

Repeated Period Doubling

Let $Q_c(x) = x^2 + c$.

1. Find the bifurcation values of c where there is the creation of 2 cycles, 4 cycles, 8 cycles, 16 cycles and 32 cycles. This is a sequence of period doubling bifurcations. (Get as many decimal places as you can, 6 or 8)
2. Discuss any pattern you see in these bifurcations. (Look at the differences in successive c values.)
3. Use your discussion in 2 to predict where the 64 cycle will start.
4. Use phaser to check your prediction in 3.

$$\begin{aligned}
 1. \quad c_2 &= -0.75 \\
 c_4 &\approx -1.25 \\
 c_8 &\approx -1.3681 \\
 c_{16} &= -1.394046 \\
 c_{32} &= -1.399633
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \frac{c_2 - c_4}{c_4 - c_8} &= \frac{0.5}{0.1181} = 4.23 \\
 \frac{c_4 - c_8}{c_8 - c_{16}} &= \frac{0.1181}{0.2595} = 4.55 \\
 \frac{c_8 - c_{16}}{c_{16} - c_{32}} &= \frac{0.2595}{0.05597} = 4.64
 \end{aligned}$$

} These numbers are close and seem to be increasing a little.

$$3. \quad \text{also } \frac{c_{16} - c_{32}}{c_{32} - c_{64}} = 4.7 \qquad c_{64} \approx \frac{c_{16} - c_{32}}{4.7} + c_{32} = -1.400822$$

$$4. \quad c_{64} = -1.400828 \quad \leftarrow \text{very close}$$