathbf{H}_i\mathbf{H}_i^\dagger\right)^{-1}\mathbf{h}_i} \sim \chi^2_{2(m-N)},$$

and Z is independent of $\mathbf{h}_i$. Denoting $X \doteq \|\mathbf{h}_i\|^2$, we get

$$\mathbf{h}_i^\dagger\left(\mathbf{H}_i\mathbf{H}_i^\dagger\right)^{-1}\mathbf{h}_i = \frac{X}{Z} = \frac{N}{m-N}\ \frac{X/2N}{Z/2(m-N)} \doteq \frac{N}{m-N}\ F,$$

where $F = \frac{X/2N}{Z/2(m-N)}$ has an $F_{2N,\ 2(m-N)}$ distribution. Therefore,

$$\Pr\left[\mathrm{SIR}_i \geq \beta\right] = \Pr\left[F \geq \beta\ \frac{m-N}{N}\right] = 1 - I(\frac{\beta}{1+\beta}; N, m-N).$$

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