

Numerical simulation of granular flows

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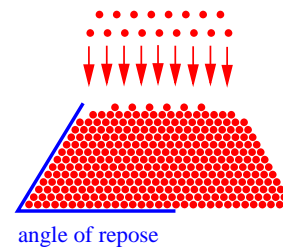
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What a granular material is: Anything consisting of a large number of individual solid particles. Typically, the size of the particles range from 10^{-7} m, for a very fine powder, to 10^{-1} m, in case of grains, gravel, pills, etc...

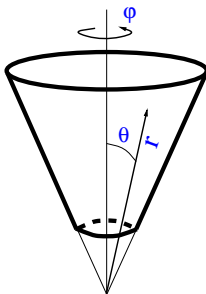
Applications: Countless materials are stored in a granular form. Difficulties in handling them translate into significant financial losses in Food Processing, in Construction, in Mining and Metallurgy as well as in the Chemical and Pharmaceutical Industries. Other applications are found in the Geosciences (Soil Mechanics).

Mathematical model: The approach taken is that of Continuum Mechanics. The mechanical properties of the grains are described by a small number of assumptions and parameters. Friction plays a fundamental role through, for instance, the [angle of repose](#).

Unlike that of a stagnant liquid, the top surface of a mass of granular material need not be horizontal. There is, however, an upper limit to the slope of the upper surface. The angle between this maximum slope and the horizontal is the angle of repose.



Hopper flows:



In many applications, granular materials are retrieved from storage bins or silos by letting them flow, under the action of gravity, through outlets located at the bottom of the structures. Difficulties in the retrieval process are common. They include irregular flows, violent vibrations and material sticking to the walls. The geometry considered here is that of a [conical hopper](#).

A spherical coordinate system (r, θ, φ) is used. The axisymmetric structure results from the absence of motion in the φ direction, in contrast with what happens for fluids.

Laws of conservation:

Contrary to fluids, the forces per unit area acting on an elementary material surface depend on the direction of that surface. For a given coordinate system, each row of the stress tensor T , which is a 3×3 matrix, corresponds to the force density vector acting on one of the three possible coordinate surfaces. Due to symmetries in the problem, T and v can be expressed with 4 and 2 components, resp.

The state of an incompressible flowing material is described by the [stress tensor](#) T and the velocity v . For steady-state flows, both unknowns only depend on r and θ . The equations expressing incompressibility and balance of forces in the material are respectively

$$\nabla \cdot v = 0 \quad \text{and} \quad \nabla \cdot T = \rho g$$

in which ρ is the density, and the vector g is the acceleration due gravity.