P 151

1. $48 = 3 \cdot 2^4$

$56 = 7 \cdot 2^3$

56 is to the left of 48 in the Sarkovskii ordering so there is a continuous map of $\mathbb{R}$ with a point of period 48 but not one of prime period 56

2. $176 = 11 \cdot 2^4$

$96 = 3 \cdot 2^5$

176 is to the left of 96 in the Sarkovskii ordering so if a continuous map of $\mathbb{R}$ has a point of prime period 176 it must also have one of period 96.

3. It cannot happen if F is continuous. But an example where F is not continuous is

$$F(x) = \begin{cases} \frac{1}{2} & \text{if } x \in [0, \frac{1}{3}) \\ 1 & \text{if } x \in [\frac{1}{3}, \frac{2}{3}] \\ 0 & \text{if } x \in (\frac{2}{3}, 1] \end{cases}$$

$F(0) = \frac{1}{2}$, $F(\frac{1}{2}) = 1$, $F(1) = 0$

P.151

4. In the first graph, $F([0,2]) = [1,3] \supset [2,3]$

and $F([2,3]) = [0,3]$ the entire interval.

This is very much like the conditions in period 3

implies chaos.

Let A_1 be a closed interval in $[2,3]$ with

$F(A_1) = [2,3]$. Let $A_2 \subset [0,2]$ be a closed interval

with $F(A_2) = A_1$, there is such since $F([1,2]) \supset A_1$.

Since $F([2,3]) \supset A_2$ $\exists$ A_3 a closed interval in $[2,3]$

with $F(A_3) = A_2$.

Now $F^3(A_3) = F^2(A_2) = F(A_1) = [2,3] \supset A_3$.

So there is a fixed point x^* for F^3 in A_3. Now

$F(x^*) \in A_2 \subset [0,2]$. $x^* \neq 2$ since 2 has prime

period 4. Thus x^* is not fixed. Its orbit is in

$[2,3]$ for two iterations and then in $[0,2]$. Thus

it has prime period 3.

By Sarkovskii's ordering it has points of every

prime period.

P. 151 #6

$F([1,2]) = [4,7]$, $F([4,7]) = [1,5]$, $F([1,5]) = [3,7]$

$F([3,7]) = [1,6]$, $F([1,6]) = [2,7]$.

$[1,2] \cap F^5([1,2]) = 2$ a point of period 7 not 5.

Similarly.

$[2,3] \cap F^5([2,3]) = [2,3] \cap [3,7] = 3$

$[3,4] \cap F^5([3,4]) = [3,4] \cap [4,7] = 4$

$[5,6] \cap F^5([5,6]) = [5,6] \cap [1,5] = 5$

$[6,7] \cap F^5([6,7]) = [6,7] \cap [1,6] = 6$.

These intersections are all period 7 not period 5.

So the only interval left is $[4,5]$.

$F([4,5]) = [3,5]$ decreasing

$F([3,5]) = [3,6]$ decreasing

$F([3,6]) = [3,6]$ decreasing

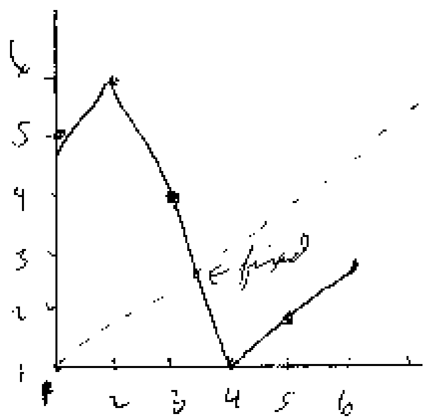
$F([3,6]) = [2,7]$ decreasing

$F([2,7]) = [1,7]$ decreasing

F^5 is decreasing on $[4,5]$ so it has only one fixed point which is the original fixed point. So no prime period 5.

P. 152

7. The integer values give a point of period 6.



$F([1, 3]) = [4, 6]$

$F([4, 6]) = [1, 3]$

Points in these two intervals go back and forth. No point in them can have odd period.

$F([3, 4]) = [1, 4]$ and it is decreasing. The points that get out must stay out, see the first argument.

For odd iterations of $[3, 4]$ the map is decreasing on what remains in $[3, 4]$. So it must have only one fixed point. But this is the original fixed point of prime period 1. Thus no point has prime period an odd number larger than 1.