On the Capacity of Distributed MIMO Systems

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Abstract—The predicted enormous capacity potential of multiple-input multiple-output (MIMO) systems is significantly limited by realistic outdoor propagation environments. In this paper, we investigate a generalized paradigm for multiple-antenna communications, called distributed MIMO (D-MIMO), which can address the problems inherent in conventional co-located MIMO (C-MIMO) systems. A comprehensive three-stage model encompassing the effects of spatial correlation and large-scale fading is introduced, which includes many prevalent C-MIMO models in literature as well as our proposed D-MIMO as special cases. Based on this model, both the dimension gain (due to channel rank improvement) and the power gain (due to macrodiversity) of D-MIMO over C-MIMO in capacity are addressed and verified.

Index Terms—Capacity, channel rank, channel conditioning, MIMO systems, macrodiversity

I. INTRODUCTION

Over the past few years, demand for broadband wireless data access has grown exponentially. Except diversity gain, array gain, and interference reduction advantages [11], the dimension gain, also called spatial multiplexing gain, has recently been recognized for multiple-input multiple-output (MIMO) wireless communication systems [4]. A MIMO channel, typically modeled as a matrix with independent and identically distributed (i.i.d.) complex Gaussian entries, provides multiple spatial dimensions for communications. At high signal to noise ratio (SNR), Shannon capacity can increase linearly with the minimum number of transmit and receive antennas \( \min(n_t, n_r) \). MIMO techniques are anticipated to be widely employed in future wireless networks to address the ever-increasing capacity demands.

However, achieving these dramatic capacity gains in practice, especially for outdoor deployment, could be problematic. The first problem is the rank deficiency and ill-conditionness of the MIMO channel matrix \( H \). This is mainly caused by the spatial correlation due to the scattering environment and the antenna configurations [12], and sometimes by the “keyhole” effect even though the fading is essentially uncorrelated on each end of the channel [2]. Therefore, the MIMO capacity may be greatly reduced and adding more (co-located) antennas only wastes resources. Secondly, the effect of the macroscopic fading (or large-scale fading), largely neglected in current MIMO study, may also induce negative impact on the anticipated system capacity.

In this paper, we investigate a generalized paradigm for multiple-antenna communications, called distributed MIMO (D-MIMO), which can address the problems inherent in conventional co-located MIMO (C-MIMO) systems. In particular, a comprehensive three-stage model is used to describe both C-MIMO and D-MIMO channels, and the rank/conditioning advantage and macrodiversity gain achieved by D-MIMO are addressed and verified based on this model.

As depicted in Fig. 1, in which a triplet \( (n_r, n_t, n_p) \) is used to represent the D-MIMO system, the key difference between D-MIMO and C-MIMO is that multiple antennas for one end (transmitter side for this downlink scenario) of communications are distributed among multiple widely-separated radio ports, and independent large-scale fading and small scale fading are experienced for each link between a mobile-port pair. In a D-MIMO system, the multiple ports may have the same functionality as base stations in today’s cellular system, or may be realized as remote antennas, i.e., small devices containing antennas and electric-optic converters which relay the radio signal to a control unit in the access network. It is reasonably assumed that the multiple ports in D-MIMO are connected by a high-speed backbone that allows information to be reliably exchanged among them, and that joint and cooperative processing is possible. Our discussion can also be extended to the case where antenna elements at both ends are widely separated in geography, like in sensor networks.

D-MIMO can also be regarded as a generalization of distributed antenna system (DAS), whose study dates back to [9], and has attracted attention recently due to its power and capacity advantage over the centralized configuration in broadband wireless network [3]. Most work on DAS so far has emphasized on its advantages in practical employment, such as
lower transmit power, larger system capacity, uniform and enhanced coverage, and ease of cell planning [10],[13]. Some theoretical study on the outage probability and outage capacity of DAS was given only recently in [7], [8], which focuses mainly on the macrodiversity gain for uncorrelated channels, whose information is not known at the transmitter.

This paper is organized as follows. After a brief overview of the capacity of MIMO systems in Section II, a realistic three-stage channel model encompassing the effects of spatial correlation and large-scale fading is presented in Section III. Then, advantages of D-MIMO over C-MIMO in channel capacity are illustrated in Section IV. Numerical results are provided in Section V to verify the main points of this paper. Finally, Section VI contains some concluding remarks.

