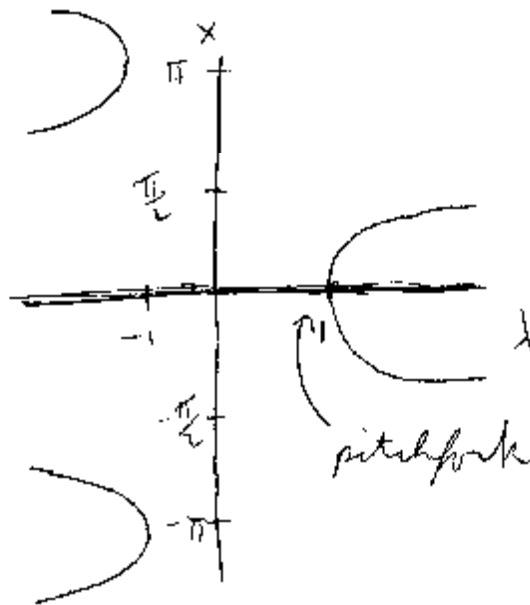


Pitchfork Bifurcation

1. a. $f_\lambda(x) = \lambda x \cos x$

$\lambda x \cos x = x$, $x = 0$ and $\lambda \cos x = 1$

$\lambda = \sec x$



$f_\lambda(x) = \lambda x \cos x$ has continuous partials ✓

$f_\lambda(0) = 1 \cdot 0 \cdot \cos 0 = 0$ ✓

$f'_\lambda(x) = \lambda \cos x - \lambda x \sin x$, $f'_\lambda(0) = 1 \cdot \cos 0 - 1 \cdot 0 \cdot \sin 0 = 1$ ✓

$f''_\lambda(x) = -\lambda \sin x - \lambda \sin x - \lambda x \cos x$, $f''_\lambda(0) = -1 \sin 0 - 1 \sin 0 - 1 \cdot 0 \cdot \cos 0 = 0$ ✓

$f'''_\lambda(x) = -2\lambda \cos x - \lambda \cos x + \lambda x \sin x$, $f'''_\lambda(0) = -3 \neq 0$ ✓

$\frac{\partial}{\partial \lambda} f'_\lambda(x) = \cos x - x \sin x$, $\left. \frac{\partial}{\partial \lambda} f'_\lambda(0) \right|_{\lambda=1} = 1 \neq 0$ ✓