

Ma551 Homework 2 Hints

1. This problem should be fairly easy using the Schröder Bernstein Theorem.
2. One way to do this is by defining a function recursively. For example, first define $f(\min A) = \min(B)$. Now suppose that $x \in A$ and f is defined for everything less than x , i.e. on $Pred(x, A)$. Define $f(x) = \min(B \setminus f(Pred(x, A)))$.
3. Suppose we want to well order A . Let \mathcal{C} be the set of all pairs (C, \prec_C) so that $C \subset A$, and \prec_C is a well order on C . Order the elements of \mathcal{C} as follows:
Let (C, \prec_C) and (B, \prec_B) both be in \mathcal{C} . Then $(C, \prec_C) \leq (B, \prec_B)$ iff
 - (a) $C \subseteq B$,
 - (b) \prec_B restricted to C is the same as \prec_C ,
and
 - (c) (C, \prec_C) is an initial segment (predecessor set) in (B, \prec_B) .

Apply **Zorn's Lemma** to this collection. Then show that a maximal element must well order all of A .