II. MIMO SYSTEM CAPACITY

For sake of illustration, we will focus on the flat-fading channel in this paper, although the extension to the wideband frequency-selective fading scenario is straightforward. A system model for the narrowband flat-fading MIMO is given as:

\[ y = Hx + n, \quad y = [y_1, \cdots, y_n]^T, \quad x = [x_1, \cdots, x_n]^T, \]

(1)

where \( y \) is the received vector corresponding to the outputs of \( n_r \) receive antennas, \( x \) contains the substreams transmitted by \( n_t \) transmit antennas, \( H \) is an \( n_r \times n_t \) channel matrix, and \( n \) is an \( n_r \)-dimensional complex Gaussian white noise vector with variance \( \sigma^2 \). The total transmit power \( E[\text{tr}(xx^H)] \) is constrained to be no larger than \( P \), and the SNR is \( \rho = P/\sigma^2 \).

The mutual information of channel (1) with Gaussian input is given by:

\[ I(H) = \log \left| I + \frac{1}{\sigma^2}HH^H \right| = \log \left( 1 + \frac{P}{\sigma^2 \lambda_i^2} \right), \]

(2)

where \( \Sigma = \text{tr}(\Sigma) = E[\text{tr}(xx^H)] \leq P \) is the covariance matrix of \( x \). The second equality in (2) comes from the singular value decomposition (SVD) of matrix \( HH^H \), with which the channel can be decomposed into \( L = \text{rank}(H) \) independent substreams, called the eigenmodes. \( \{\lambda_i\} \) are non-zero singular values of \( H \), and \( \{P_i\} \) are the powers assigned to these eigenmodes. The optimal power allocation which achieves the capacity of the instantaneous channel is through waterfilling with the constraint \( \sum \lambda_i^2 P_i = P \). This capacity is achieved when the perfect channel state information (CSI) is known at the transmitter so the transmitted signal is in the form of \( x = V\sqrt{P}\bar{u} \) with uncorrelated Gaussian transmit data \( E[\bar{u}\bar{u}^H] = I \), where \( V \) collects the \( L = \text{rank}(H) \) right singular vectors of \( H \) corresponding to non-zero singular values, and \( P = \text{diag}(\{P_i\}) \). Note that the number of simultaneously transmitted data streams (dimension) can be no larger than the rank of the channel matrix. If instead the CSI is not known at the transmitter, a simpler transmit scheme with equal power allocation \( x = (P/n_t)\bar{u} \) yields:

\[ I_{eq}(H) = \sum_{r=1}^{\text{rank}(H)} \log \left( 1 + \frac{P}{n_r \sigma^2 \lambda_i^2} \right), \]

(3)

which in general is inferior to (2) as power is wasted in eigenmodes with null gains. In the special case of rich scattering with \( \text{rank}(H) = n_r \), (3) approaches (2) in high SNR.

Finally, note that for fading channels there are two distinct notions of capacity: ergodic capacity and outage capacity. Ergodic capacity is the maximum mutual information averaged over all channel states. The outage capacity is related to the instantaneous channel capacity above as follows. The probability \( p \) that the instantaneous channel capacity is lower than some fixed rate \( R \), for which an outage may be declared, can be one-to-one mapped to the rate \( R \) through the cumulative distribution function (CDF) curve of (2) or (3), which in turn is defined as the \( p \)-outage capacity.

III. REALISTIC CHANNEL MODEL

A. Channel Matrix

From (2) and (3), in rich-scattering environments, full rank can be assumed for C-MIMO and essentially \( L = \min(n_r,n_t) \) more bits/s/Hz are obtained for every 3 dB increase in SNR. However, in some extreme environments (e.g., the keyhole problem [2]), a C-MIMO system will lose its capacity advantage (dimension gain) over a single-input single-output (SISO) system, even though other advantages like diversity and array gains may still be preserved. Another important factor influencing the MIMO capacity is the channel conditioning number \( \kappa = \max_i \lambda_i / \min_i \lambda_i \), or more generally the singular value distribution of the channel matrix. Noting that equal-power allocation among eigenmodes achieves optimal performance in high SNR, we conclude from (3) by the Jensen’s inequality that a channel with \( \kappa = 1 \) has the largest capacity, with the same total power constraint. In rich scattering environment, channel matrix \( H \) is assumed to have normalized i.i.d. complex Gaussian entries and thus is well-conditioned. In realistic environments, \( H \) may get ill-conditioned due to fading correlation, resulting from the existence of few dominant scatterers, small angle spread, and insufficient antenna spacing [12]. From (2) and (3) we see that those eigenmodes with \( \lambda_i^2 >> 1 \) are essentially of no use, even though the channel still has full rank. Therefore, originally predicted tremendous capacity gain of MIMO systems based on ideal i.i.d. Gaussian assumptions for channel matrix may be greatly limited in practice.

