MA 405 HW 1
show all your work

1. If A = $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ compute A^n for n = 1,2,...

2. Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$. Verify that
   (i) $(AB)^T = B^T A^T$ and (ii) Tr(AC) = Tr(CA)

3. Let $P = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $Q = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.
   (i) Compute $P^T P, \quad Q^T Q, \quad PP^T$ and QQ^T
   (ii) Find P^{-1} and Q^{-1}

4. Let$E^2 = E$ and A = I + bE, where $b \neq -1$.
   (i) Show that E(I - E) = 0 = (I - E)E
   (ii) $(I - E)^2 = I - E$
   (iii) $(I - 2E)^2 = I$
   (iv) $[EX(I - E)]^2 = 0$ for any conformable X.
   (v) Find a scalar c such that $A^{-1} = I + cE$

5. If (I - AB)X = I verify that (I - BA)Y = I, where Y = I + BXA

6. Let $A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4] = \begin{bmatrix} \boldsymbol{\alpha}_1^T \\ \boldsymbol{\alpha}_2^T \\ \boldsymbol{\alpha}_3^T \\ \boldsymbol{\alpha}_4^T \end{bmatrix}$
   (i) Compute the row and column sums A$\mathbf{e}$, $\mathbf{e}^T A$
   (ii) Compute $\mathbf{a}_i^T \mathbf{a}_j$ and $\boldsymbol{\alpha}_i^T \boldsymbol{\alpha}_j$ for $i \neq j$.
   (iii) What do you notice ?

7. Let V = $\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$ with a,b,c distinct, and let $\mathbf{x} = \begin{bmatrix} bc \\ -(c+b) \\ 1 \end{bmatrix}$.
   (i) Verify that V$\mathbf{x}$= $\begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix}$ for some α.
   (ii) What is the value of α ?
   (iii) Can you find vectors $\mathbf{y}$ and $\mathbf{z}$ such that V$\mathbf{y} = \begin{bmatrix} 0 \\ \beta \\ 0 \end{bmatrix}$ and V$\mathbf{z} = \begin{bmatrix} 0 \\ 0 \\ \gamma \end{bmatrix}$
   (iv) What are the values of β, γ ?