

Tangent Bifurcation

$$f_c(x) = x^3 - x + c$$

$$f'_c(x) = 3x^2 - 1, \text{ need } f'_c(x) = 0$$

$$3x^2 - 1 = 0, \quad x^2 = \frac{1}{3} \quad x = \pm \sqrt{\frac{2}{3}}$$

the corresponding c so that these are fixed points.

$$x^3 - x + c = x$$

$$c = 2x - x^3$$

$$c = 2\sqrt{\frac{2}{3}} - \left(\frac{2}{3}\right)^{3/2} \text{ for } x_0 = \sqrt{\frac{2}{3}}$$

$$c = -2\sqrt{\frac{2}{3}} + \left(\frac{2}{3}\right)^{3/2} \text{ for } x_0 = -\sqrt{\frac{2}{3}}$$