B. Comprehensive Three-Stage Channel Model

Recently proposed realistic scattering MIMO models, e.g. [5][6], can be summarized as a three-stage comprehensive one as follows, which is also generalized to include the large-scale fading statistics, largely neglected in MIMO study thus far. The physical channel between transmitter and receiver of a MIMO system can be decomposed into three stages, i.e., the local scatterings on both ends and the significant multipaths between them, as shown in Fig. 2. In each stage, a virtual line of sight (LOS) MIMO channel can be assumed. As we know, for LOS
MIMO system, the dimension gain or channel rank can be built up only if the transmit or receive array dimension is large enough relative to the distance between these two arrays, so that the receiver can separate signals from different transmit antennas. Mathematically, the channel matrix of this scattering model can be represented as

\[ H = c_0 A_r \Phi A_t^H, \]  

(4)

where \( c_0 \) is a normalization factor, \( A_r = [a_1, a_2, \ldots, a_n] \) is a \( n_r \times L \) transmitter matrix while \( A_t = [b_1, b_2, \ldots, b_L] \) is a \( n_t \times L \) receiver matrix; \( \Phi \) is a \( L \times L \) diagonal matrix representing the large-scale fading, with diagonal values: \( \Phi_{ii} = (\beta_0 d_i^{-\gamma}) S_i \), where \( \beta_0 d_i^{-\gamma} \) represents the path loss of the \( l \)th path (\( \beta_0 \) is a propagation constant and \( \gamma \) is the path loss exponent), and \( S_i \) denotes the shadow fading, typically modeled as a log-normal random variable with standard deviation of 6–10 dB. The number of significant paths \( L \) between the transmitter and receiver in stage 2 is typically determined by the dominant remote scattering objects.

Depending on the local scattering at the transmitter side, transmitter matrix \( A_t \) can be modeled in different ways. Without any local scatterer around the transmitter array, \( a_d \) is just a fixed array-response vector corresponding to the \( l \)th dominant path. For a linear array, it can be expressed as

\[ a_d = [1, \exp(j2\pi(\frac{\Delta}{\lambda})\cos \theta_1), \ldots, \exp(j2\pi(n_t-1)(\frac{\Delta}{\lambda})\cos \theta_1)]^T, \]  

(5)

where \( \Delta \) denotes transmit antenna spacing, \( \lambda \) is the wavelength, and \( \theta_1 \) represents the direction of departure (DOD) of path \( l \). With rich scattering, the elements of \( a_d \) are totally randomized, and \( A_t \) can be modeled as a white Gaussian matrix \( H_w \), each column of which is i.i.d. with the same distribution as \( h_w \sim \mathcal{CN}(0, \Sigma) \). From another viewpoint, in the virtual LOS MIMO channel of stage 1, receiver dimension is large enough to build up the channel rank (equal to \( n_r \) ) and conditioning, so the transmitter fading is uncorrelated, and the transmit antenna can be spaced as close as \( \lambda / 2 \). In practice, fading correlation is often introduced due to insufficient angle spread or antenna spacing. Usually situations are more stringent at the base station side, where antennas are elevated and unobstructed by local scatterers, and antenna spacing may be insufficient due to environmental concerns especially when the number of antennas is large. In this case, each column of \( A_t \) can be modeled to have the same distribution as \( R_{t}^{1/2} h_w \sim \mathcal{CN}(0, R_t) \) or \( A_t = R_{t}^{1/2} H_w \), where \( R_t \) denotes the transmit correlation matrix whose entries can be expressed for a linear array with uniformly distributed local scatterers as:

\[ [R_t]_{i,j} = \frac{1}{S_i} \sum_{k=1}^{S_t} \exp(-2\pi j (i-k) \frac{\Delta}{\lambda} \cos \phi_k), \]  

(6)

where \( S_i \) is the number of total transmitter scatterers and the random variable \( \phi_k \), with a uniform distribution, is the approximate DOD from the transmit array to \( k \)th scatterer. Notice that in (6) if \( S_t \to \infty \) with a large range for the distribution of \( \phi_k \) (rich scattering) and \( \Delta \geq \lambda / 2 \), \( R_t \to I \), which means that fading is uncorrelated. The above discussion readily apply to the receive matrix \( A_r \) as well. Supposing \( r_t = \text{rank}(A_t) \leq n_t \) and \( r_r = \text{rank}(A_r) \leq n_r \) with the fact \( \text{rank}(\Phi) = L \), we can get:

\[ \text{rank}(H) = \min(r_r, r_t, L). \]  

(7)

The generalized channel model (4) includes many of the interesting scenarios in practice for conventional C-MIMO where \( \Phi = \Phi \)I, as shown in the following.

