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1. d. $G_{\mu}(x) = \mu x + x^3$, $\mu = 1$, $G_{\mu}(x)$ has continuous partial derivatives,

$$G_{\mu}(x) = x + x^3$$

$$x + x^3 = x$$

$$x^3 = 0$$

$x = 0$ is a fixed point.

$$G'_{\mu}(x) = \mu + 3x^2$$

$G'_1(0) = 1 + 0 = 1$ neutral so could be a bifurcation.

$$G''_{\mu}(x) = 6x$$

$G''_1(0) = 0$ not tangent bifurcation.

$G'''_{\mu}(x) = 6 \neq 0$ could be a pitchfork.

$$\frac{\partial}{\partial \mu} \left(G'_{\mu}(x) \right) = \frac{\partial}{\partial \mu} (\mu + 3x^2) = 1 \neq 0$$

$G_{\mu}(0) = 0$, $x = 0$ is always fixed.

all the conditions for a simple pitchfork bifurcation are satisfied.