

MA 121 Summer I 2007

Test 2

Copy B

Name Key

Show your work on the test page or scrap paper. Each problem is worth ten points. Simplify your answers as much as possible.

1. Find $f'(x)$ for:

(a) $f(x) = (2x - 5)^2(x^2 + 4x - 3)^5$
 $f'(x) = 2(2x - 5)(2)(x^2 + 4x - 3) + 5(x^2 + 4x - 3)^4(2x + 4)(2x - 5)^2$

(b) $f(x) = \frac{x+2}{x^2+7}$
 $f' = \frac{(x^2+7)-(2x)(x+2)}{(x^2+7)^2}$

2. Let $y(u) = u^{\frac{4}{3}}$ and let $u(x) = x^2 - 5$.

(a) Find $u(2)$ and $y(u(2))$.
 $u(2) = -1, y(u(2)) = y(-1) = (-1)^{\frac{4}{3}} = 1$

(b) Compute $\frac{dy}{du}, \frac{du}{dx}$ and $\frac{dy}{dx}$.
 $\frac{dy}{du} = \frac{4}{3}u^{\frac{1}{3}}, \frac{du}{dx} = 2x, \frac{dy}{dx} = \frac{4}{3}(x^2 - 5)^{\frac{1}{3}}(2x)$

3. Let $f(x) = -x^3 + 27x + 5$. Find all relative maximums and minimums of $f(x)$. Find the intervals when $f(x)$ is increasing and decreasing.

(3, 59) is a relative max, (-3, -49) is a relative min, f is increasing on (-3, 3), f is decreasing on (-∞, -3) and (3, ∞)

4. Let $f(x) = x^4 - 10x^3 + 36x^2 + 2x - 7$. Find all the points of inflection of $f(x)$. Find the intervals when the function is concave up and concave down.

(2, 77) is a point of inflection, (3, 134) is a point of inflection, f is concave up on (-∞, 2) and (3, ∞), f is concave down on (2, 3)

5. Find all of the asymptotes (horizontal, vertical and oblique) of the following functions.

(a) $f(x) = \frac{3x^2+7x-2}{x+2}$
Oblique Asymptote $y = 3x + 1$, Vertical Asymptote $x = -2$

(b) $f(x) = \frac{2x-3}{5x-9}$
Horizontal asymptote $y = \frac{2}{5}$, Vertical Asymptote $x = -\frac{9}{5}$

6. Find the absolute maximum and minimum of $f(x) = -2x^3 - 9x^2 + 24x - 5$ on the closed interval $[-3, 3]$.

Absolute Max (1, 8), Absolute Min (-3, -98)

7. A movie theatre sells 250 boxes of popcorn when they charge \$2.00 a box. If they increase the price by \$1 then they sell 25 fewer boxes. How much should they charge per box to get the maximum revenue? What is the maximum revenue?

Maximum Revenue of \$900 when the price is \$6

8. Find $f'(x)$ for:

(a) $f(x) = e^{-x^3-2}$
 $f'(x) = (-3x^2)e^{-x^3-2}$

(b) $f(x) = xe^{2x}$
 $f'(x) = e^{2x} + 2e^{2x}(x)$

9. Find $f'(x)$ for:

(a) $f(x) = \ln(x^2 - 5x + 7)$
 $f'(x) = \frac{2x-5}{x^2-5x+7}$

(b) $f(x) = \ln((x+7)(2x^3-3))$
 $f'(x) = \frac{1}{x+7} + \frac{6x^2}{2x^3-3}$

10. Let $f(x) = 3e^{x^2+2x}$.

- (a) Find the equation of the line tangent to the graph of $f(x)$ when $x = 0$.

$$y = 6x + 3$$

- (b) Find all of the critical points of $f(x)$.
 $x = -1$ is the only critical point