C. Some Special Cases in C-MIMO

(1) Ideal C-MIMO channel: When fading is uncorrelated on both transmit and receive sides and there are sufficiently large numbers of independent paths in between (\( L \to \infty \)), by central limit theory the channel matrix (4) can be rewritten as \( H = \Phi H_w \), which typically is of full rank and well-conditioned.

(2) Fading Correlation: When fading is correlated on both ends due to insufficient scattering, antenna spacing, or angle spread, but \( L \to \infty \) still holds, the channel matrix (4) becomes:

\[ H = c_0 \Phi R_{t}^{1/2} H_w R_{t}^{1/2}. \]  

(8)

If correlation only exists on one end, (8) can be readily modified through replacing one of the correlation matrices with an identity matrix. Study based on this separable correlation model has revealed that fading correlation reduces the channel rank \( r \) and/or increases the condition number \( \kappa \) of the channel matrix, thus decreases the channel capacity.

(3) Keyhole: When the distance between the transmitter and receiver is much larger than the radii of local scatterers on both ends and no significant remote scatterers exist in between, there is essentially only one narrow “pipe” for the signals to travel through in the virtual LOS channel of stage 2. In this case, \( L = 1 \) and the channel matrix can be modeled as:

\[ H = c_0 a_d^H. \]  

(9)
Therefore, the channel has rank one and exhibits no dimension gain, even though at the same time fading on both ends can be uncorrelated and thus the diversity gain is still preserved.

(4) Downlink of outdoor cellular macrocell: In this scenario, antenna arrays at the base stations are elevated above urban clusters and far away from local scattering, while mobile terminals are surrounded by rich scatterers, and the number of independent paths $L$ is limited by few far-field reflectors. Therefore, the downlink channel matrix can be modeled as:

$$H = \Phi \Phi^T \mathbf{A}^t,$$

where $\mathbf{A}$ collects the $L$ dominant transmit array response vectors. In this case, channel rank depends on $L$, and the channel matrix may be both rank-deficient and ill-conditioned, determined by the propagation and system parameters.

IV. CAPACITY GAINS IN D-MIMO

As indicated by (8)–(10), in many practical situations, C-MIMO may experience very low channel rank or large conditioning number, and lose much of its capacity advantage over a SISO system. On the other hand, as shown in Fig. 1, antennas in D-MIMO are widely distributed among radio ports with much larger antenna spacing, so the spatial correlation and the keyhole problem are largely alleviated, and even can be completely eliminated with good deployment. For a $(n_s, n_t, n_p)$ D-MIMO system, because of the independency among different radio ports, it is not difficult to prove that the channel matrix (4) can be decomposed into $n_p$ sub-matrices, each corresponding to the channel between one port-mobile pair. Therefore, the downlink channel matrix can be modeled as:

$$H = \{\Phi_1 \mathbf{H}_1, \Phi_2 \mathbf{H}_2, \ldots, \Phi_{n_p} \mathbf{H}_{n_p}\},$$

where $\{\mathbf{H}_i\}$ are mutually independent, each of which can be modeled as in the previous section, with the independent large-scale fading $\{\Phi_i\}$ explicitly expressed. Based on the geometry of distributed transmitter array structure, each radio port provides at least one independent link even in the absence of remote scattering objects. No matter what channel model among (8)–(10) is used for $\{\mathbf{H}_i\}$, the overall number of independent links in (11), given by $\sum_{i=1}^{n_p} \text{rank}(\mathbf{H}_i)$, is then guaranteed to be at least equal to $n_p$. If $n_p \geq n_t$, $H$ will always have full rank. It is found in [1] recently that unlike the SISO case, delay spread channels offer advantages over flat-fading channels in terms of ergodic capacity for C-MIMO systems. This conclusion is a result of the assumption that delay paths tend to increase the total angle spread, and thus improve the channel rank. In our proposed D-MIMO, however, the full rank can be obtained even with the flat-fading channel, due to large angle spread and wide antenna spacing inherent in D-MIMO. Furthermore, for the channel of (11), the channel conditioning will not be greatly degraded even if transmitter fading correlation happens at each radio port, as the fading between different transmitter antennas at different ports are still uncorrelated.

