

p.112

10. a. Since  $|s_i - t_i| \geq 0$ ,  $d(s, t) \geq 0$

If  $s = t$  then  $|s_i - t_i| = 0$  so  $d(s, t) = 0$

If  $s \neq t$  then  $\exists k \in \mathbb{N}$  such that  $s_k \neq t_k$ .

Thus  $|s_k - t_k| > 0$  and  $d(s, t) \geq \frac{|s_k - t_k|}{N^k}$ .

b. Since  $|s_i - t_i| = |t_i - s_i|$ ,

$$d(s, t) = \sum_{i=0}^{\infty} \frac{|s_i - t_i|}{N^i} = \sum_{i=0}^{\infty} \frac{|t_i - s_i|}{N^i} = d(t, s)$$

c. Let  $s, t, u \in \Sigma_N$ . Then

$$d(s, t) = \sum_{i=0}^{\infty} \frac{|s_i - t_i|}{N^i} = \sum_{i=0}^{\infty} \frac{|s_i - u_i + u_i - t_i|}{N^i} \leq \sum_{i=0}^{\infty} \frac{|s_i - u_i|}{N^i} + \frac{|u_i - t_i|}{N^i}$$

$$= \sum_{i=0}^{\infty} \frac{|s_i - u_i|}{N^i} + \sum_{i=0}^{\infty} \frac{|u_i - t_i|}{N^i} = d(s, u) + d(u, t)$$

11. also note since  $|s_i - t_i| \leq N-1$

$$d(s, t) \leq \sum_{i=0}^{\infty} \frac{N-1}{N^i} \leq \sum_{i=0}^{\infty} \frac{N-1}{N^i} = \frac{N-1}{1-N} = N$$

P.112

12.  $\boxed{N}$  fixed points, they are the  $N$  constant sequences.

The points of period 2 are the number of 2 blocks.

We have  $N$  choices for each, so there are

$\boxed{N^2}$  points of period 2.

$N$  of these are fixed so  $\boxed{N^2 - N}$  prime period 2.

13.  $N^k$