

MA 121 Summer I 2007

Test 2

Copy C

Name Key

Show your work on the test page or scrap paper. Each problem is worth ten points. Simplify your answers as much as possible.

1. Find $f'(x)$ for:

(a) $f(x) = (-x^2 + 3x)(2x + 7)^3$
 $f'(x) = (-2x + 3)(2x + 7)3 + 3(2x + 7)^2(2)(-x^2 + 3x)$

(b) $f(x) = \frac{x+4}{7x-1}$
 $f'(x) = \frac{(7x-4)-7(x+4)}{(7x-1)^2}$

2. Let $y(u) = u^{-\frac{1}{3}}$ and let $u(x) = x^3 + 7$.

(a) Find $u(1)$ and $y(u(1))$.
 $u(1) = 8, y(u(1)) = y(8) = 8^{-\frac{1}{3}} = \frac{1}{2}$

(b) Compute $\frac{dy}{du}, \frac{du}{dx}$ and $\frac{dy}{dx}$.
 $\frac{dy}{du} = -\frac{1}{3}u^{-\frac{4}{3}}, \frac{du}{dx} = 3x^2, \frac{dy}{dx} = -\frac{1}{3}(x^3 + 7)^{-\frac{4}{3}}(3x^2)$

3. Let $f(x) = x^3 - 3x^2 - 9x + 5$. Find all relative maximums and minimums of $f(x)$. Find the intervals when $f(x)$ is increasing and decreasing.

$(-1, 10)$ is a relative max, $(3, -22)$ is a relative min, f is increasing on $(-\infty, -1)$ and $(3, \infty)$, f is decreasing on $(-1, 3)$

4. Let $f(x) = -x^4 + 24x^2 - 7x + 3$. Find all the points of inflection of $f(x)$. Find the intervals when the function is concave up and concave down.

$(-2, 97)$ is a point of inflection, $(2, 67)$ is a point of inflection, f is concave down on $(-\infty, -1)$ and $(1, \infty)$, f is concave up on $(-2, 2)$

5. Find all of the asymptotes (horizontal, vertical and oblique) of the following functions.

(a) $f(x) = \frac{5}{2x+9}$
Horizontal asymptote $y = 0$, Vertical Asymptote $x = -\frac{9}{2}$

(b) $f(x) = \frac{x^2+3x+2}{x-1}$
Oblique Asymptote $y = x + 4$, Vertical Asymptote $x = 1$

6. Find the absolute maximum and minimum of $f(x) = x^3 - 3x^2 + 15$ on the closed interval $[-2, 5]$.

Absolute Max (5, 65), Absolute Min (-2, -5)

7. A movie theatre sells 400 boxes of popcorn when they charge \$2.50 a box. If they increase the price by \$1 then they sell 20 fewer boxes. How much should they charge per box to get the maximum revenue? What is the maximum revenue?

Maximum Revenue of \$1000 when the price is \$20

8. Find $f'(x)$ for:

(a) $f(x) = e^{-4x+3}$
 $f'(x) = (-4)e^{-4x+3}$

(b) $f(x) = x^3e^x$
 $f'(x) = (3x^2)e^x + e^x(x^3)$

9. Find $f'(x)$ for:

(a) $f(x) = \ln(x^2 + x - 9)$
 $f'(x) = \frac{2x+1}{x^2+x-9}$

(b) $f(x) = \ln\left(\frac{x^2}{3x+2}\right)$
 $f'(x) = \frac{2}{x} - \frac{3}{3x+2}$

10. Let $f(x) = 2e^{x^2-4x}$.

- (a) Find the equation of the line tangent to the graph of $f(x)$ when $x = 0$.

$$y = -8x + 2$$

- (b) Find all of the critical points of $f(x)$.

$x = 2$ is the only critical point