

MA 121 Summer I 2007

Test 2

Copy A

Name Key

Show your work on the test page or scrap paper. Each problem is worth ten points. Simplify your answers as much as possible.

1. Find $f'(x)$ for:

(a) $f(x) = (x^2 - 5)^3(5x + 2)^4$
 $f'(x) = 3(x^2 - 5)^2(5x + 2)^4 + 4(5x + 2)^3(5)(x^2 - 5)^3$

(b) $f(x) = \frac{x^2 - 3x - 5}{3x + 7}$
 $f'(x) = \frac{(2x - 3)(3x + 7) - 3(x^2 - 3x - 5)}{(3x + 7)^2}$

2. Let $y(u) = u^{\frac{1}{3}}$ and let $u(x) = x^2 + 1$.

(a) Find $u(1)$ and $y(u(1))$.
 $u(1) = 2, y(u(1)) = y(2) = 2^{\frac{1}{3}} = \sqrt[3]{2}$

(b) Compute $\frac{dy}{du}, \frac{du}{dx}$ and $\frac{dy}{dx}$.
 $\frac{dy}{du} = \frac{1}{3}u^{-\frac{2}{3}}, \frac{du}{dx} = 2x, \frac{dy}{dx} = \frac{1}{3}(x^2 + 1)^{-\frac{2}{3}}(2x)$

3. Let $f(x) = \frac{x^3}{3} + \frac{x^2}{2} - 2x + 5$. Find all relative maximums and minimums of $f(x)$. Find the intervals when $f(x)$ is increasing and decreasing.

$(-2, \frac{25}{3})$ is a relative max, $(1, \frac{29}{6})$ is a relative min, f is increasing on $(-\infty, -2)$ and $(1, \infty)$, f is decreasing on $(-2, 1)$

4. Let $f(x) = x^4 - 6x^2 + 2x - 3$. Find all the points of inflection of $f(x)$. Find the intervals when the function is concave up and concave down.

$(-1, -10)$ is a point of inflection, $(1, -6)$ is a point of inflection, f is concave up on $(-\infty, -1)$ and $(1, \infty)$, f is concave down on $(-1, 1)$

5. Find all of the asymptotes (horizontal, vertical and oblique) of the following functions.

(a) $f(x) = \frac{2x+3}{4x+5}$
Horizontal asymptote $y = \frac{1}{2}$, Vertical Asymptote $x = -\frac{5}{4}$

(b) $f(x) = \frac{3x^2 - 5x + 2}{x - 3}$
Oblique Asymptote $y = 3x + 1$, Vertical Asymptote $x = 3$

6. Find the absolute maximum and minimum of $f(x) = x^3 - 9x^2 + 15x - 2$ on the closed interval $[-2, 4]$.

Absolute Max (1, 5), Absolute Min (-2, -76)

7. A movie theatre sells 200 boxes of popcorn when they charge \$3.00 a box. If they increase the price by \$.50 then they sell 10 fewer boxes. How much should they charge per box to get the maximum revenue? What is the maximum revenue?

Maximum Revenue of \$845 when the price is \$6.50

8. Find $f'(x)$ for:

(a) $f(x) = e^{x^2+7x-5}$
 $f'(x) = (2x + 7)e^{x^2+7x-5}$

(b) $f(x) = x^2e^{-x}$
 $f'(x) = (2x)e^{-x} - e^{-x}(x^2)$

9. Find $f'(x)$ for:

(a) $f(x) = \ln(x^3 + 4x - 8)$
 $f'(x) = \frac{3x^2+4}{x^3+4x-8}$

(b) $f(x) = \ln\left(\frac{2x+5}{x-7}\right)$
 $f'(x) = \frac{2}{2x+5} - \frac{1}{x-7}$

10. Let $f(x) = 5e^{x^2-x}$.

- (a) Find the equation of the line tangent to the graph of $f(x)$ when $x = 0$.

$$y = -5x + 5$$

- (b) Find all of the critical points of $f(x)$.
 $x = \frac{1}{2}$ is the only critical point