

MA 407 Homework 1a

1. (a) (i) Compute $d = \gcd(2272, 716)$ by hand and by using Maple or Matlab.
 (ii) Find u and v such that $d = 2272u + 716v$
 (b)(i) If $n \mid (r - ab)$, $n \mid (s - a)$ and $n \mid (t - b)$ show that $n \mid (r - st)$.
 (ii) Use b-(i) to prove that $ab \bmod n = (a \bmod n)(b \bmod n) \bmod n$.
2. (a) Show that $(a, (b, c)) = (a, b, c)$.
Hint: $(a, b, c) \mid b, c \Rightarrow (a, b, c) \mid (b, c)$ or use prime factorization
 (b) How could you use (a) to show that $[a, [b, c]] = [a, b, c]$
3. Verify that each nonzero element in \mathbb{Z}_{11} has a multiplicative inverse.
4. Suppose that “+” and “.” have been defined on a set S , and that $a \in S$. Further let $a^3 = a + 1$, $a^7 = 1$ and $a + a = 0$.
 (i) Compute the powers of a : (a, a^2, \dots, a^7) as polynomials in a of degree at most two.
5. (a) Let G be a group and let $a, b, x, y \in G$.
 (i) Show that $(a^{-1})^{-1} = a$ and hence that $a = b$ iff $a^{-1} = b^{-1}$
 (ii) Find the inverse of $ab^2a^3b^4$
 (iii) Find x and y if $e = xa$ and $b = ya$
 (iv) Show that $O(a) = O(a^{-1}) = O(b^{-1}ab)$ {recall $O(a) = \min\{k > 0; a^k = e\}$ if any }.
6. Show that $G = \left\{ \begin{bmatrix} a & a \\ 0 & 0 \end{bmatrix}; a \in \mathbb{R}, a \neq 0 \right\}$ is a multiplicative group. Is it abelian ?
7. Let (G, \cdot) be a group with identity e . Show that *any* of the following forces G to be abelian.
 (i) $(ab)^{-1} = a^{-1}b^{-1}$ for all a, b in G
 (ii) $(ab)^2 = a^2b^2$ for all a, b in G
 (iii) $a^{-1}ba = b$ for all a, b in G
 (iv) $a^2 = e$ for all a in G .
 (v)* $ab = ca$ implies $b = c$, for all a, b, c in G , i.e. *whenever* $ab = (\text{something})a$ then $b = (\text{something})$.

8* With a as in Q4, solve the system $\begin{bmatrix} a^2 & a \\ a^6 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a^5 \\ a^3 \end{bmatrix}$
 (e.g. use Cramer's Rule or Row reduction)