Large-scale fading in wireless channels is position sensitive, which means that co-located transmit antennas are generally subject to the same large-scale fading factor, while those at different radio ports subject to independent ones. If this fading effect is severe, the capacity of C-MIMO will degrade significantly, since all the entries of the channel matrix are subject to the same severe fading. On the other hand, D-MIMO can provide the macrodiversity protection for this impairment. Intuitively, the probability that all sub-channels of (11) are under deep large-scale fading is much lower than C-MIMO. Macrodiversity cannot increase the mean of the received SNR, but will greatly reduce its variance. This results in power gains with respect to the outage capacity, as will be seen in the next section.

V. NUMERICAL RESULTS

To evaluate the capacity gain in D-MIMO, let us study the downlink communications in an outdoor macrocell environment without significant remote scatterers, which means $L = 1$ in (10). Suppose a $4 \times 4$ C-MIMO system is employed in this scenario, which sees a rank-1 channel. Such a C-MIMO system can also be denoted as a $(4,4,1)$ D-MIMO. To make fair comparison, the total number of antennas deployed at the radio ports of a D-MIMO system is constrained to be the same (4 in this example). We study two such D-MIMO systems, denoted as $(4,2,2)$ and $(4,1,4)$ D-MIMO, respectively, each of which is modeled according to (11) with the sub-matrices modeled according to (10). For simplicity the distances between the mobile and all ports are set equal, and log-normal shadow fading is considered with a standard deviation of 8 dB. The angles of departure are generated randomly with a uniform distribution within a range of 90 degrees.

At high SNR, it is easy to show from (2) or (3) that the MIMO capacity scales with the channel rank as $C \propto \text{rank}(H) \log \rho$. In Fig. 3, where the 10% outage capacities of the three MIMO systems are shown with both waterfilling and the equal-power allocation, we can see that C-MIMO using the model (10) with $L = 1$ sees no dimension gain (1 more bits/Hz for every 3 dB gain), while the channel rank and thus the dimension gain quickly builds up with antennas distributed among separated radio ports in D-MIMO (see $(4,2,2)$ and $(4,1,4)$). Furthermore, the results of a full-rank ideal $(4,4,1)$ C-MIMO system are also listed for comparison, which reveals another advantage of D-MIMO over C-MIMO, the power gain achieved by macrodiversity. Obviously D-MIMO assumes no advantage on ergodic capacity (dimension gain) over full-rank C-MIMO, but for 10% outage capacity, D-MIMO achieves around 5 dB gain in power, as seen from the horizontal gap between the parallel capacity curves of full-rank $(4,1,4)$ D-MIMO and the ideal $(4,4,1)$ C-MIMO in Fig. 3. Intuitively, the variance of the instantaneous channel capacity due to large-scale fading is greatly reduced in D-MIMO and thus less outage is seen, due to the macrodiversity protection inherent in D-MIMO. Another interesting observation concerns the cross point of the capacity curve of the $(4,2,2)$ D-MIMO with waterfilling power.
allocation and that of the ideal (4,4,1) C-MIMO with equal power allocation. It indicates that for transmit powers up to the crossover point (around 23 dB), the correlated (4,2,2) D-MIMO channel with transmit CSI offers higher capacity than the uncorrelated (4,4,1) C-MIMO without transmit CSI. This is partly due to transmit array gain [6], as in the former case, two antenna elements in a radio port are employed to optimally transmit one data stream, and partly due to the macrodiversity gain mentioned above. Also note that waterfilling power allocation assumes advantages over equal power allocation for rank-deficient channels, as powers are wasted in unusable eigenmodes for the latter case. Therefore, it is beneficial to assume CSI at the transmitter for rank-deficient and correlated MIMO channels.

The macrodiversity protection of D-MIMO can be further illustrated by Fig. 4. With increasing shadowing effect, the outage capacity of C-MIMO degrades significantly, while that of D-MIMO almost keeps the same.

D-MIMO are addressed and verified, which reveals the great potential of this scheme on meeting the ever-increasing capacity demands for wireless communications.

VI. CONCLUSIONS

In this paper, a generalized paradigm for multiple-antenna communications, D-MIMO, has been proposed to address the problems inherent in conventional co-located MIMO systems, especially in realistic outdoor propagation environments. Both the dimension gain and macrodiversity gain achieved by D-MIMO are addressed and verified, which reveals the great potential of this scheme on meeting the ever-increasing capacity demands for wireless communications.

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